GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

Final Exam

DATE: 11-Dec-00 COURSE: ECE 2025

NAME: ______________________________ STUDENT #: ________________
LAST, FIRST

Recitation Section: **Circle the day & time** when your Recitation Section meets:

L01:Tues-9:30am (Casinovi)	L02:Thur-9:30am (Bordelon)	L03:Tues-12:00pm (Casinovi)
L04:Thur-12:00pm (Bordelon)	L05:Tues-1:30pm (Bordelon)	L06:Thur-1:30pm (Cassinovi)
L07:Tues-3:00pm (Bordelon)	L08:Thur-3:00pm (Hayes)	L09:Tues-4:30pm (Fekri)
L10:Thur-4:30pm (Li)	L11:Tues-6:00pm (Fekri)	L12:Thur-6:00pm (Li)
L13:Mon -3:00pm (Williams)	L14:Weds-3:00pm (Bordelon)	L15:Mon -4:30pm (Verriest)
L16:Weds-4:30pm (Dansereau)	L17:Mon -6:00pm (Verriest)	L18:Weds-6:00pm (Dansereau)
L19:Weds-1:30pm (Bordelon)	L20:Mon -1:30pm (Williams)	

- Write your name on your exam, and **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted. However, one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes permitted. OK to write on both sides.
- Justify your reasoning clearly to receive any partial credit.
  Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
8	20	

Problem F-00-F.3.1:

(a) Define $x(t) = 2\cos(20\pi t + \pi/4) + 3\cos(20\pi(t - .025))$. Use phasor addition to express $x(t)$ in the form $x(t) = A\cos(\omega_0 t + \phi)$ by finding numerical values for ω_0, A, and ϕ.

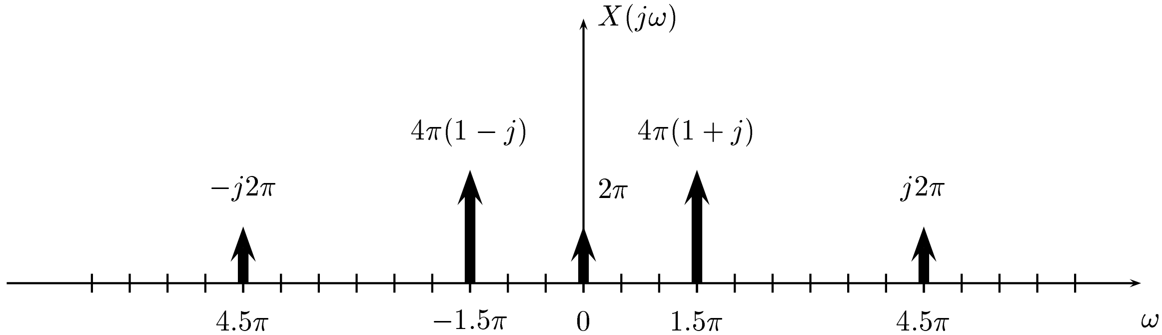
(b) Evaluate the following expression, $|(3 - 2j)e^{j(0.2t)}|^2 =$

(c) Evaluate the following integral, $\int_{t-1}^{t+1} \delta(\tau - 2) d\tau$

(d) Evaluate the following integral, $\int_{0}^{1/2} \sin(2\pi t) e^{-j4\pi t} dt$. **Note:** Your answer should be either a real number, or a complex number in polar form.

Problem F-00-F.3.2:

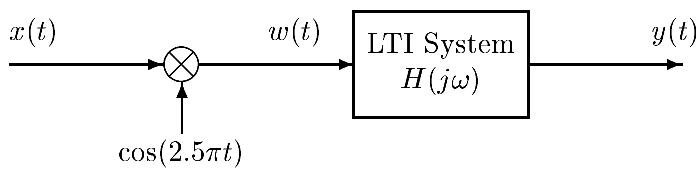
The Fourier transform of a signal $x(t)$ is shown in the following figure.



(a) Write an equation for $x(t)$ in terms of <u>cosine</u> functions.

(b) Suppose that $x(t)$ is modulated by a cosine of frequency $\omega_c = 2.5\pi$, and then lowpass filtered with a filter that has a frequency response

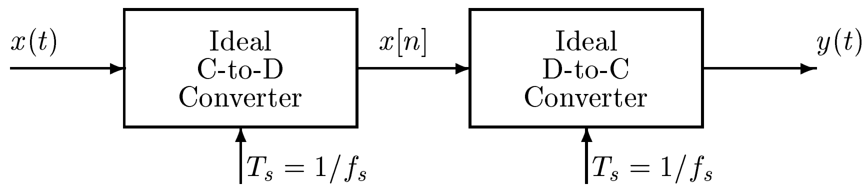
$$H(j\omega) = \begin{cases} 1 & |\omega| \le 2\pi \\ 0 & \text{otherwise} \end{cases}$$



Make a plot of the Fourier transform of $y(t)$.

Problem F-00-F.3.3:

Consider the ideal C-D and D-C converter shown in the figure below.



(a) If the Fourier transform of the input $X(j\omega)$ is equal to zero for $|\omega| > 1000\pi$, what is the largest value for the sampling period T_s that can be used with no aliasing?

$T_s =$

(b) If $x[n] = 3\cos(0.2\pi n + \pi/6)$, and the sampling frequency is $f_s = 1000$ Hz, find the signal $x(t)$ that would produce the given $x[n]$, and that would produce an output of the D-C converter such that $y(t) = x(t)$.

(c) If $x(t) = \text{Re}\left\{1 + \frac{1}{2}e^{-j(1000\pi t)} + \frac{1}{4}e^{j(3000\pi t)}\right\}$, what is the Nyquist rate? **Note: Your answer should be expressed in Hertz.**

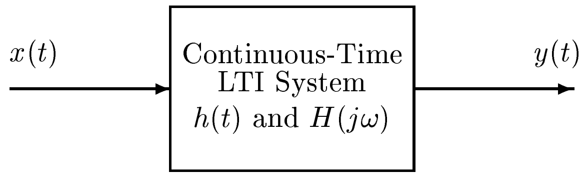
Problem F-00-F.3.4:

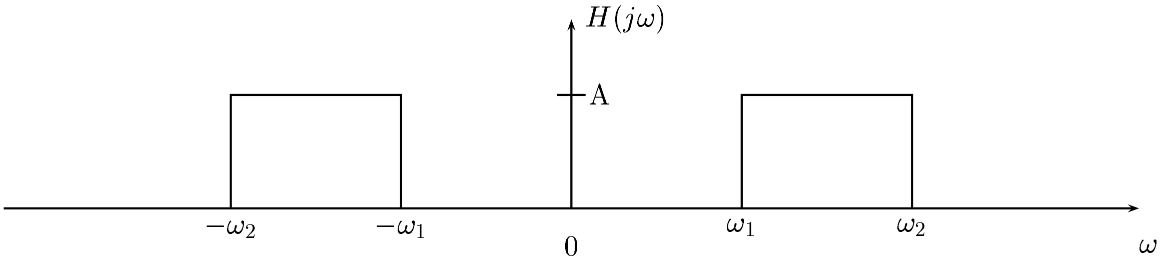
(a) Consider the signal $x(t) = \dfrac{10\sin(4\pi t)}{\pi t}$. Make a carefully labeled sketch of $x(t)$.

(b) Find the Fourier transform of $y(t) = x(t) * \delta(t-10)$, where $x(t)$ is the signal defined in part (a).

Problem F-00-F.3.5:

A periodic signal $x(t)$ that has a period $T_s = 10^{-3}$ seconds is input to a bandpass filter as illustrated below.





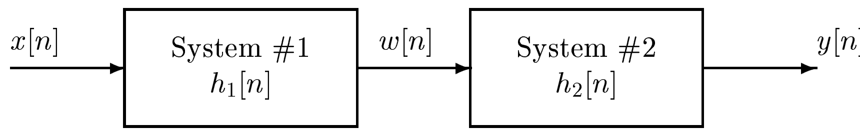
The Fourier series coefficients of $x(t)$ are

$$a_k = \begin{cases} 10 & k = 0 \\ \dfrac{2j}{k} & k \neq 0. \end{cases}$$

Find the frequencies of the bandpass filter, ω_1 and ω_2, and the gain A so that the output signal is

$$y(t) = \cos(2\pi(2000)t + \phi)$$

Problem F-00-F.3.6:
Shown in the figure below is a cascade of two linear time-invariant systems with impulse responses $h_1[n]$ and $h_2[n]$.



The impulse responses of the two systems are

$$h_1[n] = \delta[n] - \tfrac{1}{2}\delta[n-1] \qquad\qquad h_2[n] = (0.25)^n u[n]$$

(a) If $x[n] = \delta[n] + \delta[n-1]$, what is the output of the first system, $w[n]$?

(b) Determine the system function $H(z)$ for the cascade of the two systems. In other words, if $y[n] = x[n] * h[n]$, what is $H(z)$?

(c) Make a plot of the poles and zeros of $H(z)$ in the z-plane, where $H(z)$ is the system function found in part (b).

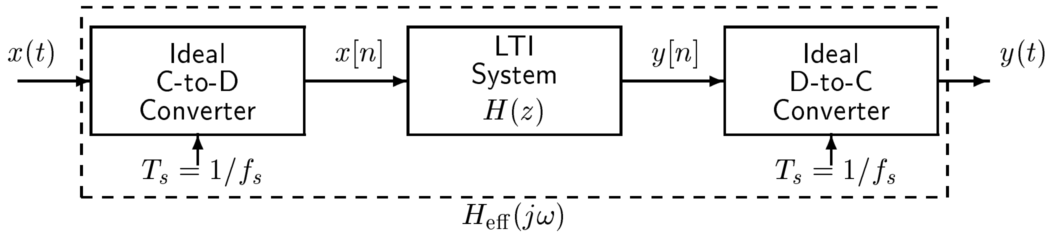
Problem F-00-F.3.7:

For each of the system functions listed on the left, find the corresponding impulse response or difference equation on the right, and enter the number in the answer box:

System Function	**Impulse Response or Difference Equation**
(a) $H(z) = 1 - z^{-2}$ ANS =	1. $y[n] = -0.2y[n-1] + x[n-2]$
	2. $h[n] = \delta[n] - \delta[n-2]$
	3. $h[n] = (0.2)^{n+2}u[n+2]$
(b) $H(z) = \dfrac{1}{1 - 0.2z^{-1}}$ ANS =	4. $h[n] = \delta[n] + 1.2u[n-1]$
	5. $y[n] = 0.2y[n-1] + x[n]$
(c) $H(z) = \dfrac{z^{-2}}{1 + 0.2z^{-1}}$ ANS =	6. $h[n] = (-0.2)^n u[n]$
	7. $y[n] = -y[n-1] + x[n] + 0.2x[n-1]$
	8. $y[n] = x[n] + 0.2x[n-1]$
(d) $H(z) = 1 + 0.2z^{-1}$ ANS =	
(e) $H(z) = \dfrac{1 + 0.2z^{-1}}{1 - z^{-1}}$ ANS =	

Problem F-00-F.3.8:

A discrete-time linear time-invariant filter with an impulse response $h[n]$ is used to filter a continuous-time signal $x(t)$ as illustrated below.



The sampling frequency of the C-D and D-C converters is

$$f_s = 10^4 \text{ Hz}$$

and the system function of the LTI system is

$$H(z) = \frac{1 + 2z^{-1}}{1 - 0.8z^{-1}}$$

The overall system (dashed box) behaves as a continuous-time LTI system, i.e., $Y(j\omega) = H_{\text{eff}}(j\omega)X(j\omega)$.

(a) Give an expression for the overall effective frequency response of the continuous-time LTI system, $H_{\text{eff}}(j\omega)$.

(b) Make a carefully labeled plot of the magnitude of the effective frequency response, $|H_{\text{eff}}(j\omega)|$. Make sure that you label the maximum and minimum values of $|H_{\text{eff}}(j\omega)|$ and the frequencies at which these occur.

Problem Fall-00-F.1:

(a) Define $x(t) = 4\cos(100\pi t - \pi/4) + 5\cos(100\pi(t - .005))$. Use phasor addition to express $x(t)$ in the form $x(t) = A\cos(\omega_0 t + \phi)$ by finding numerical values for ω_0, A, and ϕ.

$$x(t) = 4\cos(100\pi t - \pi/4) + 5\cos(100\pi t - \pi/2)$$

$$= \mathrm{Re}\left\{4e^{-j\pi/4}e^{j100\pi t} + 5e^{-j\pi/2}e^{j100\pi t}\right\} = \mathrm{Re}\left\{A e^{j100\pi t}\right\}$$

where

$$A = 4e^{-j\pi/4} + 5e^{-j\pi/2} = 2.828 - j7.828 = 8.324e^{-j0.39\pi}$$

so

$$x(t) = 8.324\cos(100\pi t - 0.39\pi)$$

(b) Evaluate the following expression, $|(2-5j)e^{-j(100\pi t)}|^2 =$

$$|(2-5j)e^{-j(100\pi t)}|^2 = \underbrace{|2-5j|^2}_{29} \cdot \underbrace{|e^{-j(100\pi t)}|^2}_{1} = 29$$

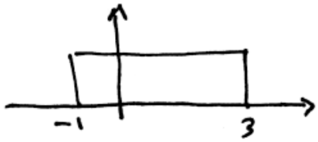
(c) Evaluate the following integral, $\int_{t-2}^{t+2} \delta(\tau - 1)d\tau$

The integral is equal to one when the limits include the point $\tau = 1$. Thus, it is one when

$$t+2 \leq 1 \quad \text{and} \quad t-2 \geq 1$$

or

$$-1 \leq t \leq 3$$



Therefore,

$$\int_{t-2}^{t+2} \delta(\tau-1)d\tau = \begin{cases} 1 & -1 \leq t \leq 3 \\ 0 & \text{else} \end{cases}$$

(d) Evaluate the following integral, $\int_0^{1/2} 5\sin(2\pi t)e^{-j4\pi t}dt$. **Note:** Your answer should be either a real number, or a complex number in polar form.

$$\int_0^{1/2} 5\sin(2\pi t)e^{-j4\pi t}dt = \frac{5}{2j}\int_0^{1/2}\left[e^{j2\pi t} - e^{-j2\pi t}\right]e^{-j4\pi t}dt$$

$$= \frac{5}{2j}\int_0^{1/2} e^{-j2\pi t}dt - \frac{5}{2j}\int_0^{1/2} e^{-j6\pi t}dt$$

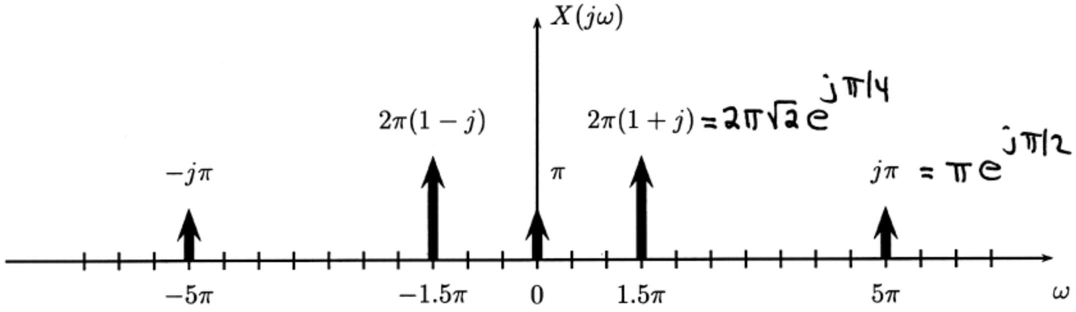
$$= \frac{5}{2j}\frac{1}{-j2\pi}e^{-j2\pi t}\Big|_0^{1/2} - \frac{5}{2j}\frac{1}{-j6\pi}e^{-j6\pi t}\Big|_0^{1/2}$$

$$= \frac{5}{4\pi}(-1-1) - \frac{5}{12\pi}(-1-1)$$

$$= -\frac{5}{2\pi} + \frac{5}{6\pi} = -\frac{5}{3\pi}$$

Problem Fall-00-F.2:

The Fourier transform of a signal $x(t)$ is shown in the following figure.



(a) Write an equation for $x(t)$ in terms of <u>cosine</u> functions.

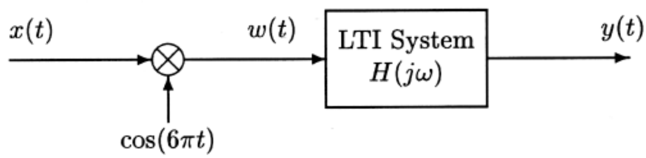
Recall (Table 12.1): $A\cos(\omega_0 t+\phi) \longleftrightarrow \pi A e^{j\phi}\delta(\omega-\omega_0)+\pi A e^{-j\phi}\delta(\omega+\omega_0)$

Therefore, we have

$$x(t) = \frac{1}{2} + 2\sqrt{2}\cos(1.5\pi t + \pi/4) + \cos(5\pi t + \pi/2)$$

(b) Suppose that $x(t)$ is modulated by a cosine of frequency $\omega_c = 6\pi$, and then lowpass filtered with a filter that has a frequency response

$$H(j\omega) = \begin{cases} 1 & |\omega| \le 2\pi \\ 0 & \text{otherwise} \end{cases}$$

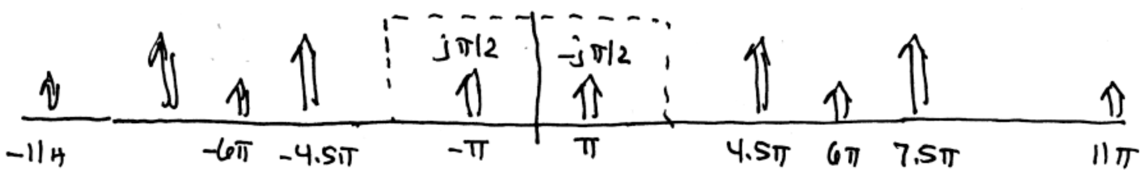


Make a plot of the Fourier transform of $y(t)$.

Recall the modulation property

$$x(t)\cos(\omega_c t) \longleftrightarrow \frac{1}{2}X(j(\omega-\omega_c)) + \frac{1}{2}X(j(\omega+\omega_c))$$

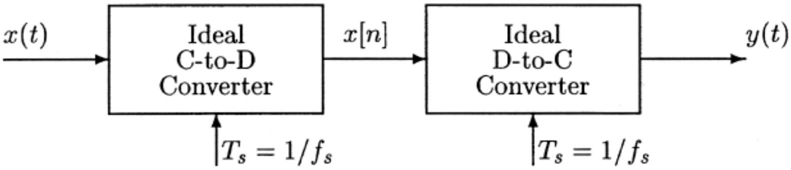
Therefore, the F.T. of $x(t)$ is shifted <u>up</u> in frequency and <u>down</u> in frequency by $\omega_c = 6\pi$.



The lowpass filter removes everything except the impulses at ±π.

Problem Fall-00-F.3:

Consider the ideal C-D and D-C converter shown in the figure below.



(a) If the Fourier transform of the input $X(j\omega)$ is equal to zero for $|\omega| > 3000\pi$, what is the largest value for the sampling period T_s that can be used with no aliasing?

Nyquist rate $= 2 \cdot (3000\pi) = 2\pi / T_s$

$T_s =$ ☐

$\Rightarrow$ maximum $T_s = 1/3000$

(b) If $x[n] = 4\cos(0.4\pi n - \pi/4)$, and the sampling frequency is $f_s = 4000$ Hz, find the signal $x(t)$ that would produce the given $x[n]$, and that would produce an output of the D-C converter such that $y(t) = x(t)$.

$$x[n] = x(nT_s) = 4\cos\left(0.4\pi \cdot \frac{1}{T_s} nT_s - \pi/4\right)$$

$$= 4\cos(0.4\pi f_s nT_s - \pi/4)$$

$$\Rightarrow x(t) = 4\cos(0.4\pi f_s t - \pi/4) = 4\cos(1600\pi t - \pi/4)$$

(c) If $x(t) = \text{Re}\left\{4 + \frac{1}{2}e^{-j(250\pi t)} + \frac{1}{4}e^{j(750\pi t)}\right\}$, what is the Nyquist rate? **Note: Your answer should be expressed in Hertz.**

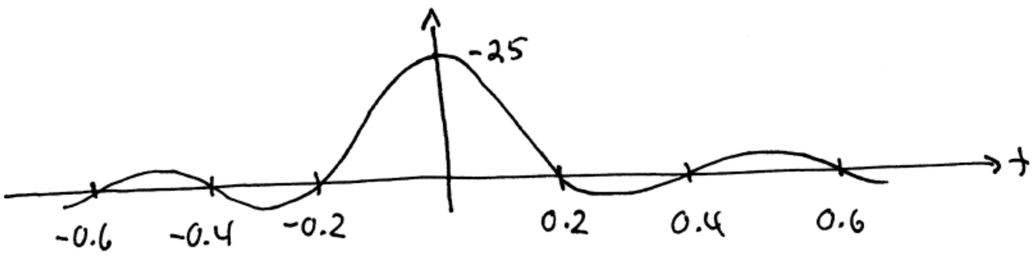
The maximum frequency in $x(t)$ is $\omega = 750\pi$. Therefore, the Nyquist rate is

$$\omega_s = 2 \cdot (750\pi)$$

$$\Rightarrow f_s = 750 \text{ Hz}$$

Problem Fall-00-F.4:

(a) Consider the signal $x(t) = \dfrac{5\sin(5\pi t)}{\pi t}$. Make a carefully labeled sketch of $x(t)$.



(b) Find the Fourier transform of $y(t) = x(t) * \delta(t-8)$, where $x(t)$ is the signal defined in part (a).

Use the F.T. pair

$$\frac{\sin(\omega_c t)}{\pi t} \longleftrightarrow u(\omega+\omega_c) - u(\omega-\omega_c)$$

and the delay property

$$x(t-t_0) \longleftrightarrow e^{-j\omega t_0} X(j\omega)$$

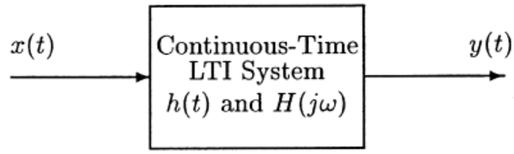
to write

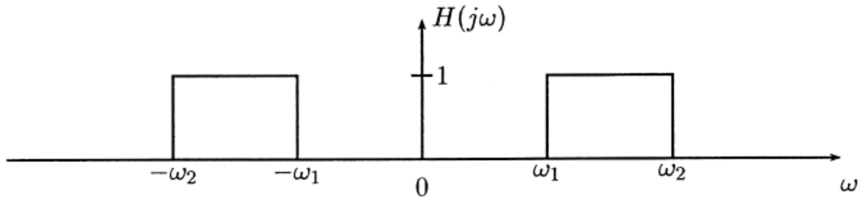
$$y(t) = x(t) * \delta(t-8) = x(t-8)$$

$$\Rightarrow \quad Y(j\omega) = 5\left[u(\omega+5\pi) - u(\omega-5\pi)\right] e^{-j8\omega}$$

Problem Fall-00-F.5:

A periodic signal $x(t)$ that has a period $T_s = 10^{-3}$ seconds is input to a bandpass filter as illustrated below.





The Fourier series coefficients of $x(t)$ are

$$a_k = \begin{cases} 10 & k = 0 \\ \dfrac{2j}{k} & k \neq 0. \end{cases}$$

(a) Find frequencies for the bandpass filter, ω_1 and ω_2, so that the output signal has the form

$$y(t) = A\cos(2\pi(4000)t + \phi)$$

With $T_s = 10^{-3}$, the fundamental frequency of $x(t)$ is $f = 1000$ Hz. Therefore, to pass a cosine of frequency 4000 Hz, we need $3000 < f_1 < 4000$ and $4000 < f_2 < 5000$. So, one choice is

$$\omega_1 = 2\pi(3500) = 7000\pi \quad \& \quad \omega_2 = 2\pi(4500) = 9000\pi$$

(b) Given that the gain of the bandpass filter is equal to one, find A and ϕ for the signal $y(t)$ in part (a).

Since we are passing the fourth harmonic in $x(t)$, and $a_4 = 2j/4 = 0.5e^{j\pi/2}$, then

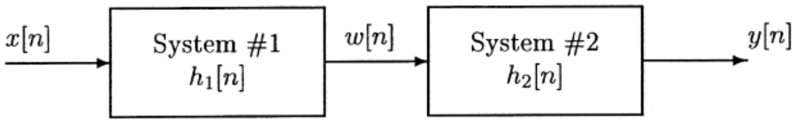
$$y(t) = a_4 e^{j8000\pi t} + a_{-4} e^{-j8000\pi t}$$

$$= \tfrac{1}{2} e^{j8000\pi t} e^{j\pi/2} + \tfrac{1}{2} e^{-j8000\pi t} e^{-j\pi/2} = \cos(8000\pi t + \pi/2)$$

So $A = 1$ and $\phi = \pi/2$.

Problem Fall-00-F.6:

Shown in the figure below is a cascade of two linear time-invariant systems with impulse responses $h_1[n]$ and $h_2[n]$.



The impulse responses of the two systems are

$$h_1[n] = \delta[n] - \tfrac{1}{4}\delta[n-1] \qquad\qquad h_2[n] = (0.75)^n u[n]$$

(a) If $x[n] = 2\delta[n] - \delta[n-1]$, what is the output of the first system, $w[n]$?

$$w[n] = x[n] * h_1[n] = \{2\delta[n] - \delta[n-1]\} * h_1[n]$$

$$= 2h_1[n] - h_1[n-1] = 2\delta[n] - \tfrac{1}{2}\delta[n-1] - \left[\delta[n-1] - \tfrac{1}{4}\delta[n-2]\right]$$

$$= 2\delta[n] - \tfrac{3}{2}\delta[n-1] + \tfrac{1}{4}\delta[n-2]$$

(b) Determine the system function $H(z)$ for the cascade of the two systems. In other words, if $y[n] = x[n] * h[n]$, what is $H(z)$?

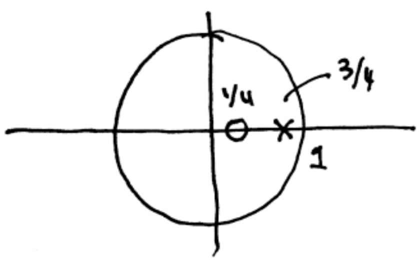
$$H_1(z) = 1 - \tfrac{1}{4}z^{-1} \quad \text{and} \quad H_2(z) = \frac{1}{1 - 0.75z^{-1}}$$

so

$$H(z) = H_1(z)\,H_2(z) = \frac{1 - \frac{1}{4}z^{-1}}{1 - 0.75z^{-1}}$$

(c) Make a plot of the poles and zeros of $H(z)$ in the z-plane, where $H(z)$ is the system function found in part (b).

$$H(z) = \frac{1 - \frac{1}{4}z^{-1}}{1 - 0.75z^{-1}} = \frac{z - 1/4}{z - 3/4} \implies \begin{array}{l}\text{zero at } z = 1/4 \\ \text{pole at } z = 3/4\end{array}$$



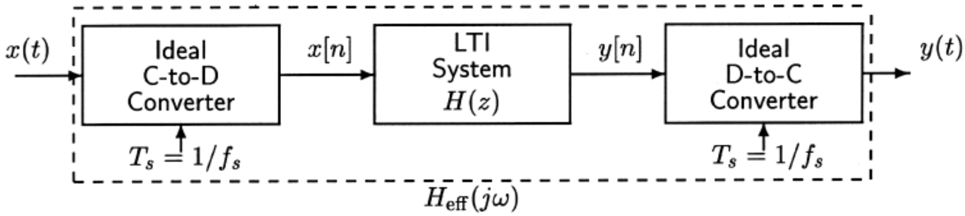
Problem Fall-00-F.7:

For each of the system functions listed on the left, find the corresponding impulse response or difference equation on the right, and enter the number in the answer box:

System Function	**Impulse Response or Difference Equation**
(a) $H(z) = 1 - z^{-2}$ ANS = 5	1. $h[n] = (-0.2)^n u[n]$ $\quad \frac{1}{1+0.2z^{-1}}$
	2. $y[n] = -y[n-1] + x[n] + 0.2x[n-1]$ $\quad \frac{1+0.2z^{-1}}{1+z^{-1}}$
	3. $y[n] = x[n] + 0.2x[n-1]$ $\quad 1+0.2z^{-1}$
(b) $H(z) = \frac{1}{1-0.2z^{-1}}$ ANS = 8	4. $y[n] = -0.2y[n-1] + x[n-2]$ $\quad \frac{z^{-2}}{1+0.2z^{-1}}$
	5. $h[n] = \delta[n] - \delta[n-2]$ $\quad 1 - z^{-2}$
(c) $H(z) = \frac{z^{-2}}{1+0.2z^{-1}}$ ANS = 4	6. $h[n] = (0.2)^{n+2} u[n+2]$ $\quad \frac{z^{2}}{1-0.2z^{-1}}$
	7. $h[n] = \delta[n] + 1.2u[n-1]$ $\quad 1+\frac{1.2}{1-z^{-1}} = \frac{1+0.2z^{-1}}{1-z^{-1}}$
	8. $y[n] = 0.2y[n-1] + x[n]$ $\quad \frac{1}{1-0.2z^{-1}}$
(d) $H(z) = 1 + 0.2z^{-1}$ ANS = 3	
(e) $H(z) = \frac{1+0.2z^{-1}}{1-z^{-1}}$ ANS = 7	

Problem Fall-00-F.8:

A discrete-time linear time-invariant filter with an impulse response $h[n]$ is used to filter a continuous-time signal $x(t)$ as illustrated below.



The sampling frequency of the C-D and D-C converters is

$$f_s = 10^2 \text{ Hz}$$

and the system function of the LTI system is

$$\frac{1 + 0.5z^{-1}}{1 - 0.5z^{-1}}$$

The overall system (dashed box) behaves as a continuous-time LTI system, i.e., $Y(j\omega) = H_{\text{eff}}(j\omega)X(j\omega)$.

(a) Give an expression for the overall effective frequency response of the continuous-time LTI system, $H_{\text{eff}}(j\omega)$.

$$H(e^{j\hat{\omega}}) = \frac{1+0.5e^{-j\hat{\omega}}}{1-0.5e^{-j\hat{\omega}}}$$

Recall that

$$H_{eff}(j\omega) = \begin{cases} H(e^{j\omega T_s}) & |\omega| < \omega_s/2 \\ 0 & \text{else} \end{cases}$$

Therefore,

$$H_{eff}(j\omega) = \frac{1+0.5e^{-j\omega/100}}{1-0.5e^{-j\omega/100}} \quad \text{for} \quad |\omega| < 200\pi$$

(b) Make a carefully labeled plot of the magnitude of the effective frequency response, $|H_{\text{eff}}(j\omega)|$. Make sure that you label the maximum and minimum values of $|H_{\text{eff}}(j\omega)|$ and the frequencies at which these occur.