GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

Final Exam

DATE: Dec-00 COURSE: ECE 2025

NAME: ______________________________ STUDENT #: ________________

LAST, FIRST

Recitation Section: **Circle the day & time** when your Recitation Section meets:

- L01:Tues-9:30am (Casinovi)
- L02:Thur-9:30am (Bordelon)
- L03:Tues-12:00pm (Casinovi)
- L04:Thur-12:00pm (Bordelon)
- L05:Tues-1:30pm (Bordelon)
- L06:Thur-1:30pm (Cassinovi)
- L07:Tues-3:00pm (Bordelon)
- L08:Thur-3:00pm (Hayes)
- L09:Tues-4:30pm (Fekri)
- L10:Thur-4:30pm (Li)
- L11:Tues-6:00pm (Fekri)
- L12:Thur-6:00pm (Li)
- L13:Mon -3:00pm (Williams)
- L14:Weds-3:00pm (Bordelon)
- L15:Mon -4:30pm (Verriest)
- L16:Weds-4:30pm (Dansereau)
- L17:Mon -6:00pm (Verriest)
- L18:Weds-6:00pm (Dansereau)
- L19:Weds-1:30pm (Bordelon)
- L20:Mon -1:30pm (Williams)

- Write your name on your exam, and **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted. However, one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes permitted. OK to write on both sides.
- Justify your reasoning clearly to receive any partial credit.
  Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
8	20	

Problem F-00-F.1:

For each of the following problems, **SIMPLIFY** your answer as much as possible.

(a) Define $x(t) = 2\cos(20\pi t + \pi/4) + 3\cos(20\pi(t - .025))$. Use phasor addition to express $x(t)$ in the form $x(t) = A\cos(\omega_0 t + \phi)$ by finding numerical values for ω_0, A, and ϕ.

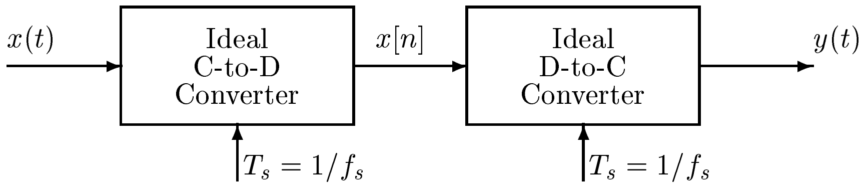
(b) Evaluate the following expression, $|1 + e^{j\pi/3}|^2 =$

(c) Evaluate the following integral, $\int_{-\infty}^{\infty} \delta(t-7)\sin(t)e^{-j\pi t}dt$

(d) Evaluate the following integral, $\int_{-\infty}^{\infty} e^{-2\pi t}u(t)e^{-j2\pi t}dt.$

Problem F-00-F.2:

Consider the ideal C-D and D-C converter shown in the figure below.



(a) If the Fourier transform of the input $X(j\omega)$ is equal to zero for $|\omega| > 1000\pi$, what is the largest value for the sampling period T_s that can be used with no aliasing?

$\boxed{T_s = \quad\quad}$

(b) If $x[n] = 3\cos(0.2\pi n + \pi/6)$, and the sampling frequency is $f_s = 1000$ Hz, find the signal $x(t)$ that would produce the given $x[n]$, and that would produce an output of the D-C converter such that $y(t) = x(t)$.

(c) If $x(t) = \text{Re}\left\{1 + \frac{1}{2}e^{-j(1000\pi t)} + \frac{1}{4}e^{j(3000\pi t)}\right\}$, what is the Nyquist rate? **Note: Your answer should be expressed in Hertz.**

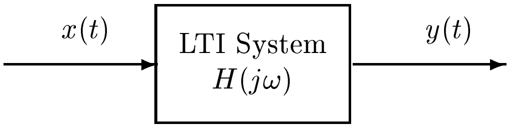
Problem F-00-F.3:

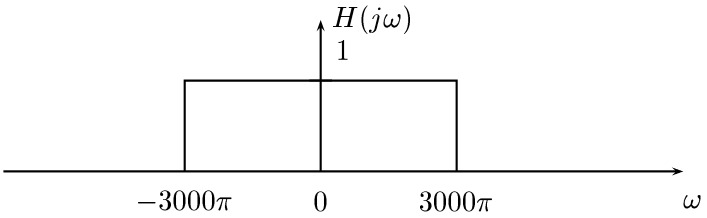
(a) Make a carefully labeled sketch of the frequency response $X(e^{j\hat{\omega}}) = \dfrac{6\sin(5\hat{\omega}/2)}{\sin(\hat{\omega}/2)}$.

(b) Make a pole-zero plot in the z-plane of the system function $H(z) = \dfrac{1+z^{-2}}{1+z^{-1}}$. **Note:** Make sure you include **ALL** poles and zeros.

Problem F-00-F.4:

(a) A periodic signal $x(t)$ with a period $T_s = 10^{-3}$ seconds is input to a lowpass filter that has a frequency response $H(j\omega)$ as illustrated below.



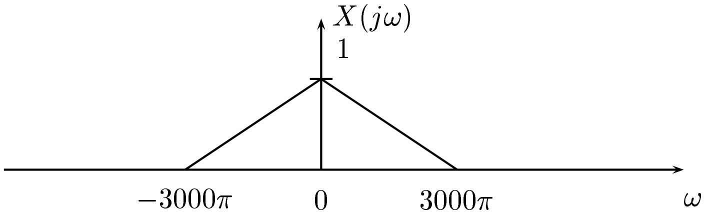


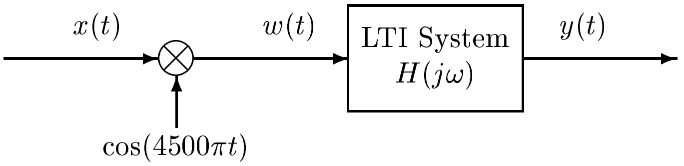
The Fourier series coefficients of $x(t)$ are

$$a_k = \begin{cases} 10 & k = 0 \\ \dfrac{2j}{k} & k \neq 0. \end{cases}$$

Find the output signal, $y(t)$, making sure to express your answer in terms of cosines.

(b) Let $x(t)$ be a bandlimited signal with a Fourier transform $X(j\omega)$ as illustrated in the figure below.





Suppose that $x(t)$ is modulated by a cosine of frequency $\omega_c = 4500\pi$, and then lowpass filtered with a filter that has a frequency response $H(j\omega)$ as defined in part (a), i.e.,

$$H(j\omega) = \begin{cases} 1 & |\omega| \leq 3000\pi \\ 0 & \text{otherwise} \end{cases}$$

Make a plot of the Fourier transform of $y(t)$.

Problem F-00-F.5:

In each of the following problems, find the Fourier transform, or inverse Fourier transform. Give your answer as a $\boxed{\text{simple}}$ formula or a plot. $\boxed{\textbf{Explain}}$ each answer by stating which property and transform pair you used.

(a) Find $X(j\omega)$ when $x(t) = 3\delta(t-1) + e^{-2t}u(t-1)$.

(b) Find $h(t)$ when $H(j\omega) = \dfrac{j\omega}{4 + 3j\omega}$.

(c) Find $V(j\omega)$ when $v(t) = \begin{cases} 1 & 3 \le t < 7 \\ 0 & \text{otherwise} \end{cases}$.

Problem F-00-F.6:

For each of the systems defined on the left, find the corresponding frequency response on the right. Pick the correct frequency response and enter the number in the answer box:

Impulse Response or Difference Equation

(a) $h[n] = (-\frac{1}{2})^n u[n]$

ANS =

(b) $h[n] = \delta[n] - (\frac{1}{2})^n u[n-1]$

ANS =

(c) $h[n] = \delta[n] + \delta[n-2]$

ANS =

(d) $y[n] = \frac{1}{2} y[n-1] + x[n]$

ANS =

(e) $y[n] = \Big((-\frac{1}{2})^n u[n]\Big) * \Big(\delta[n] + \delta[n-1]\Big)$

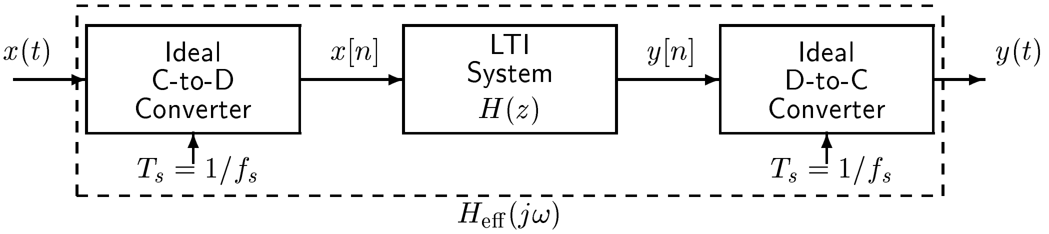
ANS =

Frequency Response

1. $H(e^{j\hat{\omega}}) = \dfrac{1 + e^{-j\hat{\omega}}}{1 + \frac{1}{2}e^{-j\hat{\omega}}}$
2. $H(e^{j\hat{\omega}}) = 1 + \frac{1}{2}e^{-j\hat{\omega}}$
3. $H(e^{j\hat{\omega}}) = \dfrac{1 - e^{j\hat{\omega}}}{1 - \frac{1}{2}e^{-j\hat{\omega}}}$
4. $H(e^{j\hat{\omega}}) = \dfrac{1}{1 - \frac{1}{2}e^{-j\hat{\omega}}}$
5. $H(e^{j\hat{\omega}}) = 2e^{-j\hat{\omega}}\cos(\hat{\omega})$
6. $H(e^{j\hat{\omega}}) = 1 + e^{-j\hat{\omega}}$
7. $H(e^{j\hat{\omega}}) = \dfrac{1}{1 + \frac{1}{2}e^{-j\hat{\omega}}}$
8. $H(e^{j\hat{\omega}}) = \dfrac{1 + \frac{1}{2}e^{-j\hat{\omega}}}{1 + e^{-j\hat{\omega}}}$

Problem F-00-F.7:

A discrete-time linear time-invariant filter with an impulse response $h[n]$ is used to filter a continuous-time signal $x(t)$ as illustrated below.



The sampling frequency of the C-D and D-C converters is

$$f_s = 10^4 \text{ Hz}$$

and the system function of the LTI system is

$$H(z) = 1 + z^{-2}$$

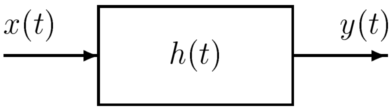
The overall system (dashed box) behaves as a continuous-time LTI system, i.e., $Y(j\omega) = H_{\text{eff}}(j\omega)X(j\omega)$.

(a) Give an expression for the overall effective frequency response of the continuous-time LTI system, $H_{\text{eff}}(j\omega)$.

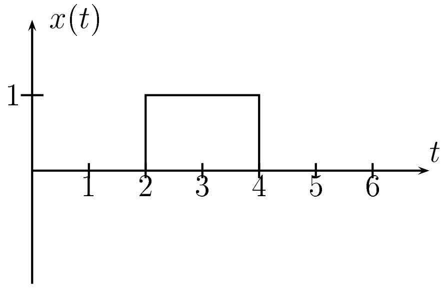
(b) Make a carefully labeled plot of the magnitude of the effective frequency response, $|H_{\text{eff}}(j\omega)|$. Make sure that you label the maximum and minimum values of $|H_{\text{eff}}(j\omega)|$ and the frequencies at which these occur.

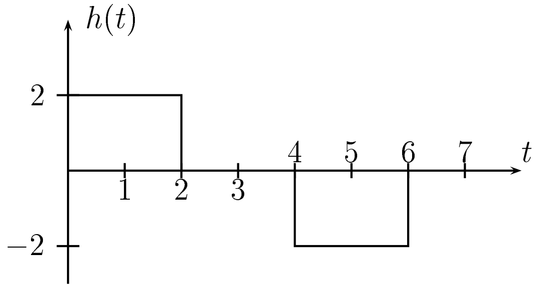
Problem F-00-F.8:

Consider the linear time-invariant system shown below.



The input $x(t)$ and the impulse response $h(t)$ are shown in the figures below.





(a) Draw $x(1-t)$.

(b) Make a carefully labeled plot of the filter output, $y(t)$.

Problem F-00-F.1:

For each of the following problems, **SIMPLIFY** your answer as much as possible.

(a) Define $x(t) = 2\cos(20\pi t + \pi/4) + 3\cos(20\pi(t - .025))$. Use phasor addition to express $x(t)$ in the form $x(t) = A\cos(\omega_0 t + \phi)$ by finding numerical values for ω_0, A, and ϕ.

$$x(t) = 2\cos(20\pi t + \pi/4) + 3\cos(20\pi t - \pi/2)$$

$$= \mathrm{Re}\left\{2e^{j\pi/4} \cdot e^{j20\pi t} + 3e^{-j\pi/2} \cdot e^{j20\pi t}\right\}$$

$$= \mathrm{Re}\left\{Ae^{j20\pi t}\right\} \quad \text{where} \quad A = 2e^{j\pi/4} + 3e^{-j\pi/2}$$

Expressing A in polar form we have

$$A = 2e^{j\pi/4} + 3e^{-j\pi/2} = 2.1248\,e^{-j0.2682\pi}$$

so

$$x(t) = \mathrm{Re}\left\{Ae^{j20\pi t}\right\} = 2.1248\cos(20\pi t - 0.2682\pi)$$

(b) Evaluate the following expression, $|1 + e^{j\pi/3}|^2 =$

$$|1 + e^{j\pi/3}|^2 = (1 + e^{j\pi/3})(1 + e^{-j\pi/3})$$

$$= 1 + e^{j\pi/3} + e^{-j\pi/3} + 1$$

$$= 2 + 2\cos(\pi/3) = 2 + 2(1/2) = 3$$

(c) Evaluate the following integral, $\int_{-\infty}^{\infty} \delta(t-7)\sin(t)e^{-j\pi t/2}dt$

$$\int_{-\infty}^{\infty} \delta(t-7)\sin(t)\, e^{-j\pi t/2}\, dt = \sin(7)\, e^{-j\pi 7/2}$$

$$= \sin(7)\, e^{+j\pi/2} = +j\sin(7)$$

(d) Evaluate the following integral, $\int_{-\infty}^{\infty} e^{-2\pi t}u(t)e^{-j2\pi t}dt$.

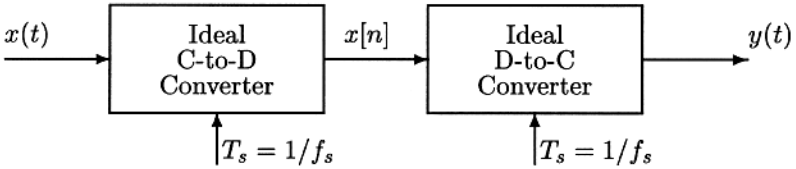
$$\int_{-\infty}^{\infty} e^{-2\pi t} u(t)\, e^{-j2\pi t}\, dt = \int_{0}^{\infty} e^{-2\pi(1+j)t}\, dt$$

$$= \frac{1}{-2\pi(1+j)}\, e^{-2\pi(1+j)t}\Big|_{0}^{\infty}$$

$$= \frac{1}{2\pi(1+j)} = \frac{1-j}{4\pi} = \frac{\sqrt{2}}{4\pi}\, e^{-j\pi/4}$$

Problem F-00-F.2:

Consider the ideal C-D and D-C converter shown in the figure below.



(a) If the Fourier transform of the input $X(j\omega)$ is equal to zero for $|\omega| > 1000\pi$, what is the largest value for the sampling period T_s that can be used with no aliasing?

The highest frequency is $\omega = 1000\pi = 2\pi(500)$

$\boxed{T_s = 1/1000}$ so the Nyquist rate is $f_s = 2\cdot(500) = 1000$

(b) If $x[n] = 3\cos(0.2\pi n + \pi/6)$, and the sampling frequency is $f_s = 1000$ Hz, find the signal $x(t)$ that would produce the given $x[n]$, and that would produce an output of the D-C converter such that $y(t) = x(t)$.

$$x[n] = 3\cos\left(0.2\pi\cdot 1000\cdot\underbrace{\frac{n}{1000}}_{nT_s} + \pi/6\right)$$

So

$$x(t) = 3\cos\left(200\pi t + \pi/6\right)$$

(c) If $x(t) = \text{Re}\left\{1 + \frac{1}{2}e^{-j(1000\pi t)} + \frac{1}{4}e^{j(3000\pi t)}\right\}$, what is the Nyquist rate? **Note: Your answer should be expressed in Hertz.**

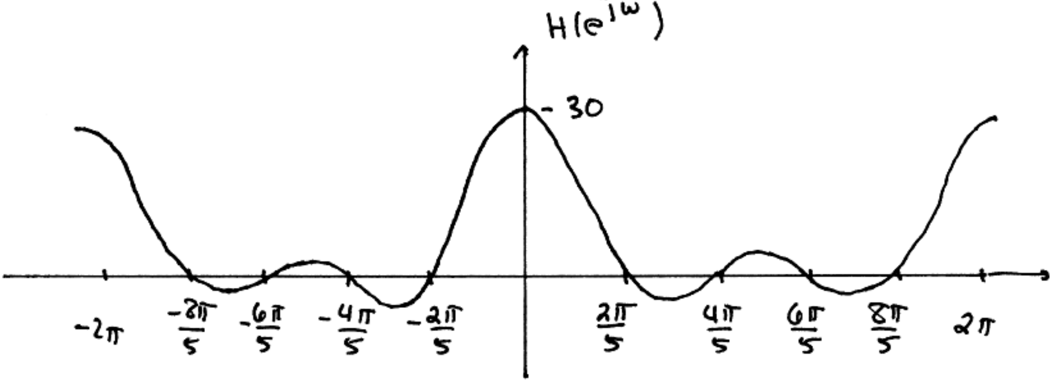
The highest frequency in $x(t)$ is $\omega = 3000\pi = 2\pi(1500)$

so the Nyquist rate is

$$f_s = 3000$$

Problem F-00-F.3:

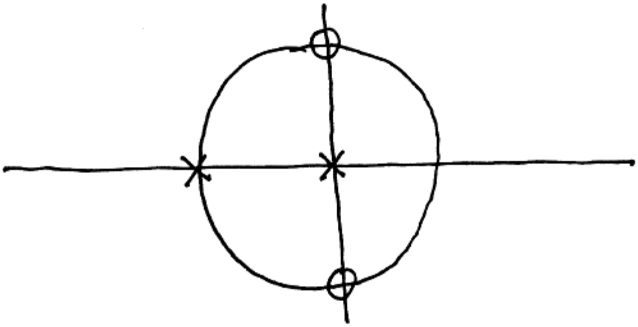
(a) Make a carefully labeled sketch of the frequency response $H(e^{j\hat{\omega}}) = \dfrac{6\sin(5\hat{\omega}/2)}{\sin(\hat{\omega}/2)}$.



(b) Make a pole-zero plot in the z-plane of the system function $H(z) = \dfrac{1+z^{-2}}{1+z^{-1}}$. **Note:** Make sure you include **ALL** poles and zeros.

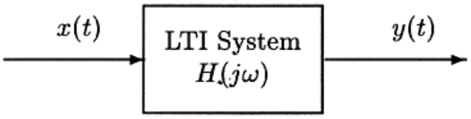
$$H(z) = \frac{1+z^{-2}}{1+z^{-1}} = \frac{z^2+1}{z(z+1)} \Rightarrow$$

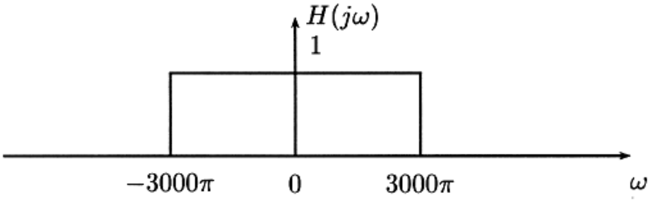
zeros at $z = \pm j$
poles at $z = 0, -1$



Problem F-00-F.4:

(a) A periodic signal $x(t)$ with a period $T_s = 10^{-3}$ seconds is input to a lowpass filter that has a frequency response $H(j\omega)$ as illustrated below.





The Fourier series coefficients of $x(t)$ are

$$a_k = \begin{cases} 10 & k = 0 \\ \dfrac{2j}{k} & k \neq 0. \end{cases}$$

Find the output signal, $y(t)$, making sure to express your answer in terms of cosines.

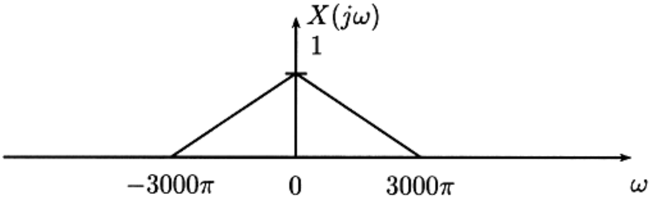
The fundamental frequency of $x(t) = 1000$ Hz. Since the lowpass filter blocks any harmonics above $\omega = 3000\pi$ or $f = 1500$, only the $k=0$ and $k=\pm 1$ terms are passed. Thus

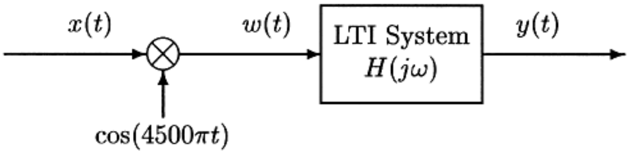
$$y(t) = a_0 + a_1 e^{j2\pi(1000)t} + a_{-1} e^{-j2\pi(1000)t}$$

$$= 10 + 2e^{j\pi/2} \cdot e^{j2\pi(1000)t} + 2e^{-j\pi/2} \cdot e^{j2\pi(1000)t}$$

$$= 10 + 4\cos(2000\pi t + \pi/2)$$

(b) Let $x(t)$ be a bandlimited signal with a Fourier transform $X(j\omega)$ as illustrated in the figure below.



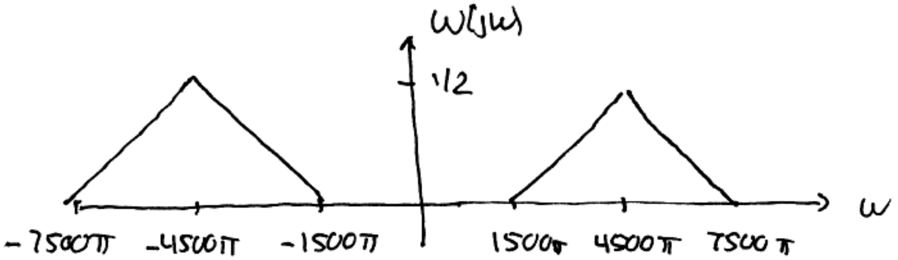


Suppose that $x(t)$ is modulated by a cosine of frequency $\omega_c = 4500\pi$, and then lowpass filtered with a filter that has a frequency response $H(j\omega)$ as defined in part (a), i.e.,

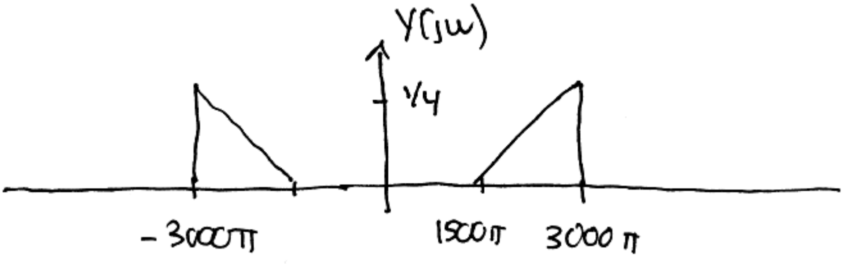
$$H(j\omega) = \begin{cases} 1 & |\omega| \leq 3000\pi \\ 0 & \text{otherwise} \end{cases}$$

Make a plot of the Fourier transform of $y(t)$.

Modulating by $w_c = 4500\pi$ shifts the spectrum of x(t) up and down by w_c. So W(jw) looks like



The lowpass filter removes all frequencies above $w = 3000\pi$. Thus, for Y(jw) we have



Problem F-00-F.5:

In each of the following problems, find the Fourier transform, or inverse Fourier transform. Give your answer as a $\boxed{\text{simple}}$ formula or a plot. $\boxed{\textbf{Explain}}$ each answer by stating which property and transform pair you used.

(a) Find $X(j\omega)$ when $x(t) = 3\delta(t-1) + e^{-2t}u(t-1)$.

$$\delta(t) \longleftrightarrow 1 \quad \text{and} \quad e^{-at}u(t) \longleftrightarrow \frac{1}{1+aj\omega}$$

Using linearity and the delay property we have:

$$x(t) = 3\delta(t-1) + e^{-2}e^{-2(t-1)}u(t-1)$$

$$X(j\omega) = 3e^{-j\omega} + e^{-2}e^{-j\omega}\frac{1}{2+j\omega},$$

(b) Find $h(t)$ when $H(j\omega) = \dfrac{j\omega}{4+3j\omega}$.

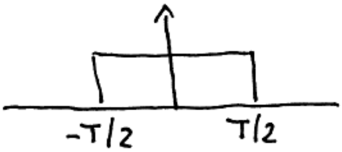
$$H(j\omega) = \frac{j\omega}{3} \cdot \frac{1}{\frac{4}{3}+j\omega}$$

(use the derivative prop)

$$h(t) = \frac{d}{dt}\left\{\frac{1}{3}e^{-\frac{4}{3}t}u(t)\right\} = -\frac{4}{9}e^{-\frac{4}{3}t}u(t) + \frac{1}{3}\overset{1}{\cancel{e}}\,\delta(t)$$

(c) Find $V(j\omega)$ when $v(t) = \begin{cases} 1 & 3 \le t < 7 \\ 0 & \text{otherwise} \end{cases}$.

Recall



$$\longleftrightarrow \quad \frac{\sin(\omega T/2)}{\omega/2}$$

Using the delay property, and noting that T=4 we have

$$V(j\omega) = e^{-j5\omega} \cdot \frac{\sin(2\omega)}{\omega/2}$$

Problem F-00-F.6:

For each of the systems defined on the left, find the corresponding frequency response on the right. Pick the correct frequency response and enter the number in the answer box:

Impulse Response or Difference Equation

(a) $h[n] = (-\frac{1}{2})^n u[n]$

ANS =

7

(b) $h[n] = \delta[n] - (\frac{1}{2})^n u[n-1]$

ANS =

3

(c) $h[n] = \delta[n] + \delta[n-2]$

ANS =

5

(d) $y[n] = \frac{1}{2} y[n-1] + x[n]$

ANS =

4

(e) $y[n] = \left((-\frac{1}{2})^n u[n]\right) * \left(\delta[n] + \delta[n-1]\right)$

ANS =

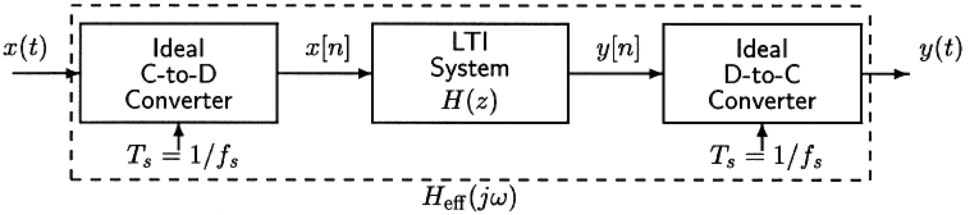
1

Frequency Response

1. $H(e^{j\hat{\omega}}) = \dfrac{1 + e^{-j\hat{\omega}}}{1 + \frac{1}{2} e^{-j\hat{\omega}}}$
2. $H(e^{j\hat{\omega}}) = 1 + \frac{1}{2} e^{-j\hat{\omega}}$
3. $H(e^{j\hat{\omega}}) = \dfrac{1 - \overset{\overset{-}{\downarrow}}{e}{}^{j\hat{\omega}}}{1 - \frac{1}{2} e^{-j\hat{\omega}}}$
4. $H(e^{j\hat{\omega}}) = \dfrac{1}{1 - \frac{1}{2} e^{-j\hat{\omega}}}$
5. $H(e^{j\hat{\omega}}) = 2e^{-j\hat{\omega}} \cos(\hat{\omega})$
6. $H(e^{j\hat{\omega}}) = 1 + e^{-j\hat{\omega}}$
7. $H(e^{j\hat{\omega}}) = \dfrac{1}{1 + \frac{1}{2} e^{-j\hat{\omega}}}$
8. $H(e^{j\hat{\omega}}) = \dfrac{1 + \frac{1}{2} e^{-j\hat{\omega}}}{1 + e^{-j\hat{\omega}}}$

Problem F-00-F.7:

A discrete-time linear time-invariant filter with an impulse response $h[n]$ is used to filter a continuous-time signal $x(t)$ as illustrated below.



The sampling frequency of the C-D and D-C converters is

$$f_s = 10^4 \text{ Hz}$$

and the system function of the LTI system is

$$H(z) = 1 + z^{-2}$$

The overall system (dashed box) behaves as a continuous-time LTI system, i.e., $Y(j\omega) = H_{\text{eff}}(j\omega)X(j\omega)$.

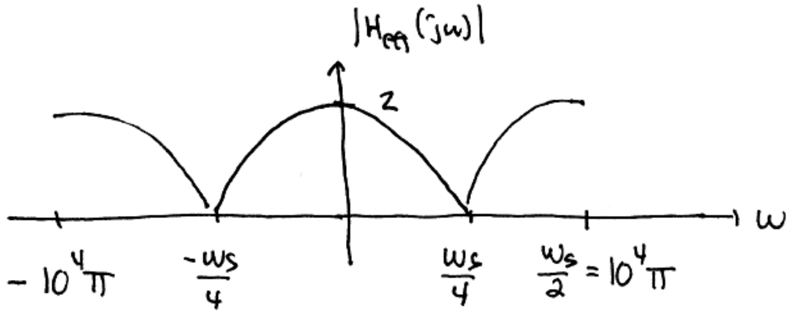
(a) Give an expression for the overall effective frequency response of the continuous-time LTI system, $H_{\text{eff}}(j\omega)$.

Since $H_{\text{eff}}(j\omega) = \begin{cases} H(e^{j\omega T_s}) & |\omega| < \omega_s/2 \\ 0 & \text{else} \end{cases}$

with $H(e^{j\hat{\omega}}) = 1 + e^{-j2\hat{\omega}}$ we have

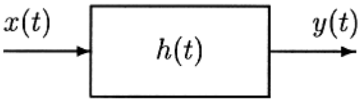
$$H_{\text{eff}}(j\omega) = \begin{cases} 1 + e^{-j2\omega T_s} = 1 + e^{-j\omega(2\cdot 10^{-4})} & |\omega| < 10^4\pi \\ 0 & \text{else} \end{cases}$$

(b) Make a carefully labeled plot of the magnitude of the effective frequency response, $|H_{\text{eff}}(j\omega)|$. Make sure that you label the maximum and minimum values of $|H_{\text{eff}}(j\omega)|$ and the frequencies at which these occur.

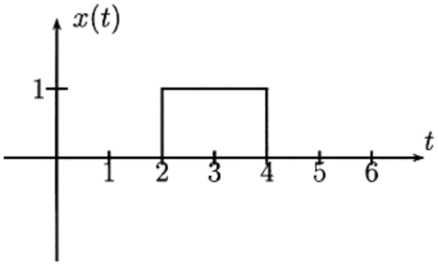


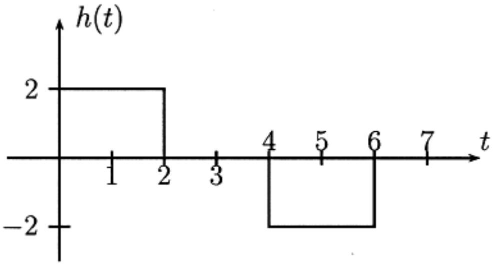
Problem F-00-F.8:

Consider the linear time-invariant system shown below.

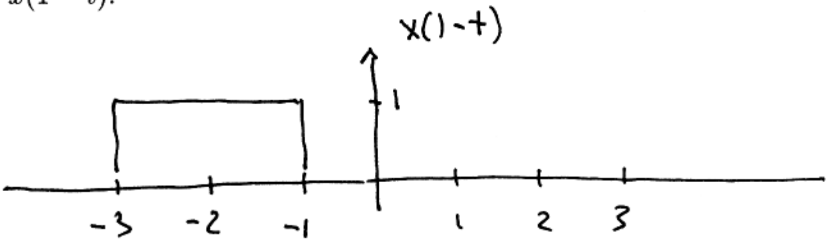


The input $x(t)$ and the impulse response $h(t)$ are shown in the figures below.





(a) Draw $x(1-t)$.



(b) Make a carefully labeled plot of the filter output, $y(t)$.

Recall that the convolution of two pulses of the same length gives a triangle. So, we have