ECE 2025, Spring 2006, Problem Set #1

- Problem 1.1 Signal Processing First, Appendix A, Problem P-A.1, page 441.

  Convert the following to polar form:

$$(a) \quad z = 0 + j2 = 2\ e^{j\pi/2}$$

$$(b) \quad z = (-1, 1) = \sqrt{2}\ e^{j3\pi/4}$$

$$(c) \quad z = -3 - j4 = 5\ e^{j(\pi+\arctan(4/3))} = 5\ e^{j(\pi+0.9273)} = 5\ e^{j4.0689}$$

$$(d) \quad z = (0, -1) = e^{-j\pi/2}$$

- Problem 1.2: Convert the following to rectangular form (by using Euler's formula). Give numerical values for the real and imaginary parts.

  (a)

$$z = 9\ e^{j(5\pi/4)} = 9\cos(5\pi/4) + 9j\sin(5\pi/4) = -6.3640 - 6.3640j$$

  (b)

$$z = 5\ e^{j(11\pi/2)} = 5\cos(11\pi/2) + 5j\sin(11\pi/2) = -5j$$

  (c)

$$z = 44\angle(5\pi/6) = 44\cos(5\pi/6) + 44j\sin(5\pi/6) = -38.1051 + 22.0000j$$

  (d)

$$z = \sqrt{2}\angle(17\pi) = \sqrt{2}\cos(17\pi) + \sqrt{2}j\sin(17\pi) = -\sqrt{2} = -1.4142$$

- Problem 1.3. Evaluate the following and give the answer in both rectangular and polar form. In all cases, assume that the complex numbers are $z_1 = 1 - j$ and $z_2 = \sqrt{2}e^{j(11\pi/2)}$.

  (a)

$$z_1^* = 1 + j = \sqrt{2}\ e^{j\pi/4}$$

(b)

$$jz_2 = \sqrt{2}\ e^{j(11\pi/2)}\ e^{j\pi/2} = \sqrt{2}\ e^{j6\pi} = \sqrt{2}\ e^{j\times 0} = \sqrt{2}$$

(c)

$$z_2/z_1 = \frac{\sqrt{2}\ e^{j(11\pi/2)}}{\sqrt{2}\ e^{-\pi/4}} = e^{j23\pi/4} = e^{-j\pi/4} = 0.7071 - 0.7071j$$

(d)

$$z_2^2 = 2\ e^{j(11\pi)} = 2\ e^{-j\pi/2} = -2$$

(e)

$$z_1^{-1} = \frac{1}{1-j} = \frac{1+j}{1+1} = 0.5 + 0.5j = \frac{\sqrt{2}}{2}\ e^{j\pi/4}$$

(f)

$$z_1 z_2 = \sqrt{2}\, e^{-j\pi/4}\, \sqrt{2}\, e^{j(11\pi/2)} = 2\, e^{j21\pi/4} = 2\, e^{-j3\pi/4} = -1.4142 - 1.4142j$$

(g)

$$z_1 + z_2^* = 1 - j + \sqrt{2}\, e^{-j11\pi/2} = 1 - j + \sqrt{2}j = 1 + 0.4142j = 1.0824\, e^{j\times 0.3927}$$

(h)

$$|z_2|^2 = (\sqrt{2})^2 = 2 = 2\ e^{j\times 0}$$

(i)

$$z_2 + z_2^* = 2\sqrt{2}\cos(11\pi/2) = 2\sqrt{2}\times 0 = 0$$

- Problem 1.4. Signal Processing First, Appendix 2, Problem P-2.1, page 31.

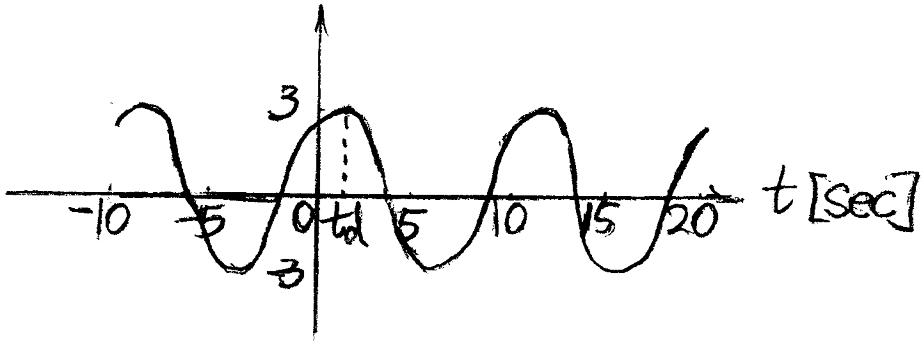
  Define $x(t)$ as

  $$x(t) = 3\cos(\omega_0 t - \pi/4)$$

  For $\omega_0 = \pi/5$, make a plot of $x(t)$ that is valid over the range $-10 \leq t \leq 20$.

  Solution: From $\omega_0 = \pi/5$, we find $f_0 = \omega_0/(2\pi) = 0.1$ Hz, and thus $T = 1/f_0 = 10$ sec.

From $\phi = -\pi/4 = -\omega_0 t_d$, we find $t_d = 1.25$ sec.



- Problem 1.5. Signal Processing First, Appendix 2, Problem P-2.2, page 31.

  Figure P-2.2 is a plot of a sinusoidal wave. From the plot, determine values for the amplitude (A), phase (ϕ), and frequency (ω_0) needed in the represenation

  $$x(t) = A\cos(\omega_0 t + \phi)$$

  Given the answers as numerical values, including the units where applicable.

  Solution: We find: $A = 20$;

  $T = 25$ msec, thus $f_0 = 1000/25 = 40$ Hz, $\omega_0 = 2\pi f_0 = 80\pi$ rads/sec.

  $t_d = 5$ msec, thus $\phi = -\omega_0 t_d = -80\pi \times 5 \times 10^{-3} = -0.4\pi$ rads.