HW#2

2.1 (a) $x_a(t) = 5\cos(100\pi t - \pi/4) - 4\cos(100\pi t - 3\pi/4)$
$- 5\cos(99\pi t - \pi/4) + 4\cos(99\pi t - 3\pi/4)$

$x_a(t) = A\cos(100\pi t + \varphi) + B\cos(99\pi t + \theta)$

$$Ae^{j\varphi} = 5e^{-j\frac{\pi}{4}} - 4e^{-j3\pi/4} = 3.5355(1-j) - 4(-1-j)$$
$$= 6.364 - 0.7071j$$
$$= 6.402\, e^{j*(-0.1107)}$$

$$Be^{j\theta} = -5e^{-j\frac{\pi}{4}} + 4e^{-j\frac{3\pi}{4}} = -3.5355(1-j) - 2.8284(1+j)$$

$$Be^{j\theta} = -6.364 + 0.07071i$$
$$= 6.402\, e^{j(3.03)}$$

$\therefore$ $x_a(t) = 6.402\cos(100\pi t + (-0.1107))$
$+ 6.402\cos(99\pi t + 3.03)$ ※

(b) $x_b(t) = 4\sqrt{2}\cos(\pi t + 63\pi) - 20\cos(\pi t - 63.25\pi) + 4\sqrt{2}\cos(\pi t + 63.5\pi)$

$x_b(t) = A\cos(\pi t + \varphi)$

$$Ae^{j\varphi} = 4\sqrt{2}\, e^{j63\pi} - 20\, e^{j(-63.25\pi)} + 4\sqrt{2}\, e^{j63.5\pi}$$
$$= (-5.6569) + (14.141 - 14.141j) + (-5.6569j)$$
$$= 8.4853 - 19.799j = 21.5407\, e^{j(-1.1659)}$$

$\therefore$ $x_b(t) = 21.5407 * \cos(\pi t - 1.1659)$ ※

2.1

(c) $x_c(t) = 22\cos(13t - 2\pi/7) + 22\cos(13t + \pi/21) + 22\cos(13t + 8\pi/21)$

$x_c(t) = A\cos(13t + \varphi)$

$$Ae^{j\varphi} = 22e^{j(-2\pi/7)} + 22e^{j\frac{\pi}{21}} + 22e^{j8\pi/21}$$

$$= 22(0.6235 - 0.7818j + 0.9888 + 0.149j + 0.3653 + 0.9309j)$$

$$= 22(1.9777 + 0.2981j)$$

$$= (22\times 2)e^{j0.1496} = 44e^{j(0.1496)}$$

$\therefore x_c(t) = 44\cos(13t + 0.1496)$ ※

2.2 Define $x(t) = \sqrt{3}\cos(\omega_0 t - 3\pi/4) + 3\cos(\omega_0 t + 3\pi/4)$

(a) find $z_1(t)$ such that $Re\{z_1(t)\} = 77\cos(\omega_0 t - 2\pi/7)$

$$\therefore z_1(t) = 77 e^{-j2\pi/7} e^{j\omega_0 t} \quad \#$$

(b) find $z(t)$ such that $x(t) = Re\{z(t)\}$

$$x(t) = \sqrt{3}\cos(\omega_0 t - 3\pi/4) + 3\cos(\omega_0 t + 3\pi/4)$$
$$= A\cos(\omega_0 t + \varphi)$$
$$Ae^{j\varphi} = \sqrt{3}\, e^{-j(\frac{3\pi}{4})} + 3\, e^{j(3\pi/4)}$$
$$= -1.2247(1+j) + 2.1213(-1+j)$$
$$= -3.3461 + 0.8966j$$
$$= 3.4641\, e^{j(2.8798)}$$

$$\therefore z(t) = 3.4641\, e^{j(2.8798)} e^{j\omega_0 t} \quad \#$$

(c) $\because \omega_0 = 3\pi$ rad/sec $\qquad x(t) = Re\{(-\sqrt{3} - j3)e^{j\omega_0 t}\}$

$$x(t) = Re\{-2\sqrt{3}\left(\tfrac{1}{2} + \tfrac{\sqrt{3}}{2}j\right)e^{j\omega_0 t}\}$$
$$= Re\{2\sqrt{3}\, e^{j\frac{4\pi}{6}} e^{j\omega_0 t}\}$$
$$\therefore z(t) = 2\sqrt{3}\, e^{j\frac{4\pi}{6}} e^{j\omega_0 t} = 2\sqrt{3}\, e^{j\frac{2\pi}{3}} e^{j3\pi t} \quad \#$$

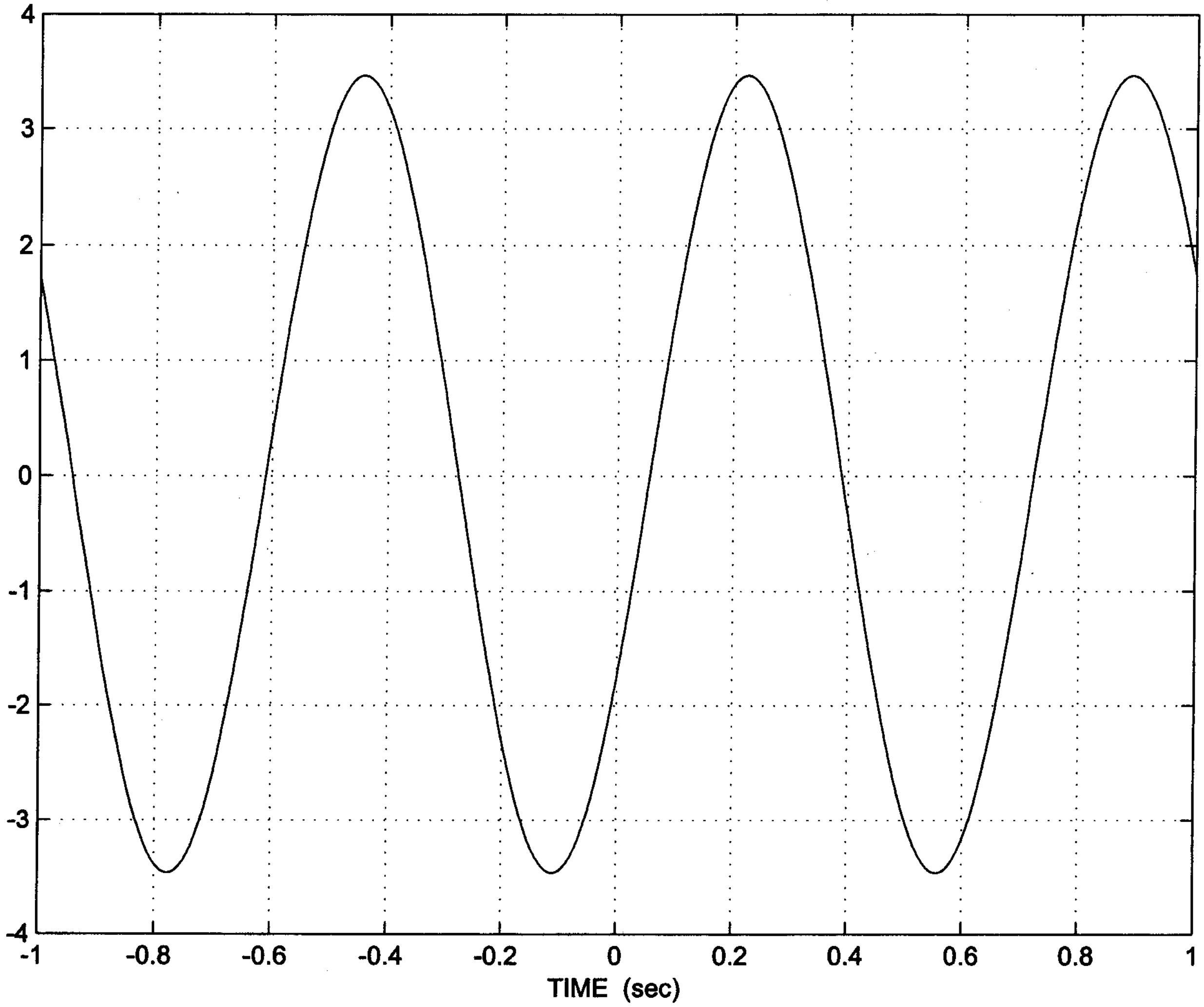
$$f_0 = \frac{3\pi}{2} = 1.5\text{ Hz} \qquad T = \frac{1}{1.5} = 0.667\text{ sec}$$

for plot of $x(t)$ $\qquad -1 \le t < 1 \qquad$ see attachment figure

$\therefore$ there are $\frac{2}{0.667} = 3$ periods $\#$

problem 2.2(c) plot of $x(t) = Re\{2\sqrt{3}\, e^{j\frac{2\pi}{3}} e^{j\omega_0 t}\}$

$\omega_0 = 3\pi$



2.3 Consider $z(t) = Ze^{j13\pi t}$ where $Z = 13e^{-j3\pi/4}$

(a) $\int_0^{0.5} z(t)\,dt = Z\int_0^{0.5} e^{j13\pi t}\,dt = 13e^{-j3\pi/4}\int_0^{0.5} e^{j13\pi t}\,dt$

$= 13e^{-j3\pi/4}\left(\frac{1}{j13\pi}\right) e^{j13\pi t}\Big|_0^{0.5}$

$= e^{-j3\pi/4}\left(\frac{1}{j\pi}\right)\left(e^{j6.5\pi t} - 1\right) = -0.4502j$ ※

(b) $\int_{-0.5}^{0.5} z^2(t)\,dt = \int_{-0.5}^{0.5} (Ze^{j13\pi t})(Ze^{j13\pi t})\,dt$

$= Z^2 \int_{-0.5}^{0.5} e^{j26\pi t}\,dt = 169\,e^{-j3\pi/2}\left(\frac{1}{j26\pi}\right)e^{j26\pi t}\Big|_{-0.5}^{0.5}$

$= \left(\frac{13}{j2\pi}e^{-j3\pi/2}\right)\left(e^{j13\pi} - e^{-j13\pi}\right) = 0$ ※

(c) $\int_0^2 z^* z\,dt = \int_0^2 [(13e^{-j3\pi/4})e^{j13\pi t}]^* [13e^{+j3\pi/4}e^{-j13\pi t}]\,dt$

$= \int_0^2 169\, e^0 e^0\,dt = 169 \times 2 = 338$ ※

2.4 Find A_1, A_2 and φ_1, φ_2

$$2\cos(\omega_0 t + 2\pi/3) = A_1 \cos(\omega_0 t + \varphi_1) + A_2 \cos(\omega_0 t + \varphi_2) \quad \text{—(1)}$$

$$2\cos(\omega_0 t + \pi) = A_1 \cos(\omega_0 t + \varphi_1) - A_2 \cos(\omega_0 t + \varphi_2) \quad \text{--- (2)}$$

Let eq.(1) − eq(2)

$$\Rightarrow \quad 2\cos(\omega_0 t + 2\pi/3) - 2\cos(\omega_0 t + \pi) = 2A_2 \cos(\omega_0 t + \varphi_2)$$

$$2(e^{j2\pi/3} - e^{j\pi}) = 2A_2 e^{j\varphi_2}$$

$$\frac{1}{2} + \frac{\sqrt{3}}{2} j = A_2 e^{j\varphi_2} = e^{j\pi/3}$$

$$\Rightarrow \quad A_2 = 1, \quad \varphi_2 = \pi/3 \quad \#$$

Let eq(1) − eq(2)

$$\therefore \quad 2\cos(\omega_0 t + 2\pi/3) + 2\cos(\omega_0 t + \pi) = 2A_1 \cos(\omega_0 t + \varphi_1)$$

$$2(e^{j2\pi/3} + e^{j\pi}) = 2A_1 e^{j\varphi_1}$$

$$\frac{-3}{2} + \frac{\sqrt{3}}{2} j = A_1 e^{j\varphi_1} = \sqrt{3}\, e^{j5\pi/6}$$

$$\Rightarrow \quad A_1 = \sqrt{3}, \quad \varphi_1 = 5\pi/6 \quad \#$$

2.5 $F_0 = 7\ Hz$

zz = 15 * exp(j* (2*pi *Fo *(xt +0.875)))

xx = Real (zz)

$zz = 15\ \exp(j * 14\pi t)\ \exp(j * 12.25\pi)$

$$= 15 e^{j14\pi t} e^{j\frac{\pi}{4}} = 15 e^{j\frac{\pi}{4}} e^{j14\pi t}$$

$$x(t) = Re\{zz\} = 15 \cos(14\pi t + \tfrac{\pi}{4})$$

$$\because x(t) = A \cos(\omega t + \varphi)$$

$$A = 15, \quad \omega = 14\pi, \quad \varphi = \tfrac{\pi}{4}$$

See the attached plot in MATLAB

Problem. 2.5
MATLAB PLOT