GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2006
Problem Set #4

Assigned: 27-Jan-06
Due Date: Week of 6-Feb-06

Quiz #1 will be held in lecture on Friday 10-Feb-06. It will cover material from Chapters 2 and 3, as represented in Problem Sets #1, #2, #3 and #4.
Closed book, calculators permitted, and one hand-written formula sheet ($8\frac{1}{2}'' \times 11''$, both sides)

Reading: In *SP First*, Chapter 3: *Spectrum Representation,* all.

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.
Please follow the format guidelines (cover page, etc.) for homework.

PROBLEM 4.1*:

A linear-FM "chirp" signal is one that sweeps in frequency from $\omega_1 = 2\pi f_1$ to $\omega_2 = 2\pi f_2$ as time goes from $t = 0$ to $t = T_2$.

(a) One way to write the chirp is to use the following MATLAB code:

```
tt = 0:0.0001:1.2;
aa = 800*pi;
bb = 200*pi;
cc = -0.5*pi;
psi = aa*tt.*tt + bb*tt + cc;
xx = real( exp(j*psi) );
```

In other words, the chirp is written using the complex exponential notation:

$$x(t) = \Re e\left\{ e^{j(at^2+bt+c)} \right\}$$

Using the values in the MATLAB code and the signal defined by `xx` and the *angle function* `psi`, draw a sketch of the instantaneous frequency as time goes from $t = 0$ to $t = 1.2$ sec. Label the axes to make it clear whether frequency is in hertz or rad/s.

(b) Write MATLAB code (similar to that above) for a chirp that starts at $f_1 = 3000$ Hz and ends at $f_2 = 600$ Hz over a duration of 0.8 secs.

PROBLEM 4.2*:

Suppose that a periodic signal is defined (over one period) as: $x(t) = \begin{cases} e^{|t|} & \text{for } -2 \le t \le 2 \\ 0 & \text{for } 2 < t < 5 \end{cases}$

(a) Assume that the period of $x(t)$ is 7 sec. Draw a plot of $x(t)$ over the range $-10 \le t \le 10$ sec.

(b) Determine the DC value of $x(t)$ from the Fourier series integral.

PROBLEM 4.3*:

Continuation of previous problem:

(a) Write the Fourier integral expression for the coefficient a_k in terms of the specific signal $x(t)$ defined above. Set up all the specifics of the integral (e.g., limits of integration, integrand).

(b) Evaluate the integral in the previous part and obtain an expression for a_k that is valid for all k. Simplify your final formula for a_k so that it is purely real.

PROBLEM 4.4*:

A signal is defined by the equation

$$x(t) = \cos^3(22\pi t - 0.5\pi)$$

(a) Determine the fundamental frequency of the signal.

(b) Determine the Fourier Series coefficients of this signal.
Hint: Avoid integration. Use the expansion of $(a+b)^3$ to quickly obtain an exponential form equivalent to $\cos^3$.

(c) Plot the spectrum of $x(t)$. Most of $\{a_k\}$ are zero, but the nonzero ones can be used to make it is clear which harmonic components are present in the signal's spectrum.

PROBLEM 4.5*:

We have seen that musical tones can be modeled mathematically by sinusoidal signals. If you read music or play the piano[1] you are aware of the fact that the piano keyboard is divided into octaves, with the tones in each octave being twice the frequency of the corresponding tones in the next lower octave. To calibrate the frequency scale, the reference tone is the A above middle-C, which is usually called A440 since its frequency is 440 Hz. Each octave contains 12 tones, and the ratio between the frequencies of successive tones is constant. Since middle C is 9 tones below A440, its frequency is approximately $(440)2^{-9/12} \approx 262$ Hz. In musical notation the tones are called notes; the names of the notes in the octave starting with middle-C and ending with high-C are:

note name	C	$C^\#$	D	$E^\flat$	E	F	$F^\#$	G	$G^\#$	A	$B^\flat$	B	C
note number	40	41	42	43	44	45	46	47	48	49	50	51	52
frequency													

(a) Explain why the ratio of the frequencies of successive notes must be $2^{1/12}$.

(b) Make a table of the frequencies of the tones in the octave beginning with middle-C assuming that A above middle C is tuned to 440 Hz.

(c) The above notes on a piano are numbered 40 through 52. If n denotes the note number, and f denotes the frequency of the corresponding tone, give a simple formula for the frequency of the tone as a function of the note number.

[1] If you don't read music, the problem description still gives enough information to define the keyboard layout.