GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2005
Problem Set #5

Assigned: 10-Feb-05
Due Date: Week of 20-Feb-05

Reading: In *SP First*, Chapter 4: *Sampling and Aliasing*

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.
Please follow the format guidelines (cover page, etc.) for homework.

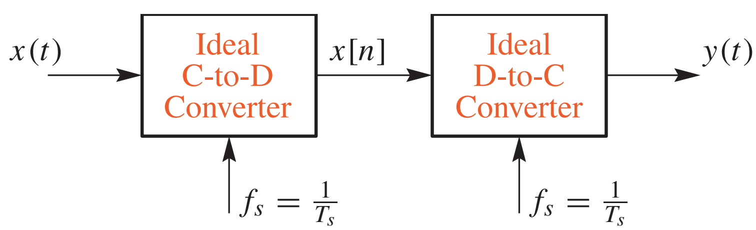


Figure 1: Ideal sampling and reconstruction systems. An ideal C-to-D converter samples $x(t)$ with a sampling period $T_s = 1/f_s$ to produce the discrete-time signal $x[n]$. The ideal D-to-C converter then forms a continuous-time signal $y(t)$ from the samples $x[n]$.

PROBLEM 5.1*:

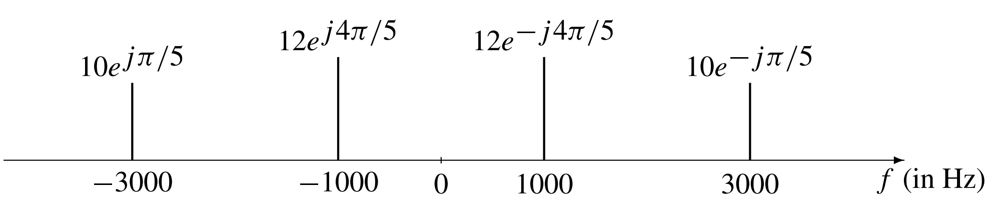
Consider the ideal sampling and reconstruction system shown in Fig. 1.

(a) Suppose that the discrete-time signal $x[n]$ in Fig. 1 is given by the formula

$$x[n] = 10\cos(0.35\pi n - 0.6\pi)$$

If the sampling rate of the C-to-D converter is $f_s = 10000$ samples/second, many *different* continuous-time signals $x(t) = x_\ell(t)$ could have been inputs to the above system. Determine two such inputs with their frequencies between 10000 and 20000 Hz; i.e., find $x_1(t) = A_1\cos(\omega_1 t + \phi_1)$ and $x_2(t) = A_2\cos(\omega_2 t + \phi_2)$ such that $x[n] = x_1(nT_s) = x_2(nT_s)$ if $T_s = 1/10000$ secs.

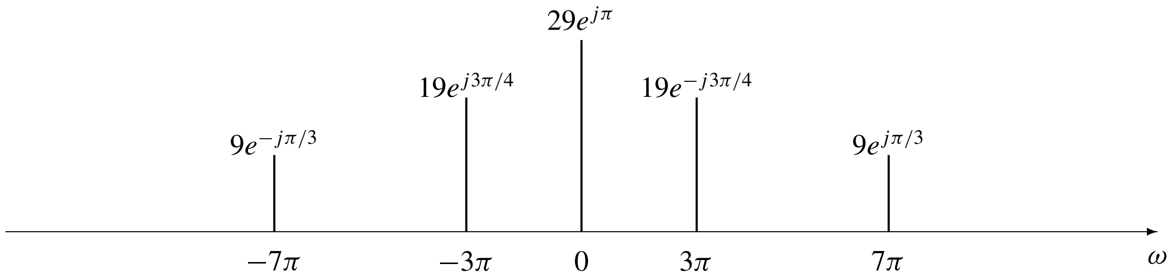
(b) Now suppose that the input $x(t)$ to the system in Fig. 1 has the two-sided spectrum representation shown below, what is the *minimum* sampling rate f_s such that the output $y(t)$ is equal to the input $x(t)$?



(c) Using the signal $x(t)$ from part (b), determine the spectrum for $x[n]$ when $f_s = 1000$ samples/sec. Simplify your as much as possible and make a plot for your answer, but label the frequency, and complex amplitude (magnitude and phase) of each spectral component.

PROBLEM 5.2*:

A real signal $x(t)$ has the following two-sided spectrum:



(a) Plot the spectrum of the discrete-time signal formed when $x(t)$ is sampled at a rate of $f_s = 11$ samples/sec.

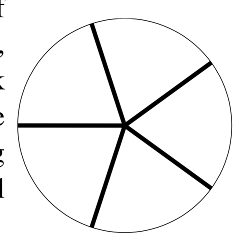
(b) Plot the spectrum of the discrete-time signal formed when $x(t)$ is sampled at a rate of $f_s = 6$ samples/sec.

(c) Determine the spectrum of the continuous-time signal reconstructed from the discrete-time signal in part (b). Assume the D-to-C converter is operating at a rate of $f_s = 6$ samples/sec.

PROBLEM 5.3*:

When watching old TV movies, all of us have seen the phenomenon where a wagon wheel appears to move backwards. The same illusion can also be seen in automobile commercials, when the hubcaps of a car or truck have a spoked pattern. Both of these are due to the 30 frames/sec sampling used in transmitting TV images.

In the figure to the right, a five-spoked wheel is shown. Assume that the diameter of this wheel is 0.75 meters, which might be the tire diameter of a big truck. In addition, assume that the wheel is rotating clockwise, so that if attached to a truck, the truck would be traveling to the right *at a constant speed.* However, when seen on TV the spoke pattern of the truck wheel appears to stand still. How fast is the truck traveling (in kilometers per hour)? Derive a general equation that will make it easy to give all possible answers.



PROBLEM 5.4*:

Chirps and FM signals are very useful signals for probing the behavior of sampling and reconstruction systems (such as Fig. 1). Consider the following MATLAB code:

```
%-- make an FM signal and display its spectrogram
%--
fs = 10000;
tt = 0:1/fs:1;
psi = 1000*pi*tt + 40*exp(2*pi*tt);
xx = cos(psi);
plotspec(xx+j*1e-11,fs,128), grid on, shg  %-- specgram could be used here
```

(a) The MATLAB code can interpreted as equivalent to the system in Fig. 1. Determine the mathematical expressions for $x(t)$ and $x[n]$, the signals at the input and output of the C-to-D converter. Write your answer assuming that f_s is a parameter whose value is not yet assigned.

(b) When using the spectrogram, it turns out that you are essentially calculating the spectrogram of the output signal, $y(t)$. If the sampling rate is $f_s = 10000$ Hz, then the output signal $y(t)$ will have time-varying frequency content. Use mathematics to determine the analog *instantaneous* frequency (in Hz) versus time of the signal $y(t)$ **after reconstruction**, and then draw a graph of what the spectrogram should look like. Comment on whether or not the sampling theorem is satisfied when $f_s = 10000$ Hz. *Hint:* this could be checked in MATLAB by using the code above.

PROBLEM 5.5*:

Here are some operations that are often done in MATLAB. In each case, the length of the `nn` or `tt` vector is huge, so the code cannot be run in MATLAB. Therefore, you should analyze the code and determine the answer via mathematics.

(a) Suppose that a student enters the following MATLAB code:

```
nn = 0:4480099;
xx = (3/pi) * cos(2*pi*0.8*nn + 14.92);
soundsc(xx,40000)
```

Determine the analog frequency (in Hertz) that will be heard.

(b) Suppose that a student writes the following MATLAB code to generate a sine wave:

```
tt = 0:1/16000:10000;
xx = sin(2*pi*1000*tt+pi/3);
soundsc(xx,fsamp);
```

Although the sinusoid was not written to have a frequency of 1600 Hz, it is possible to play out the vector `xx` so that it sounds like a 1600 Hz tone. Determine the value of `fsamp` that should be used to play the vector `xx` as a 1600 Hz tone.

(c) Consider the following piece of MATLAB code:

```
tt = 0:(1/48000):48;
xx = cos(2*pi*440*tt);
soundsc(xx,16000);
```

Determine the duration (in seconds) of the final played tone. (Assume that the computer has an infinite amount of memory so that we don't need to worry about running out.)