Problem 5.1

a.

$$x[n] = 10\cos(0.35\pi n - 0.6\pi) = \mathrm{Re}\{10e^{-j0.6\pi}e^{j0.35\pi n}\} = \mathrm{Re}\{10e^{-j0.6\pi}e^{j0.35\pi n + j2k\pi n}\}, \quad k \text{ integer}$$

$$0.35\pi + k2\pi = \hat{\omega} + k2\pi = 2\pi\frac{f}{f_s} = 2\pi\frac{f}{10000} \quad , \quad f = 10000(0.175 + k)\ \text{Hz}$$

In (10000, 20000), $f = 10000(0.175 + 1) = 11750\ \text{Hz} \quad \Rightarrow \ x_1(t) = 10\cos(2\pi 11750t - 0.6\pi)$

$$x[n] = 10\cos(0.35\pi n - 0.6\pi) = \mathrm{Re}\{10e^{j0.6\pi}e^{-j0.35\pi n}\} = \mathrm{Re}\{10e^{j0.6\pi}e^{-j0.35\pi n + j2k\pi n}\}, \quad k \text{ integer}$$

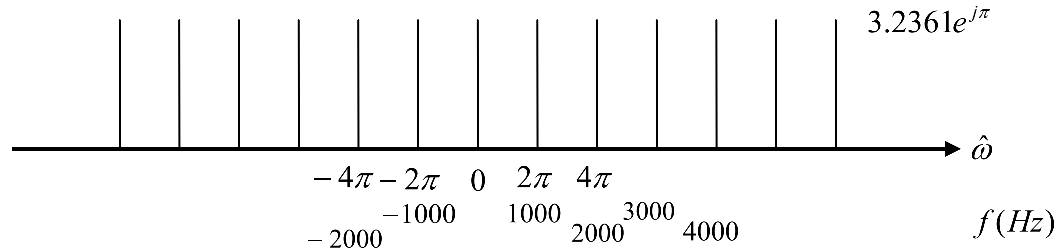
$$-0.35\pi + k2\pi = \hat{\omega} + k2\pi = 2\pi\frac{f}{f_s} = 2\pi\frac{f}{10000}, \quad f = 10000(-0.175 + k)\ \text{Hz}$$

$$f = 10000(-0.175 + 2) = 18250\ \text{Hz} \quad \Rightarrow \ x_2(t) = 10\cos(2\pi 18250t + 0.6\pi)$$

b. 6000Hz

c.

Spectrum of the discrete time signal



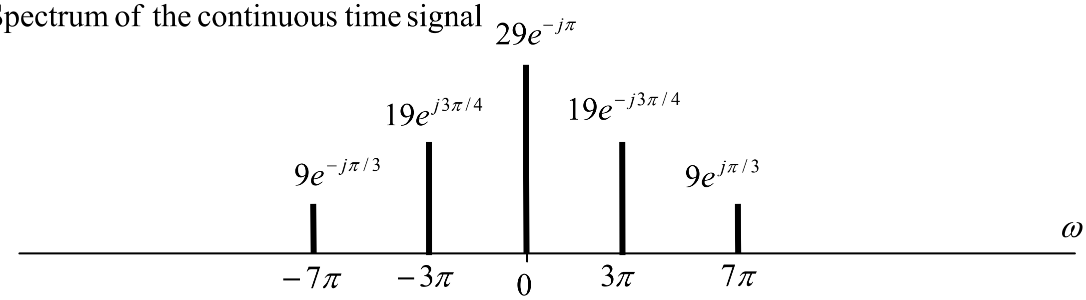

The original spectrum will be shifted left and right by integer multiples of 1000 Hz (2π). As a result, for $f = k1000$ (or $\hat{\omega} = k2\pi$ ), $k = \ldots,-2,-1,0,1,2,\ldots$, all four components in the original spectrum come together and have to be added up. That produces

$$A + A^* + B + B^* = 24\cos\frac{4\pi}{5} + 20\cos\frac{\pi}{5} = -3.2361 \quad \text{where } A = 12e^{-j4\pi/5} \text{ and } B = 10e^{-j\pi/5}$$

The spectrum of the sampled signal is thus as plotted above.

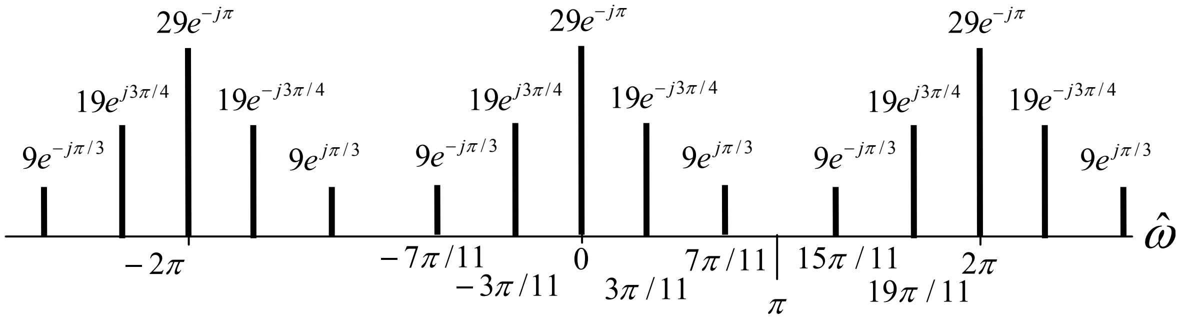
Problem 5.2

Spectrum of the continuous time signal



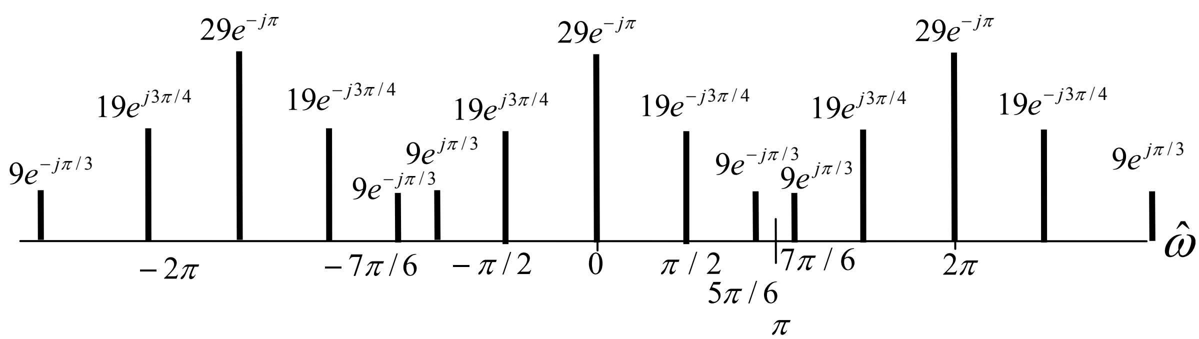
a.

Spectrum of the discrete time signal




b.

Spectrum of the discrete time signal




c. $y(t) = -29 + 38\cos(3\pi t - 3\pi/4) + 18\cos(5\pi t - \pi/3)$

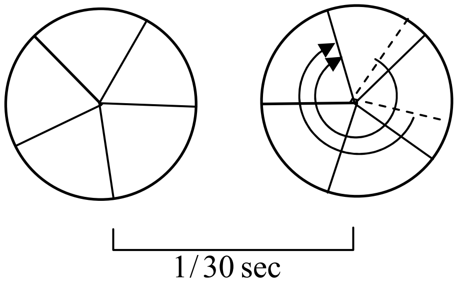
Problem 5.3

Within the sampling interval, there could have been $l$ complete turns plus integer multiples of 1/5 turns for the wheel to appear still.

\# turns per sampling period $= l + (k/5)$ $\quad l$ : (non - negative) integer; $k$ : integer in [0,4]

turns per sec $= 30l + 6k$; turns per hour $= (30l + 6k) * 3600$

distance per turn $= 0.75\pi = 2.33m$




$speed = 2.33 * 3.6 * (30l + 6k) \quad km/hr$

for $l = 0, k = 3$,

speed $= 2.33 * 3.6 * 18 = 150.98$ km/hr

for $l = 0, k = 2$,

speed $= 2.33 * 3.6 * 12 = 100.66$ km/hr

Problem 5.4

a. $x(t) = \cos(1000\pi t + 40e^{2\pi t})$, $\quad x[n] = \cos\left(2\pi \frac{500}{f_s} n + 40e^{2\pi n / f_s}\right)$

b. $\phi(t) = 1000\pi t + 40e^{2\pi t}$ $\quad \frac{d}{dt}\phi(t) = 1000\pi + 80\pi e^{2\pi t}$ $\quad f_x(t) = 500 + 40e^{2\pi t}$

Find all t when $f_x(t)$ reaches multiples of 5000Hz;

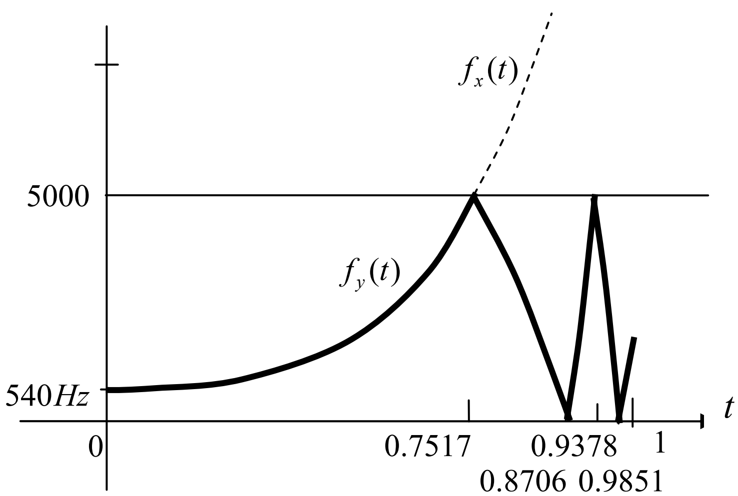
$t = 0.7517 \quad f_x(t) = 5000 = f_s/2 \quad \rightarrow \quad 0 \le t \le 0.7517, \quad f_y(t) = 500 + 40e^{2\pi t}$

$t = 0.8706 \quad f_x(t) = 10000 = f_s \quad \rightarrow \quad 0.7517 < t \le 0.8706, \quad f_y(t) = 10000 - (500 + 40e^{2\pi t})$

$t = 0.9378 \quad f_x(t) = 15000 = 3f_s/2 \rightarrow 0.8706 < t \le 0.9378, \quad f_y(t) = -10000 + (500 + 40e^{2\pi t})$

$t = 0.9851 \quad f_x(t) = 20000 = 2f_s \rightarrow 0.9378 < t \le 0.9851, \quad f_y(t) = 20000 - (500 + 40e^{2\pi t})$

finally $\quad 0.9851 < t \le 1, \quad f_y(t) = -20000 + (500 + 40e^{2\pi t})$




Problem 5.5

a. $x[n] = \frac{3}{\pi}\cos(2\pi \bullet 0.8n + 14.92) \quad \frac{f}{f_s} = 0.8 = ((1-0.8)) = \frac{f}{40000}, \quad f = 8000Hz$

b. $\frac{1000}{16000} = \frac{1600}{f_s'}, \quad f_s' = 16 * 1600 = 25600Hz$

c. tt has length equivalent to 48 seconds (unit time), sample at 48 KHz. When played back at 16 KHz, the duration becomes 3*48s=144s.