GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2006
Problem Set #6

Assigned: 17-Feb-06
Due Date: Week of 27-Feb-06

Reading: In *SP First*, Chapter 5: *FIR Filters*

⟹ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.
Please follow the format guidelines (cover page, etc.) for homework.

PROBLEM 6.1*:

A linear time-invariant discrete-time system is described by the difference equation

$$y[n] = -x[n] + x[n-2] - 3x[n-4] + 3x[n-5]$$

(a) Determine the impulse response $h[n]$ for this system, and express your answer as a sum of shifted impulses with weights.

(b) Determine the filter coefficients b_k in the causal FIR representation: $y[n] = \sum_{k=0}^{M} b_k x[n-k]$.

(c) Determine the *order* of the filter (M), and the *length* of the filter (L).

(d) Make a plot of the shifted unit-step signal $s[n] = -5u[n-1]$; plot enough to show its essential behavior.

(e) Use convolution to determine the output due to the input $-5u[n-1]$. Use the convolution table, but look for patterns. Plot the output sequence $y[n]$ for $0 \le n \le 10$.

PROBLEM 6.2*:

For each of the following systems, the signal $x[n]$ is the input and $y[n]$ is the output.

1. $y[n] = \pi\, x[n-1]$ (Multiplier)
2. $y[n] = 2^{-x[n+3]}$ (Exponential)

(a) Find the impulse response for both systems. Give your answers as plots.

(b) Determine if the systems are (1) linear; give a proof or counterexample.

(c) Determine if the systems are (2) time-invariant; give a proof or counterexample.

(d) Determine if the systems are (3) causal; give a proof or counterexample.

PROBLEM 6.3*:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

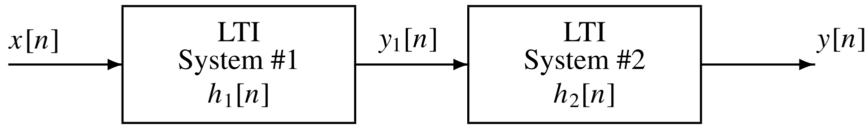

Figure 1: Cascade connection of two LTI systems.

Suppose that System #1 is a filter described by the impulse response:

$$h_1[n] = 5u[n] - 5u[n-3]$$

and System #2 is described by the difference equation

$$y_2[n] = 3y_1[n-3] + 3y_1[n-1]$$

(a) Determine the impulse response sequence, $h_2[n]$, of the second system. Plot $h_2[n]$ versus n.

(b) Determine the impulse response sequence, $h[n] = h_1[n] * h_2[n]$, of the overall cascade system.

(c) If the input signal is

$$x[n] = \cos(0.5\pi n - 0.25\pi)\,(u[n] - u[n-12])$$

determine the output signal, $y[n]$, and make a plot of $y[n]$ versus n. Exploit cascade rearrangements to simplify your work. Indicate regions where the output is zero; and regions where it is nonzero.

PROBLEM 6.4*:

Consider a system defined by $y[n] = \displaystyle\sum_{k=n-100}^{n-1} \frac{x[k]}{n-k}$, where the input signal is zero for $n < 0$.

(a) Determine the impulse response, $h[n]$. Note that $x[n] = \delta[n]$ satisfies the condition that $x[n]$ be zero for $n < 0$.

(b) Determine the filter length and the filter coefficients, $\{b_k\}$.

(c) Suppose that the input $x[n]$ has positive values in the interval $50 \le n \le 300$, but is zero for $n < 50$ and $n > 300$. Determine the interval where the output $y[n]$ will be nonzero. Give the indices of the first and last non-zero values in the output sequence $y[n]$.
Note: it is not possible to calculate the values of $y[n]$, but you can determine when $y[n]$ is zero and nonzero by using the "sliding window" view of FIR filtering (or convolution).

(d) For the previous part, determine the lengths of the input signal, impulse response and output signal (in samples). Then verify that the relationship among these lengths is $L_y = L_x + L_h - 1$, where L_y is the length of $y[n]$, L_x the length of $x[n]$, and L_h the length of $h[n]$.

PROBLEM 6.5*:

Evaluate the following

(a) $y[n] = \delta[n-31] * \delta[n-41] * \delta[n-59]$.

(b) $y[n] = u[n] * u[n]$. Use a convolution table, but then determine the general formula.

(c) Generalize the previous part to get a simple formula for the following 3-way convolution:
$y[n] = u[n] * u[n] * u[n]$.