GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2006
Problem Set #7

Assigned: 24-Feb-06
Due Date: Week of 6-March-06

Quiz #2 on 17-March-2006. Coverage includes Chapters 4, 5, 6 and 7, as well as HW #5, #6, #7 and #8.

Reading: In *SP First*, Chapter 6: *Frequency Response of FIR Filters*

⟹ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.
Please follow the format guidelines (cover page, etc.) for homework.

PROBLEM 7.1*:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

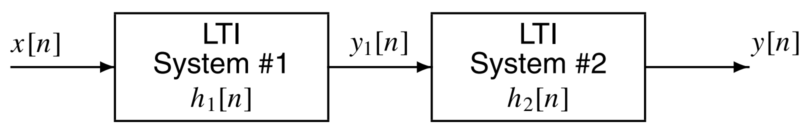


Figure 1: Cascade connection of two LTI systems.

Suppose that System #1 is a filter described by the impulse response

$$h_1[n] = \alpha\delta[n] + \delta[n-2]$$

and System #2 is described by the difference equation

$$y_2[n] = y_1[n-1] + \alpha y_1[n-3],$$

(a) Determine the frequency response, $H_2(e^{j\hat{\omega}})$, of the second system.

(b) Determine the frequency response, $H(e^{j\hat{\omega}})$, of the overall cascade system.

(c) For the case where $\alpha = \frac{1}{2}$, plot the magnitude of the overall frequency response of the cascaded system.

(d) When $\alpha = \frac{1}{2}$ and the input to this system is

$$x[n] = 10\cos(\tfrac{1}{2}\pi n + 0.25\pi)$$

Use the frequency response to compute the values of $y[n]$, over the range $-\infty \le n \le \infty$.

PROBLEM 7.2*:

Consider the linear time-invariant system given by the difference equation

$$y[n] = x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4] + x[n-5] + x[n-6] + x[n-7] = \sum_{k=0}^{7} x[n-k]$$

(a) Find an expression for the frequency response $H(e^{j\hat{\omega}})$ of the system.

(b) Show that your answer in (a) can be expressed in the form

$$H(e^{j\hat{\omega}}) = \frac{\sin(8\hat{\omega}/2)}{\sin(\hat{\omega}/2)} e^{-j3.5\hat{\omega}}$$

(c) Sketch the frequency response (magnitude and phase) as a function of frequency from the formula above. You might want to check your plot by doing it in MATLAB with `freekz( )` or `freqz( )`.

(d) Suppose that the input is

$$x[n] = 100 + 200\cos(\hat{\omega}_0 n) \qquad \text{for } -\infty < n < \infty$$

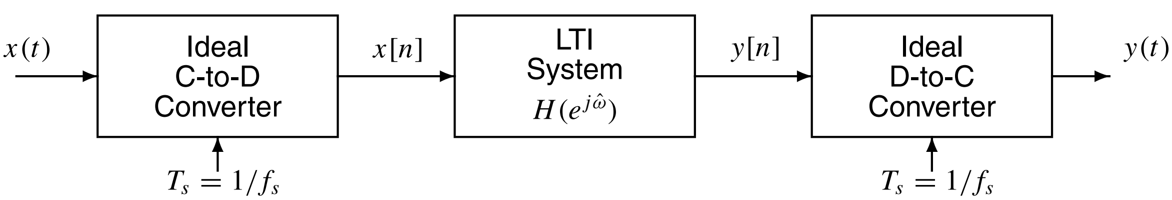
Find all possible non-zero frequencies $0 < \hat{\omega}_0 < \pi$ for which the output $y[n]$ is a constant for all n, i.e.,

$$y[n] = c \qquad \text{for } -\infty < n < \infty$$

and find the value for c. (In other words, the sinusoid is removed by the filter.)

PROBLEM 7.3*:

Consider the following system for discrete-time filtering of a continuous-time signal:



In this problem, assume that the frequency response of the discrete-time system is

$$H(e^{j\hat{\omega}}) = 1 + e^{-j2\hat{\omega}}$$

(a) Make a plot of the frequency response magnitude for $H(e^{j\hat{\omega}})$ over the frequency range $-\pi < \hat{\omega} \leq \pi$.

(b) In this part, assume that the input is

$$x(t) = 500 + 400\cos(800\pi t) \quad \text{for } -\infty < t < \infty$$

For a sampling rate of $f_s = 1000$ samples/sec, draw the spectrum of $x[n]$, the discrete-time signal after the C-to-D converter.

(c) For the same $x(t)$ as in the previous part, and the same sampling rate, determine a simple formula for the output $y(t)$ for $-\infty < t < \infty$.

PROBLEM 7.4*:

A discrete-time system is defined by the input/output relation

$$y[n] = b_1 x[n-1] + b_3 x[n-3]$$

where b_1 and b_3 are constants to be determined.

(a) When the input is the signal, $x_1[n] = 10\cos(0.5\pi n)$, the output is $y_1[n] = \cos(0.5\pi(n-1))$. Also, when the input is the signal, $x_2[n] = 20$, the output is zero. Determine the values of b_1 and b_3.

(b) Obtain an expression for the frequency response of this system, using b_1 and b_3 from part (a).

(c) For the system above, determine the output $y_4[n]$ when the input is

$$x_4[n] = 13 + 17(-1)^n$$

Hint: Use the frequency response and superposition to solve this problem.

PROBLEM 7.5*:

The frequency response of a linear time-invariant filter is given by the formula

$$H(e^{j\hat{\omega}}) = (1 + e^{-j\hat{\omega}})(1 - je^{-j\hat{\omega}})(1 + je^{-j\hat{\omega}}) \quad (1)$$

(a) Write the difference equation that gives the relation between the input $x[n]$ and the output $y[n]$. *Hint:* Multiply out the factors to obtain a sum of powers of $e^{-j\hat{\omega}}$.

(b) What is the impulse response of this system?

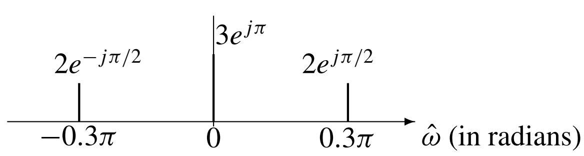
(c) Use superposition to determine the output of this system when the input is

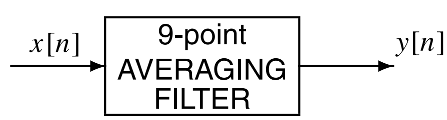
$$x[n] = 10 + 9\delta[n-2] + 8\cos(0.5\pi n) \qquad \text{for } -\infty < n < \infty$$

Hint: Divide the input into three parts and find the outputs separately each by the easiest method and then add the results. This is what it means to apply the principle of *Superposition*.

PROBLEM 7.6:

A discrete-time signal $x[n]$ has the two-sided spectrum representation shown below.





(a) Write an equation for $x[n]$. Make sure to express $x[n]$ as a real-valued signal.

(b) Determine the formula for the output signal $y[n]$.

See Problem 6.1 of Spring 1999 for solution to this problem.