GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2006
Problem Set #8

Assigned: 4-March-06
Due Date: Week of 13-March-06

Reading: In *SP First*, Chapter 7: *z-Transform*

⟹ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.
Please follow the format guidelines (cover page, etc.) for homework.

PROBLEM 8.1*:

We now have ***four ways*** of describing an LTI system: the difference equation; the impulse response, $h[n]$; the frequency response, $H(e^{j\hat{\omega}})$; and the system function, $H(z)$. In the following, you are given one of these representations and you must find the other three.

(a) $y[n] = x[n] - 3x[n-1] + x[n-2]$

(b) $h[n] = \frac{1}{7}(\delta[n-3] - \delta[n-10])$

(c) $H(e^{j\hat{\omega}}) = j\sin(2.5\hat{\omega})e^{-j2.5\hat{\omega}}$

PROBLEM 8.2*:

We now have ***four ways*** of describing an LTI system: the difference equation; the impulse response, $h[n]$; the frequency response, $H(e^{j\hat{\omega}})$; and the system function, $H(z)$. In the following, you are given $H(z)$ and you must find the other three.

(a) $H(z) = 4/z^3$

(b) $H(z) = (1 + z^{-1})(1 - e^{j\pi/4}z^{-1})(1 - e^{-j\pi/4}z^{-1})$

(c) $H(z) = \dfrac{1 + z^{-11}}{1 + z^{-1}}$ (*Note:* this is an FIR filter, so you can divide out the fraction.)

PROBLEM 8.3*:

Suppose that the system function of an FIR filter is

$$H(z) = (1 + z^{-1})(1 - e^{j\pi/4}z^{-1})(1 - e^{-j\pi/4}z^{-1})$$

(a) Determine the zeros of $H(z)$ and make a plot of the zeros in z-plane.

(b) Make a sketch of the frequency response magnitude of the FIR filter. The sketch can be approximate, but you should be able to precisely label the values of $|H(e^{j\hat{\omega}})|$ at $\hat{\omega} = 0, \pi$, and $\pm\pi/2$. Furthermore, label the precise locations where the frequency response is equal to zero.

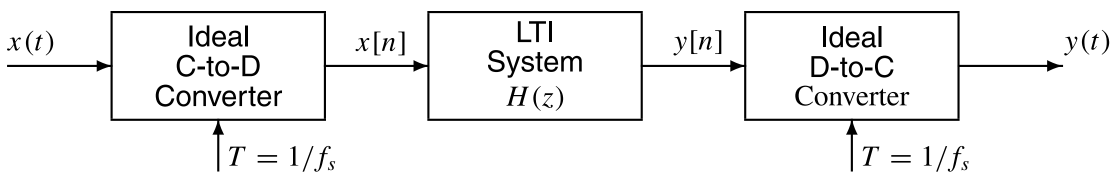
(c) If the input signal to this FIR filter is $x[n] = 3\cos(0.5\pi n - 0.1\pi)$, determine the formula for the output signal, $y[n]$.

PROBLEM 8.4*:

The input to the C-to-D converter in the figure below is

$$x(t) = 10\cos(6000\pi t - \pi/4) + 8\cos(9000\pi t - \pi/5)$$

The system function for the LTI system is $H(z) = \frac{1}{2}z^{-2} + \frac{1}{2}z^{-6}$.



(a) Determine the zeros of the FIR filter $H(z)$, and make a plot of the zeros in the z-plane.

(b) If $f_s = 8000$ samples/second, determine an expression for $y(t)$, the output of the D-to-C converter.
Hint: Recall that the frequency response $H(e^{j\hat{\omega}})$ can be obtained directly from $H(z)$.

PROBLEM 8.5*:

Consider the following MATLAB program:

```
nn = 0:40000;
xx = 50*cos(1.5*pi*nn - 0.25*pi) + 40*cos(0.25*pi*nn - 0.2*pi);
yy = conv([0,0,2,0,0,0,2],xx);
soundsc(yy,8000)
```

(a) After making the usual correspondence between `xx` and $x[n]$, and between `yy` and $y[n]$, determine the system function $H(z)$ of the FIR filter that is implemented by the `conv( )` statement.

(b) Determine the frequency response of the FIR filter, and make a plot versus $\hat{\omega}$ over the range $-\pi$ to $+\pi$. Label the values of $|H(e^{j\hat{\omega}})|$ at $\hat{\omega} = 0, \pi$, and $\pm\pi/2$, and also label the precise locations where the frequency response is equal to zero.

(c) Neglecting the end effects in the convolution, determine $y(t)$ that describes the signal produced by the `soundsc( )` statement.
Hint: The result of a previous problem might be useful here.