GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2006
Problem Set #9

Assigned: 25-Mar-06
Due Date: Week of 3-April-06

Reading: In *SP First*, Chapter 8: *IIR Filters*

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 9.1*:

Determine the z-transforms of the following signals. *Express your answer as the ratio of polynomials in* z^{-1} *by placing all terms over a common denominator.*

(a) $x_a[n] = 5\delta[n-1] + 3^n u[n-2]$

(b) $x_b[n] = 2\left(-\frac{1}{2}\right)^n u[n] + 3\left(-\frac{1}{3}\right)^n u[n]$

(c) $x_c[n] = nu[n]$ Exploit the fact that $x_c[n]$ is the convolution of two (shifted) unit-step signals.

PROBLEM 9.2*:

Given a feedback filter defined via the recursion:

$$y[n] = -0.9y[n-1] + x[n] - x[n-1] \qquad \text{(DIFFERENCE EQUATION)}$$

(a) Determine the impulse response $h[n]$, assuming the "at rest" initial condition.

(b) When the input to the system is the shifted unit-step signal, $u[n-2]$, determine the output signal $y[n]$, assuming the "at rest" initial condition (i.e., the output signal is zero for $n < 0$).

(c) Write two MATLAB statements that would first compute the output in the previous part over the range $0 \le n \le 10$, and then plot it as a stem plot. Consult `help filter`.

PROBLEM 9.3*:

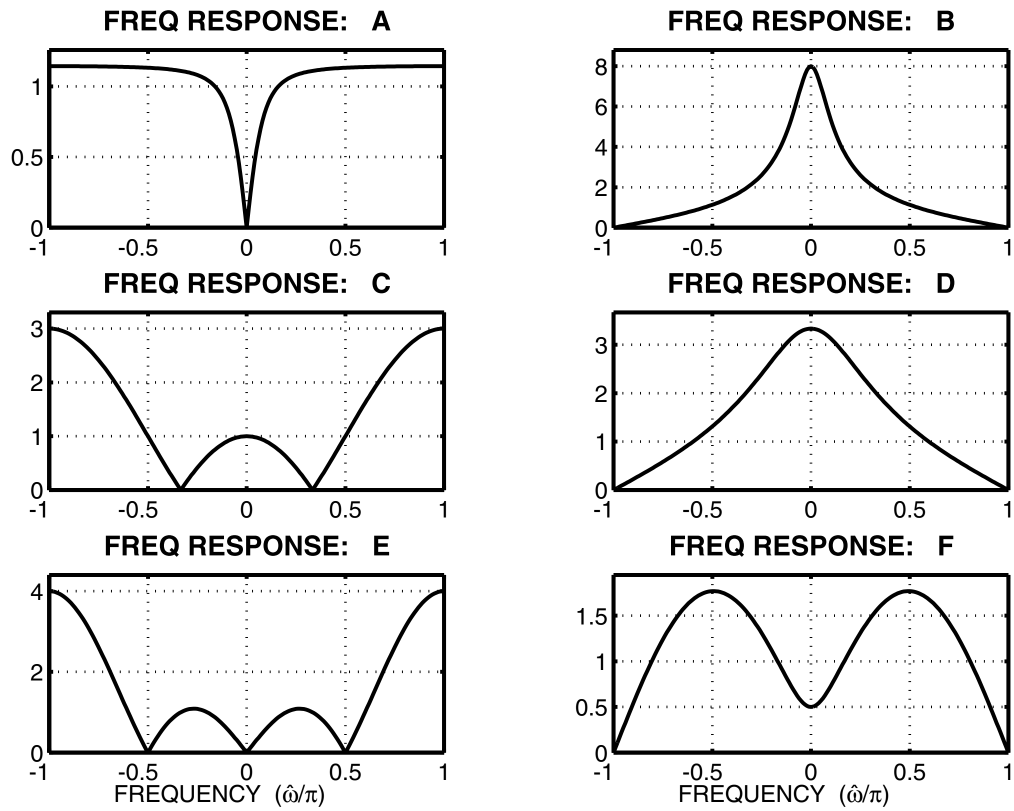
For the system: $$\boxed{H(z) = \frac{1 - z^{-1}}{1 + 0.9z^{-1}}}$$ determine various aspects of the filter's behavior:

(a) Derive a formula for the frequency response, $H(e^{j\hat{\omega}})$.

(b) Plot the magnitude of the frequency response over the range $-\pi$ to π.

(c) Determine the output signal when the input is $x[n] = 100\cos(0.8\pi n - 0.2\pi)$, for all n.

PROBLEM 9.4*:



For each of the frequency response plots (A, B, C, D, E, F), determine which one of the following systems(specified by either an $H(z)$ or a difference equation) matches the frequency response (magnitude only). NOTE: the frequency axis is **normalized**; it is $\hat{\omega}/\pi$.

A: | **B:** | **C:** | **D:** | **E:** | **F:**

Write some justification for each answer.

$\mathcal{S}_1$: $y[n] = 0.4y[n-1] + x[n] + x[n-1]$

$\mathcal{S}_2$: $H(z) = \dfrac{1 + z^{-1}}{1 - 0.75z^{-1}}$

$\mathcal{S}_3$: $y[n] = -0.75y[n-1] + x[n] - x[n-1]$

$\mathcal{S}_4$: $H(z) = \dfrac{1 - z^{-1}}{1 - 0.75z^{-1}}$

$\mathcal{S}_5$: $y[n] = x[n] - x[n-1] + x[n-2]$

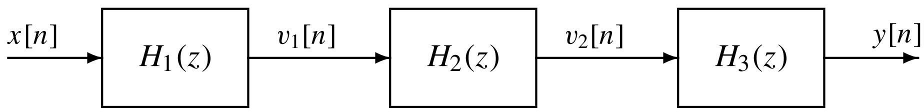
$\mathcal{S}_6$: $H(z) = 1 - z^{-1} + z^{-2} - z^{-3}$

$\mathcal{S}_7$: $y[n] = x[n] + \frac{1}{4}x[n-1] - \frac{3}{4}x[n-2]$

$\mathcal{S}_8$: $H(z) = \frac{1}{3}(1 - z^{-1})^3$

PROBLEM 9.5*:

In the following cascade of systems, all of the individual system functions, $H_i(z)$, are known.



$H_1(z) = z^{-2} + 5z^{-3}$ $\quad$ $H_2(z) = 1 + 3z^{-1}$ $\quad$ $y[n] = a_1 y[n-1] + a_2 y[n-2] + b_0 v_2[n]$

(a) When $a_1 = a_2 = 3$ and $b_0 = 4$, determine the system function, $H_3(z)$, for the last system. In addition, determine whether or not the last system is stable when it has these filter coefficients.

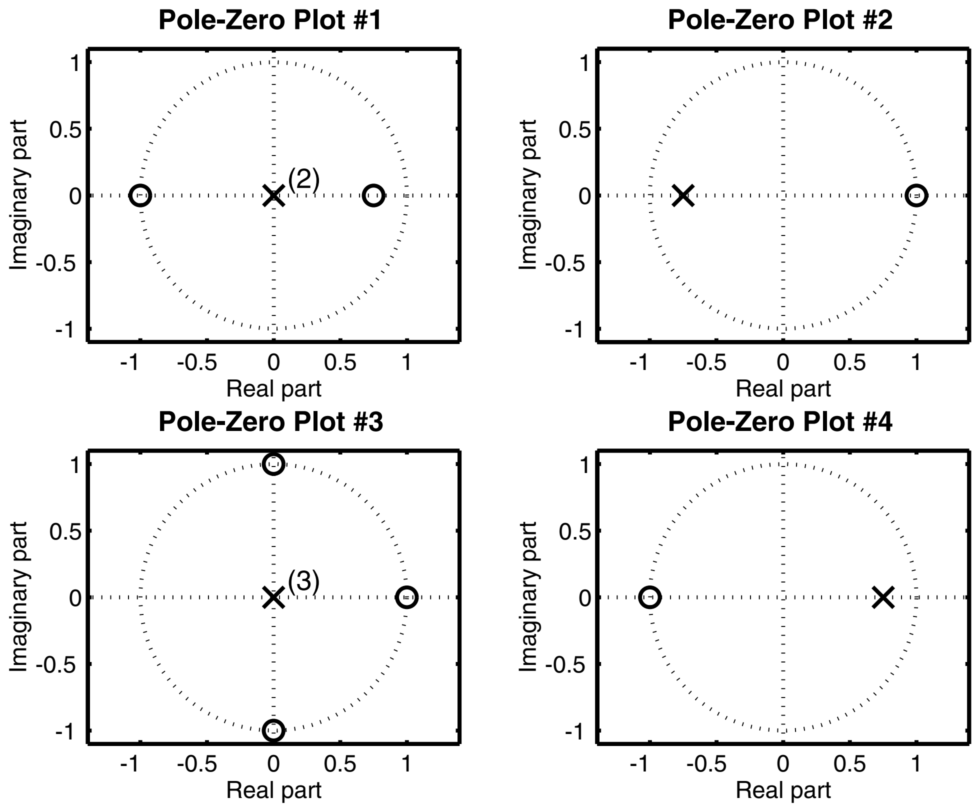
(b) When $a_1 = a_2 = 3$ and $b_0 = 4$, determine the system function, $H(z)$ for the overall cascaded system.

(c) Consider the impulse response $h[n]$ of the cascaded system, i.e., the output when the input is $x[n] = \delta[n]$. Determine values for $\{a_1, a_2, b_0\}$, as well as the time shift n_d, so that the impulse response will be

$$h[n] = 10\delta[n - n_d + 1] + 55\left(\tfrac{1}{2}\right)^{n-n_d} u[n - n_d]$$

Hint: use pole-zero cancelation to simply your work.

PROBLEM 9.6:



For each of the Pole-Zero plots plots above (#1–#4), determine which difference equation (below) defines the system.

#1:	#2:	#3:	#4:

Write some justification for each answer.

$\mathcal{S}_1: \quad y[n] = 0.4y[n-1] + x[n] + x[n-1]$

$\mathcal{S}_2: \quad y[n] = 0.75y[n-1] + x[n] + x[n-1]$

$\mathcal{S}_3: \quad y[n] = -0.75y[n-1] + x[n] - x[n-1]$

$\mathcal{S}_4: \quad y[n] = 0.75y[n-1] + x[n] - x[n-1]$

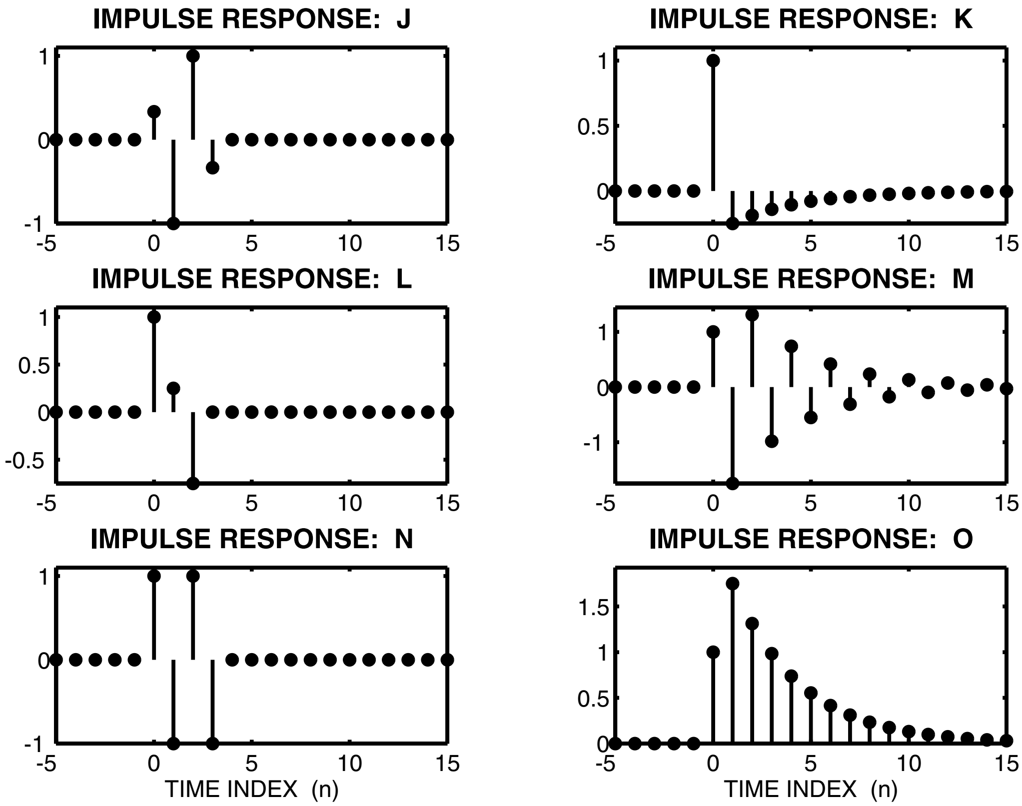
$\mathcal{S}_5: \quad y[n] = x[n] - x[n-1] + x[n-2]$

$\mathcal{S}_6: \quad y[n] = x[n] - x[n-1] + x[n-2] - x[n-3]$

$\mathcal{S}_7: \quad y[n] = x[n] + \frac{1}{4}x[n-1] - \frac{3}{4}x[n-2]$

$\mathcal{S}_8: \quad y[n] = \frac{1}{3}x[n] - x[n-1] + x[n-2] - \frac{1}{3}x[n-3]$

PROBLEM 9.7:



For each of the impulse-response plots above (J, K, L, M, N, O), determine which one of the following systems(specified by either an $H(z)$ or a difference equation) matches the impulse response.

J:	**K:**	**L:**	**M:**	**N:**	**O:**

Write some justification for each answer.

$\mathcal{S}_1$: $y[n] = 0.4y[n-1] + x[n] + x[n-1]$

$\mathcal{S}_2$: $H(z) = \dfrac{1+z^{-1}}{1-0.75z^{-1}}$

$\mathcal{S}_3$: $y[n] = -0.75y[n-1] + x[n] - x[n-1]$

$\mathcal{S}_4$: $H(z) = \dfrac{1-z^{-1}}{1-0.75z^{-1}}$

$\mathcal{S}_5$: $y[n] = x[n] - x[n-1] + x[n-2]$

$\mathcal{S}_6$: $H(z) = 1 - z^{-1} + z^{-2} - z^{-3}$

$\mathcal{S}_7$: $y[n] = x[n] + \frac{1}{4}x[n-1] - \frac{3}{4}x[n-2]$

$\mathcal{S}_8$: $H(z) = \frac{1}{3}(1-z^{-1})^3$