GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2006
Problem Set #10

Assigned: 31-Mar-06
Due Date: Week of 10-April-06

Reading: In *SP First*, Chapter 9: *Continuous-Time Signals & Systems*; Chapter 10: *Frequency Response*.

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 10.1*:
Express each of the following in a simpler form:

(a) $\frac{d}{dt}\left\{e^{-2t}\cos(3\pi t - \pi/4)u(t-1)\right\} =$

(b) $(\delta(t+2) - \delta(t-2)) * \sum_{\ell=0}^{3}(-1)^{\ell}\delta(t-2\ell) =$

(c) $\sin(\pi t/4)\sum_{\ell=0}^{3}(-1)^{\ell}\delta(t-2\ell) =$

(d) $\int_{-\infty}^{t+1} \delta(\tau - 2)d\tau =$

Note: use properties of the impulse signal $\delta(t)$ and the unit-step signal $u(t)$ to perform the simplifications. For example, recall

$$\delta(t) = \frac{d}{dt}u(t) \qquad \text{where} \qquad u(t) = \int_{-\infty}^{t}\delta(\tau)d\tau = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

Be careful to distinguish between multiplication and convolution. Convolution is denoted by a "star", as in $x(t) * \delta(t-2) = x(t-2)$ and multiplication is usually indicated as in $x(t)\delta(t-2) = x(2)\delta(t-2)$.

PROBLEM 10.2*:
Carry out the following convolutions, giving your answer as a simple formula <u>and a plot</u>.

(a) $u(t-4) * e^{-3t}u(t-2)$

(b) $q(t) * e^{-3t}u(t-2)$, where $q(t)$ is a finite-duration pulse: $q(t) = u(t-1) - u(t-4)$.

(c) $e^{-t}u(t) * e^{-t}u(t)$

PROBLEM 10.3*:

Carry out the following convolutions, giving your answer as a simple formula, or a plot.

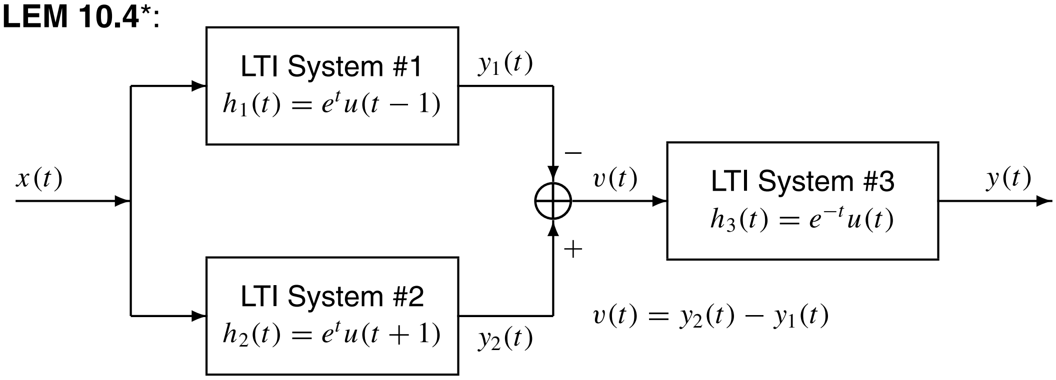
(a) $\delta(t-7) * e^{-3t}u(t-2)$

(b) $7\cos(\pi t - \pi/3) * e^{-3t}u(t)$

(c) $p_5(t-3) * p_7(t-1)$, where $p_\Delta(t)$ is a finite-duration pulse: $p_\Delta(t) = u(t) - u(t-\Delta)$.

(d) $u(t+2) * u(t-5)$

PROBLEM 10.4*:

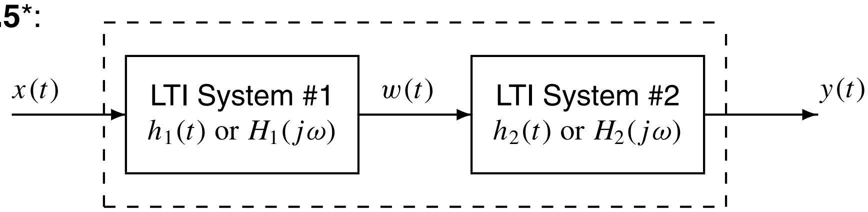


(a) Which systems (#1, #2, #3) are causal? Explain.

(b) Which systems (#1, #2, #3) are stable? Explain.

(c) Is the overall system a stable system? Explain using the overall impulse response $h(t)$ of the system.

PROBLEM 10.5*:



In the cascade of two LTI systems shown in the figure above, the first system has an impulse response

$$h_1(t) = \delta(t) + e^{-3t}u(t)$$

and the second system is described by its frequency response:

$$H_2(j\omega) = 100e^{-j\omega/10}$$

(a) Find the frequency response of the overall system.

(b) If the input signal is $x(t) = \pi\cos(3t+1)$, determine a simple formula for the output signal $y(t)$.

(c) Find the impulse response $h(t)$ of the overall system.

PROBLEM 10.6:

This is Problem 9.2 of Problem Set #9 of ECE2025 from the Fall of 2000. It is helpful to understand this problem. The best way to work it is to draw pictures with "typical" input and impulse response signals.

The impulse response of an LTI continuous-time system is such that $h(t) = 0$ for $t \le T_1$ and for $t \ge T_2$. By drawing appropriate figures as recommended for evaluating convolution integrals, show that if $x(t) = 0$ for $t \le T_3$ and for $t \ge T_4$ then $y(t) = x(t) * h(t) = 0$ for $t \le T_5$ and for $t \ge T_6$. In the process of proving this result you should obtain expressions for T_5 (the starting time) and T_6 (the ending time) in terms of T_1, T_2, T_3, and T_4.