Prob 10.1

(a) $\frac{d}{dt}\left\{ e^{-2t}\cos(3\pi t-\frac{\pi}{4})\,u(t-1)\right\}$

$$= u(t-1)\frac{d}{dt}\left\{ e^{-2t}\cos(3\pi t-\tfrac{\pi}{4})\right\} + e^{-2t}\cos(3\pi t-\tfrac{\pi}{4})\frac{d}{dt}u(t-1)$$

$$= -u(t-1)\,e^{-2t}\left\{2\cos(3\pi t-\tfrac{\pi}{4}) + 3\pi\sin(3\pi t-\tfrac{\pi}{4})\right\} - \frac{1}{\sqrt{2}}e^{-2}\delta(t-1)$$

(b) $[\delta(t+2)-\delta(t-2)] * \sum_{\ell=0}^{3}(-1)^{\ell}\delta(t-2\ell)$

$$= \sum_{\ell=0}^{3}\left[(-1)^{\ell}\delta(t-2\ell+2) - (-1)^{\ell}\delta(t-2\ell-2)\right]$$

$$= \delta(t+2)-\delta(t)-\delta(t-6)+\delta(t-8)$$

Note: $\delta(t-t_0)*\delta(t-t_1) = \delta(t-t_0-t_1)$

(c) $\sin(\frac{\pi t}{4})\sum_{\ell=0}^{3}(-1)^{\ell}\delta(t-2\ell) = 0\cdot\delta(t) - 1\cdot\delta(t-2) + 0\cdot\delta(t-4) - (-1)\delta(t-6)$

$$= -\delta(t-2)+\delta(t-6)$$

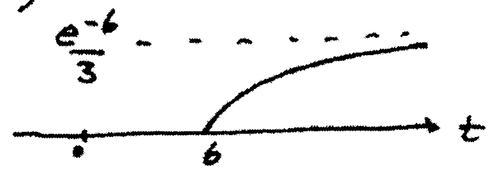
(d) $\int_{-\infty}^{t+1}\delta(\tau-2)\,d\tau = \int_{-\infty}^{t}\delta(\tau-1)\,d\tau = u(t-1)$

Prob 10.2

(a) $u(t-4) * e^{-3t}u(t-2)$

$$\int_{2}^{t-4} e^{-3\tau}d\tau = -\frac{e^{-3\tau}}{3}\Big|_{2}^{t-4} = \frac{e^{-6}-e^{-3t+12}}{3} \quad \text{for } t\geq 6$$

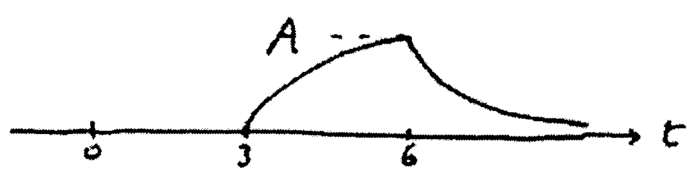
$$\therefore\ u(t-4) * e^{-3t}u(t-2) = \left(\frac{e^{-6}-e^{-3t+12}}{3}\right)u(t-6)$$



(b)
Let $f(t) = [u(t-1)-u(t-4)] * e^{-3t}u(t-2)$

$$u(t-1) * e^{-3t}u(t-2) = \frac{1}{3}(e^{-6}-e^{-3t+3})u(t-3)$$

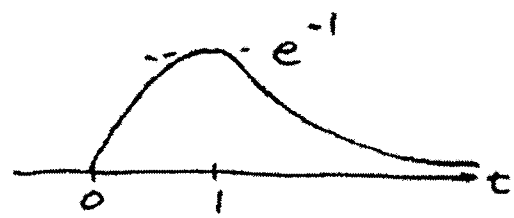
$$\therefore\ f(t) = \frac{1}{3}\left[(e^{-6}-e^{-3t+3})u(t-3) - (e^{-6}-e^{-3t+12})u(t-6)\right]$$



$A = \frac{1}{3}(e^{-6}-e^{-15})$

(c)
$e^{-t}u(t) * e^{-t}u(t)$

$$= \int_{0}^{t} e^{-t}d\tau = te^{-t} \quad \text{for } t\geq 0$$



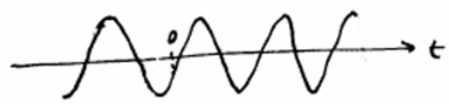
Prob 10.3

(a) $\delta(t-7) * e^{-3t}u(t-2) = e^{-3(t-7)}u(t-9)$

(b) $7\cos(\pi t - \frac{\pi}{3}) * e^{-3t}u(t) = f(t)$

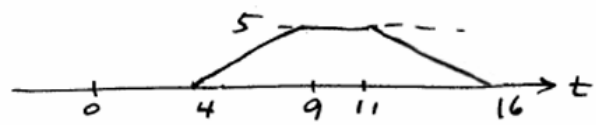
$e^{-3t}u(t) \longleftrightarrow \frac{1}{3+j\omega}$, at $\omega = \pi$, $\frac{1}{3+j\omega} = 0.2302 e^{-j0.8084}$

$f(t) = 7 \cdot 0.2302 \cos(\pi t - \frac{\pi}{3} - 0.8084) = 1.6114 \cos(\pi t - 1.8556)$

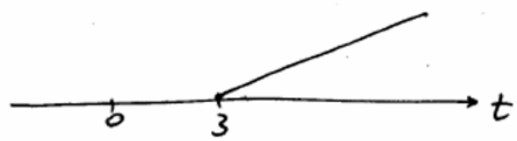


(c) $p_5(t-3) * p_7(t-1)$

$$= \begin{cases} t-4, & 4 \le t < 9 \\ 5, & 9 \le t < 11 \\ 16-t, & 11 \le t < 16 \\ 0 & \text{elsewhere} \end{cases}$$



(d) $u(t+2) * u(t-5) = (t-3)u(t-3)$



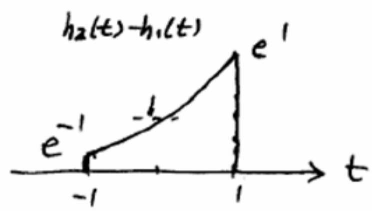
Prob 10.4

(a) #1 and #3 are causal because their impulse responses have zero values for all $t<0$; #2 is not causal because $h_2(t) \neq 0$ for some $t<0$.

(b) Only $h_3(t)$ is stable; $h_1(t)$ and $h_2(t)$ grow to infinity as $t \to \infty$.

(c) $h_2(t) - h_1(t) = e^t[u(t+1) - u(t-1)]$

Overall system is stable:



$h(t) = [h_2(t) - h_1(t)] * h_3(t)$

$= e^t[u(t+1) - u(t-1)] * e^{-t}u(t)$

$= e^{-t}\int_{-1}^{1} e^{2\tau}u(t-\tau)\,d\tau$

is essentially an exponentially decaying response.

Prob. 10.5

(a) $h_1(t) = \delta(t) + e^{-3t}u(t)$

$$H_1(j\omega) = 1 + \frac{1}{3+j\omega}, \quad H(j\omega) = H_1(j\omega)\,H_2(j\omega)$$
$$= \left(1+\frac{1}{3+j\omega}\right)\cdot 100\, e^{-j\frac{\omega}{10}}$$

(b) $x(t) = \pi\cos(3t+1)$

$\omega = 3$

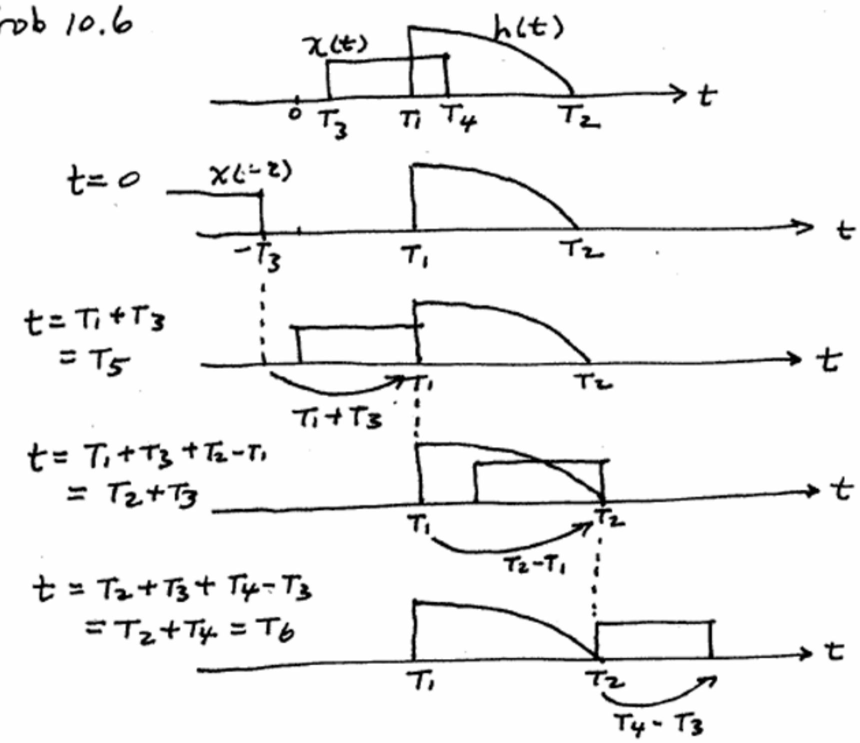
$$H(j3) = 117.85\, e^{-j0.442}$$

$$y(t) = 117.85\,\pi\cos(3t+1-0.442)$$
$$= 117.85\,\pi\cos(3t+0.558)$$

(c) $H_2(j\omega) = 100\, e^{-j\omega/10}$, $\quad h_2(t) = 100\,\delta(t-\frac{1}{10})$

$$h(t) = h_1(t) * h_2(t)$$
$$= 100\left[\delta(t-\tfrac{1}{10}) + e^{-3(t-\frac{1}{10})}u(t-\tfrac{1}{10})\right]$$

Prob 10.6



Therefore,

$\underline{T_5 = T_1 + T_3}$

$\underline{T_6 = T_2 + T_4}$