GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2006
Problem Set #12

Assigned: 17-Apr-06
Due Date: 28-Apr-06

This Homework can be turned at the last lecture on ***Friday, 28-April before Noon,*** or earlier that week.

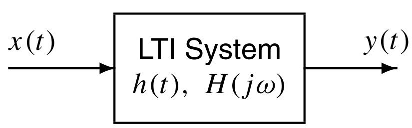
Final Exam will be given on Friday, 5-May at 2:50 PM; *Review on Thursday, 4-May at 6 PM.*
One page ($8\frac{1}{2} \times 11''$) of **handwritten** notes allowed. Calculators OK.

Reading: In *SP First*, Chapter 11: *Continuous-Time Fourier Transform*
Chapter 12: *Filtering, Modulation and Sampling,* (applications of the Fourier Transform).

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

PROBLEM 12.1*:



The impulse response of the above system is $h(t) = \dfrac{\sin(\omega_{\text{co}}t)\cos(80\pi t)}{\omega_{\text{co}}t}$,

and the input to this system is a periodic signal (with period $T_0 = 0.1$) given by the following equation:

$$x(t) = \sum_{n=-\infty}^{\infty} \pi\delta(t - n/10)$$

(a) Determine the Fourier transform $X(j\omega)$ of the input signal. Plot $X(j\omega)$ over the range $|\omega| \leq 100\pi$.

(b) For the case $\omega_{\text{co}} = 3\pi$, determine $H(j\omega)$ and plot $|H(j\omega)|$ on the same graph as $X(j\omega)$

(c) For the case $\omega_{\text{co}} = 3\pi$, use the plot in (b) to determine $Y(j\omega)$ and then $y(t)$, the output of the LTI system for the given input $x(t)$ above.

PROBLEM 12.2*:

Signal Processing First, Chapter **12**, Problem **4**, page 381–382. (Single-Sideband Modulation)

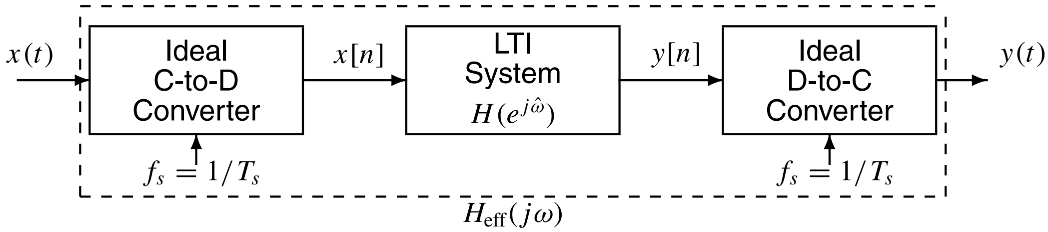
PROBLEM 12.3*:

Signal Processing First, Chapter **12**, Problem **5**, page 382. (Single-Sideband Modulation)

PROBLEM 12.4*:

This type of problem has often appeared on the Final Exam.

Consider the following system for discrete-time filtering of a continuous-time signal:

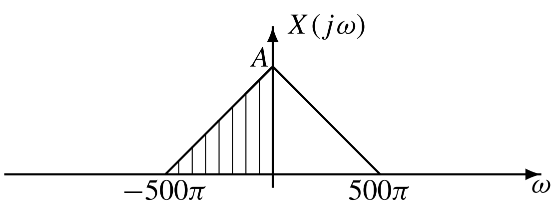


(a) Suppose that the discrete-time system is defined by the difference equation

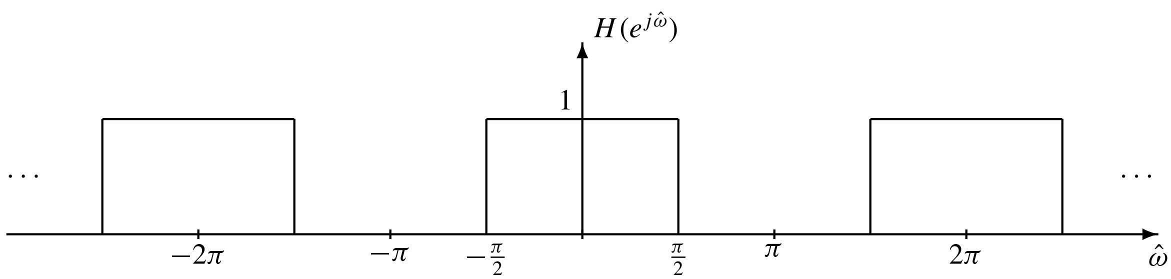
$$y[n] = -0.9y[n-2] + x[n] + x[n-2],$$

and the sampling rate of the two converters is $f_s = 1000$ samples/second. Determine an expression for $H_{\text{eff}}(j\omega)$, the overall effective frequency response of the above system. The IIR filter is a notch filter, so determine the analog frequency that is removed by the notch in the frequency response.

(b) In this part, the sampling frequency is a variable to be minimized. Assume that the input signal $x(t)$ has a bandlimited Fourier transform $X(j\omega)$ as depicted below. For this input signal, what is the *smallest* value of the sampling frequency f_s such that the Fourier transforms of the input and output satisfy the relation $Y(j\omega) = H_{\text{eff}}(j\omega)X(j\omega)$?



(c) Assume that the discrete-time system has frequency response $H(e^{j\hat{\omega}})$ defined by the following plot:

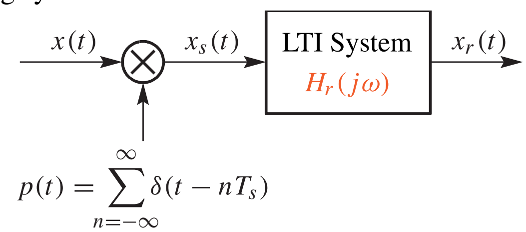


Determine the *smallest* value of the sampling rate such that the input signal (given in part (b)) passes through the lowpass filter unaltered; i.e., find the minimum f_s such that $y(t) = x(t)$, or equivalently, $Y(j\omega) = X(j\omega)$.

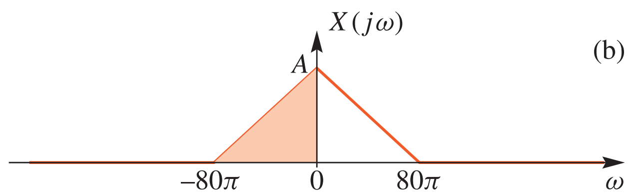
PROBLEM 12.5*:

The derivation of the Sampling Theorem involves the operations of impulse train sampling and reconstruction as shown in the following system:

(a)



(b)



A "typical" bandlimited Fourier transform of the input is also shown above.

(a) If $\omega_s = 2\pi/T_s = 100\pi$ in the above system and $X(j\omega)$ is as depicted above, plot the Fourier transform $X_s(j\omega)$ and show that aliasing occurs. There will be an infinite number of shifted copies of $X(j\omega)$, so indicate the general pattern versus ω.

(b) If $\omega_s = 2\pi/T_s = 200\pi$ in the above system and $X(j\omega)$ is as depicted above, plot the Fourier transform $X_s(j\omega)$ and show that aliasing does not occur. There will be an infinite number of shifted copies of $X(j\omega)$, so indicate the general pattern versus ω.

(c) For the conditions of part (b), i.e., $T_s = 1/100$, determine and sketch the Fourier transform of the output, $X_r(j\omega)$, if the frequency response of the LTI system is

$$H_r(j\omega) = \begin{cases} T_s & |\omega| \le \pi/T_s \\ 0 & |\omega| > \pi/T_s \end{cases}$$

PROBLEM 12.6:

The following Fourier transform pair states that the Fourier transform of a Gaussian is another Gaussian:

$$g(t) = e^{-t^2} \quad \longleftrightarrow \quad G(j\omega) = \sqrt{\pi}\, e^{-\omega^2/4}$$

Use this transform pair, to determine the Fourier transform of the following *differentiated Gaussian* signal:

$$x(t) = -100t\, e^{-25t^2}$$

In your derivation of $X(j\omega)$, list all the Fourier transform properties that have to be used.