GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

FINAL

DATE: 12-Dec-03 COURSE: ECE 2025

NAME: ______________________________ GT LOGIN: ____________

LAST, FIRST

Recitation Section: **Circle the day & time** when your Recitation Section meets:

L01:Tues-9:30 (G. Li) L02:Thur-9:30 (G-K. Chang)

L03:Tues-12:00 (G. Li) L04:Thur-12:00 (G-K. Chang)

L05:Tues-1:30 (M. Richards) L06:Thur-1:30 (T. Zhou)

L07:Tues-3:00 (M. Richards) L08:Thur-3:00 (T. Zhou)

L09:Tues-4:30 (Y. Altunbasak) L10:Thur-4:30 (G. Casinovi)

L11:Tues-6:00 (Y. Altunbasak) L13:Mon-3:00 (J. McClellan)

L14:Wed-3:00 (R. Butera) L16:Wed-4:30 (R. Butera)

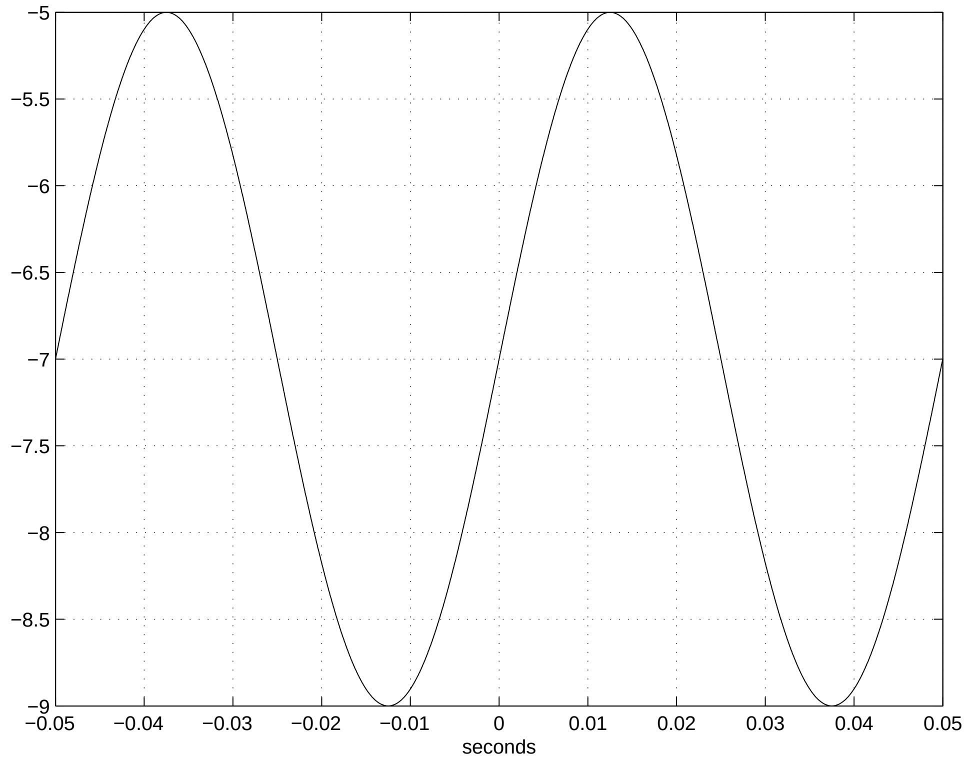
Savannah (G. AlRegib)

- Write your name on the front page ONLY. **Remove the Fourier and z-transform tables attached to the back of the test.**
- Closed book, but a calculator is permitted. However, one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes permitted. OK to write on both sides.
- Unless stated otherwise, justify your reasoning clearly to receive any partial credit. Explanations are also REQUIRED to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers, or write them in the boxes provided. If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*	*Problem*	*Value*	*Score*
1	20		6	20	
2	20		7	20	
3	20		8	20	
4	20		9	20	
5	20		10	20	

Problem F.1:

Consider a periodic continuous-time signal $x(t)$ whose graph between -0.05 seconds and 0.05 seconds looks like:



(a) What is the *DC value* of this $x(t)$?

(b) What is the *fundamental frequency* of the $x(t)$ in *radians per second*?

(c) What is the *minimum sampling rate* f_s in *Hertz* at which we could sample this signal without aliasing?

Problem F.2:

(a) The sum of sinusoids $x(t) = 130\cos(42\pi t) + 130\sin(42\pi t)$ can be simplified into an expression of the form $x(t) = A\cos(\omega_0 t + \phi)$. Find A, ω_0, and ϕ.

(b) Find the impulse response $h[n]$ of the system with frequency response $H(e^{j\hat{\omega}}) = 20j\ \sin(4\hat{\omega})e^{-j2\hat{\omega}}$. Is this system causal?

Problem F.3:

Consider the unit height triangle wave signal $x(t)$ with fundamental period $T = 0.2$ seconds (i.e., a fundamental frequency of 10π radians per second):

$$x(t) = \begin{cases} 2t/0.2 & \text{for } 0 \leq t < 0.1 \\ 2(0.2 - t)/0.2 & \text{for } 0.1 \leq t < 0.2 \end{cases}$$

The Fourier coefficients of this $x(t)$ are

$$a_k = \begin{cases} \frac{-2}{\pi k^2} & \text{for } k \text{ odd} \\ 1/2 & \text{for } k = 0 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find $X(j\omega)$, the Fourier transform of $x(t)$. Express your answer as a formula containing delta functions. (If you get stuck, remember the magic formula sheet is your friend!)

(b) Find c_k, the Fourier coefficients of

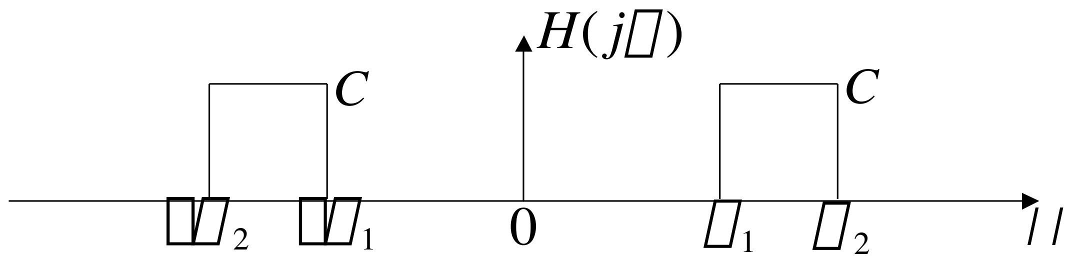
$$z(t) = \begin{cases} 16t/0.2 & \text{for } 0 \leq t < 0.1 \\ 16(0.2 - t)/0.2 & \text{for } 0.1 \leq t < 0.2 \end{cases},$$

in terms of the Fourier coefficients a_k.

So you don't have to go flipping back to the previous page, for your convenience, here's the Fourier coefficients of $x(t)$, the unit height triangle wave with fundamental period 0.2 seconds (fundamental frequency 10π radians per second) again:

$$a_k = \begin{cases} \frac{-2}{\pi k^2} & \text{for } k \text{ odd} \\ 1/2 & \text{for } k = 0 \\ 0 & \text{otherwise} \end{cases}$$

(c) Suppose we put $x(t)$ into a filter with the real-valued frequency response $H(j\omega)$ given by:



What value of the constant C (where $C > 0$), *range* of values of ω_1, and *range* of values of ω_2 would yield the output:

$$y(t) = -\cos(50\pi t) = \cos(50\pi t + \pi) = \cos(50\pi t - \pi)$$

Notice that the frequency of $y(t)$ corresponds the "fifth harmonic." Specify your ranges in terms of inequalities which must be satisfied. Give the widest ranges you can.

(Hint: usually, we give you the input $x(t)$ and the filter $H(j\omega)$ and ask you to find the output $y(t)$. Here, we instead give you the input $x(t)$ and the output $y(t)$, and you are basically being asked to find the filter $H(j\omega)$.)

Problem F.4:

The two subparts of this problem are completely independent of one another.

(a) When two finite-duration signals are convolved, the result is a finite-duration signal. In this subpart, let $h(t) = t^5[u(t-15) - u(t-9)]$ and $x(t) = 13[u(t-2) - u(t-14)]$.

Determine the length of the output signal $y(t) = x(t) * h(t)$.

(b) A scaled unit-step signal,

$$h(t) = 8u(t-2),$$

is convolved with a pulse signal,

$$x(t) = -[u(t-3) - u(t-9)].$$

The result of the convolution turns out to be:

$$y(t) = -8(t - T_{12})[u(t - T_{12}) - u(t - T_{23})] + Cu(t - T_{23}),$$

where the times T_{12} and T_{23} can be determined. (C is a constant.)

According to the formula above, the convolution integral produces an output signal that consists of three regions:

- Region #1: $y(t) = 0$ occurs when $t \le T_{12}$.
- Region #2: $y(t) = -8(t - T_{12})$ occurs when $T_{12} < t < T_{23}$.
- Region #3: $y(t) = C$ occurs when $t \ge T_{23}$. (C is a constant.)

Calculate the value of C, which is the amplitude in region #3, and the times T_{12} and T_{23}.

$C =$ ______	$T_{12} =$ ______	$T_{23} =$ ______

Problem F.5:

(a) Simplify this expression: $\int_{-\infty}^{t} \tau^2 \delta(\tau - 3) d\tau =$

(b) Find the Fourier transform of $x(t) = \sin(100\pi t)\cos(700\pi t)$. You may give your answer as a formula or as a *carefully labeled* sketch.

(c) Find the Fourier transform of $x(t) = \dfrac{\sin(100\pi t)}{\pi t}\cos(700\pi t)$. You may give your answer as a formula or as a *carefully labeled* sketch.

Problem F.6:

These problems would look like a horrible mess to someone form outside this class, but they are readily solved using your ninja ECE2025 knowledge. The patterns should look strangely familiar...

(a) Suppose $x(t)$ is given by

$$x(t) = \sum_{k=-\infty}^{\infty} \left\{ \frac{1}{5} \int_0^5 (5-u) \exp[-j(0.4)\pi k u] du \right\} \exp[j(0.4)\pi k t]$$

Sketch $x(t)$ for t between -5 and 10. Explain your reasoning. (Hint: In a way, the answer is sitting right in front of you.)

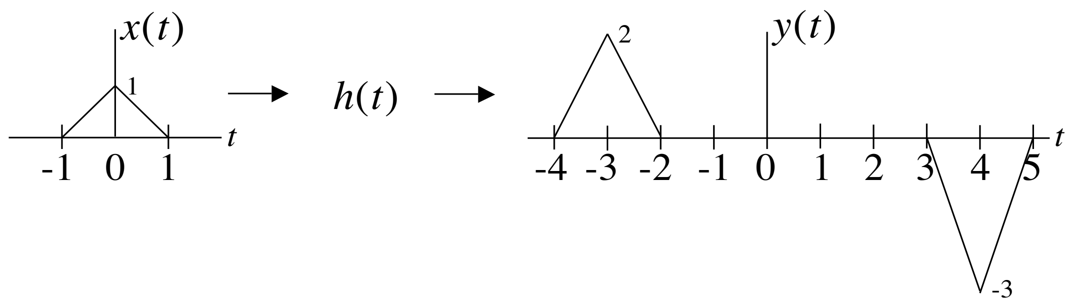
(b) Suppose $x(t)$ is given by

$$x(t) = \sum_{n=-\infty}^{\infty} \sin(0.07\pi n) \frac{\sin\left[\frac{\pi}{0.01}(t-(0.01)n)\right]}{\frac{\pi}{0.01}(t-(0.01)n)}$$

Find a simple expression for $x(t)$.

Problem F.7:

(a) Below, we see a signal $x(t)$ being imput to a mysterious LTI system with impulse response $h(t)$, yielding an output $y(t)$:



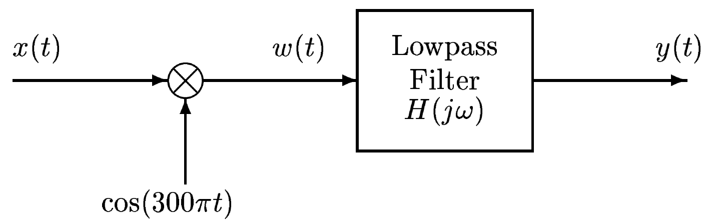
Find $h(t)$.

(b) Compute the convolution

$$\left[u\left(t+\frac{1}{300}\pi\right)+u\left(t-\frac{1}{300}\pi\right)\right] * \left[\frac{1}{\sqrt{3}}\cos(100\pi t+\pi/3)\right].$$

There is an Easy Way and a Hard Way to do this problem. Choose your path carefully!

Problem F.8:



In the above modulation/filtering system, assume that the input signal $x(t)$ has a bandlimited Fourier transform $X(j\omega)$, as depicted in Figure A below.

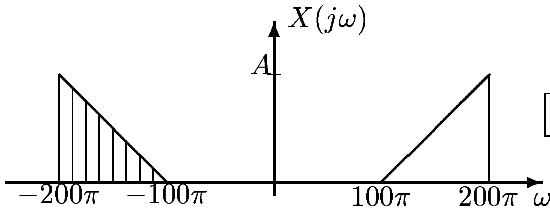
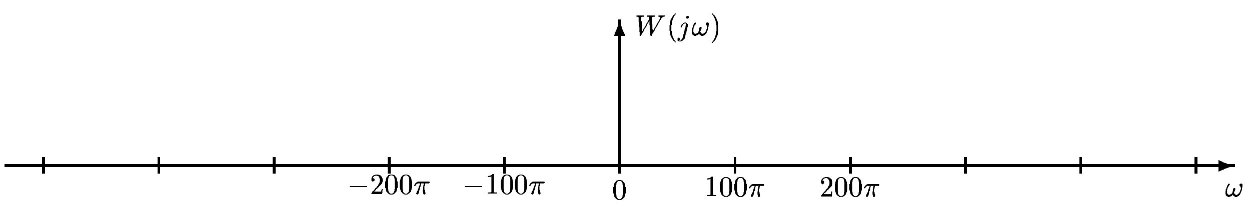


Figure A

(a) First give the general equation that expresses $W(j\omega)$, the Fourier transform of $w(t) = x(t)[\cos(300\pi t)]$, in terms of $X(j\omega)$.

$W(j\omega) =$ ________________________________

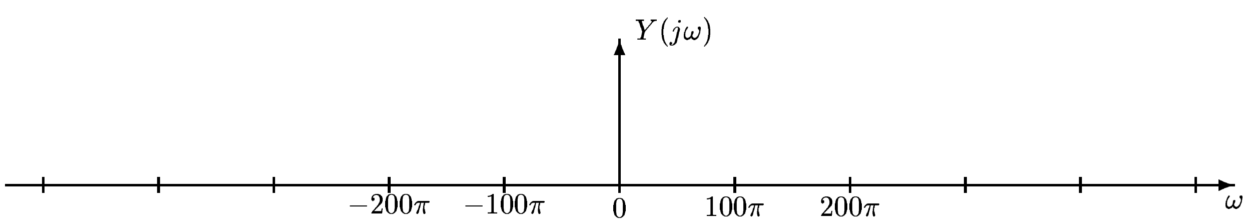
(b) Now **carefully** plot the Fourier transform $W(j\omega)$ for the specific input $x(t)$ whose Fourier transform $X(j\omega)$ is given above in Figure A. *Note that part of the Fourier transform $X(j\omega)$ is shaded. Mark the corresponding shaded region or regions in your plot of $W(j\omega)$, and be sure to carefully label both amplitudes and frequencies.*



(c) The frequency response of the lowpass filter is

$$H(j\omega) = \begin{cases} 2 & |\omega| \leq 100\pi \\ 0 & |\omega| > 100\pi \end{cases}$$

Plot the Fourier transform $Y(j\omega)$ below for the $X(j\omega)$ given in Figure A above. *Be sure to carefully label both amplitudes and frequencies and be sure to shade the region corresponding to the original shaded region in the input spectrum.*



Problem F.9:

(a) Suppose a student enters the following MATLAB code:

```
nn = 0:4480099;
xx = (3/pi) * cos(2*pi*0.8*nn + 14.92);
soundsc(xx,44800)
```

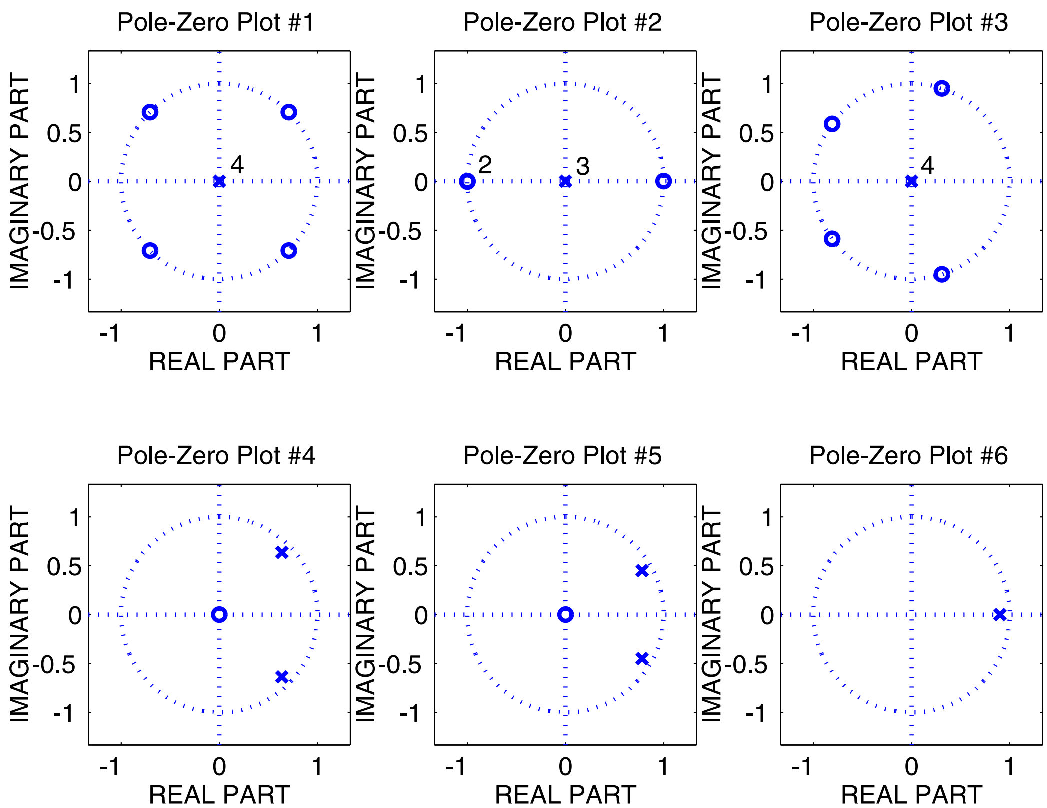
What analog frequency will be heard (in Hertz)? (Notice the folded aliasing!)

(b) Consider the following piece of MATLAB code:

```
tt = 0:(1/11025):6400;
xx = cos(2*pi*360*tt);
soundsc(xx,44100);
```

What is the duration (in seconds) of the final played tone? (We're supposing that the computer has so much memory that we don't need to worry about running out.)

Problem F.10:



For each of the pole-zero plots (#1, #2, $\cdots$, #6), determine which one of the following systems (specified by either $H(z)$, a difference equation, or a MATLAB statement) matches the pole-zero plot.

$\mathcal{S}_1$: `y = conv([1,0,0,0,1],x);`

$\mathcal{S}_2$: $y[n] = \sum_{k=0}^{5} x[n-k]$

$\mathcal{S}_3$: $y[n] = \sum_{k=0}^{4} x[n-k]$

$\mathcal{S}_4$: $y[n] = x[n] + 2x[n-1] + 3x[n-2] + 2x[n-3] + x[n-4]$

$\mathcal{S}_5$: $H(z) = \dfrac{z^{-1}}{1 - 1.273z^{-1} + 0.81z^{-2}}$

$\mathcal{S}_6$: `y = filter([0,1],[1,-1.559,0.81]);`

$\mathcal{S}_7$: $H(z) = (1 + z^{-1})^3$

$\mathcal{S}_8$: $H(z) = 1 + z^{-1} - z^{-2} - z^{-3}$

$\mathcal{S}_9$: $y[n] = 0.9y[n-1] + x[n-1]$