GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

Final Exam

DATE: 10-Dec-02 COURSE: ECE 2025

NAME: ______________________ STUDENT #: ______________

LAST, FIRST

Recitation Section: **Circle the day & time** when your Recitation Section meets:

NOTE: Failure to do so will result in a deduction of **five points** from your grade.

L01:Tues-9:30am (Casinovi)	L02:Thur-9:30am (Taylor)	L03:Tues-12:00pm (J. Michaels)
L04:Thur-12:00pm (Taylor)	L05:Tues-1:30pm (J. Michaels)	L06:Thur-1:30pm (T. Michaels)
L07:Tues-3:00pm (Lanterman)	L08:Thur-3:00pm (T. Michaels)	L09:Tues-4:30pm (Lanterman)
L10:Thur-4:30pm (Casinovi)	L13:Mon-3:00pm (Mersereau)	L14:Weds-3:00pm (Casinovi)
L15:Mon-4:30pm (Fan)	L17:Mon-6:00pm (Fan)	L20:Mon-1:30pm (Mersereau)
RPK (GTREP)		

- Write your name on the front page ONLY, and circle your recitation section (this is worth **FIVE POINTS**). **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted. However, one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes permitted. OK to write on both sides.
- Justify your reasoning clearly to receive any partial credit.
  Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
4	20	
5	20	

Problem	*Value*	*Score*
6	20	
7	20	
8	20	
TOTAL	160	

Problem F-02-Q.1.1:

For each of the following problems, **SIMPLIFY** your answer as much as possible.

(a) Find $Im\Big\{x[n]x^*[n-1]\Big\}$ when $x[n] = e^{j(0.5)\pi n}$ where $Im(\cdot)$ is the imaginary part.

$$Im\left\{ e^{j\frac{\pi}{2}n} \cdot e^{-j\frac{\pi}{2}(n-1)} \right\} = Im\left\{ e^{j\frac{\pi}{2}} \right\} = Im\{j\} = 1$$

(b) Evaluate the following expression: $|(3+2j)e^{j(0.4t^2)}|^2$.

$$\left|(3+2j)e^{j(0.4t^2)}\right|^2 = |3+2j|^2 = 9+4 = 13$$

(c) Evaluate the following derivative: $\frac{d}{dt}\Big[\cos(500\pi t - \pi/4)u(t)\Big]$.

USING THE PRODUCT RULE

$$\frac{d}{dt}\left[\cos\left(500\pi t - \frac{\pi}{4}\right)u(t)\right] = \cos\left(500\pi t - \frac{\pi}{4}\right)\delta(t) - 500\pi \sin\left(500\pi t - \frac{\pi}{4}\right)u(t)$$

$$= \cos\frac{\pi}{4}\,\delta(t) - 500\pi \sin\left(500\pi t - \frac{\pi}{4}\right)u(t)$$

$$= \frac{1}{\sqrt{2}}\,\delta(t) - 500\pi \sin\left(500\pi t - \frac{\pi}{4}\right)u(t)$$

(d) Evaluate the following integral: $\int_{-\infty}^{t} e^{-3\tau}\delta(\tau-2)d\tau$.

$$= \begin{cases} e^{-6}, & \text{IF } t>2 \\ 0, & \text{IF } t<2 \end{cases} = e^{-6}u(t-2)$$

(e) Evaluate the following convolution: $e^{-4jt} * \left[\delta(t-1) + e^{-t}u(t)\right]$.

$$e^{-j4t} * \delta(t-1) + e^{-j4t} * e^{-t}u(t) = e^{-j4(t-1)} + e^{-j4t} * e^{-t}u(t)$$

SINCE

$$e^{-t}u(t) \longleftrightarrow \frac{1}{j\omega+1}$$

THEN

$$e^{-j4t} * e^{-t}u(t) = \frac{1}{-j4+1} e^{-j4t} = \frac{1}{\sqrt{17}} e^{-j\tan^{-1}4} \cdot e^{-j4t}$$

$$\therefore \text{RESULT} = e^{-j4(t-1)} + \frac{1}{\sqrt{17}} e^{-j\tan^{-1}4} \cdot e^{-j4t}$$

Problem F-02-Q.1.2:
Which of the following signals are periodic? For those that are periodic, find the fundamental period, T_0?

(a) $x(t) = \sqrt{2}\cos(250\pi t - \pi/4) + 2e^{j350\pi t}$

Periodic? Yes or No. (Circle answer). If YES, $\boxed{T_0 = .04 \text{ SEC.}}$

$\omega_0 = 50\pi$

$T_0 = \frac{2\pi}{50\pi} = .04$

(b) $x(t) = 3 + \sin(199\pi t - \pi/2) + 2\cos(200\pi t - \pi/5)$

Periodic? Yes or No. (Circle answer). If YES, $\boxed{T_0 = 2 \text{ SEC.}}$

$\omega_0 = \pi$

$T_0 = \frac{2\pi}{\pi} = 2$

(c) $x(t) = \cos(\sqrt{2}\pi t + \pi/5) + \cos(\pi t - \pi/5)$

Periodic? Yes or No (Circle answer). If YES, $\boxed{T_0 = \quad}$

π AND $\sqrt{2}\pi$ HAVE NO COMMON DIVISOR.

(d) $x(t) = \sum_{k=-\infty}^{\infty} \frac{1}{|k|+1} e^{-j30\pi t(k)^2}$

Periodic? Yes or No. (Circle answer). If YES, $\boxed{T_0 = \frac{1}{15} \text{ SEC.}}$

$\omega_0 = 30\pi$

$T_0 = \frac{2\pi}{30\pi} = \frac{1}{15}$ SIGNAL IS COMPLEX.

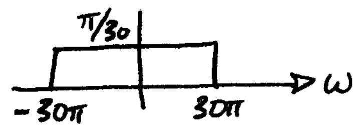
Problem F-02-Q.1.3:

(a) The sampling theorem is concerned with the reconstruction of a signal from its samples, and refers to something known as the Nyquist rate. Is it possible, in theory, to reconstruct the signal

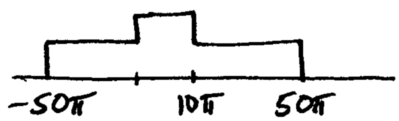
$$x(t) = \frac{\sin(30\pi t)}{30t}\cos(20\pi t)$$

from its samples, $x(nT_s)$? If so, what conditions are there on T_s? If not, explain why not.

$$\frac{\sin 30\pi t}{30t} \longleftrightarrow$$



$$x(t) \longleftrightarrow$$



SINCE $x(t)$ IS BANDLIMITED, IT IS POSSIBLE TO RECOVER IT FROM ITS SAMPLES, IF

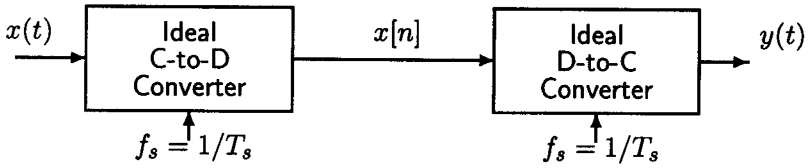
$$\frac{\pi}{50\pi} > T_s \implies T_s < 0.04 \text{ SEC.}$$

(b) Repeat part (a) for the signal

$$x(t) = u(t) - u(t-5)$$

THIS SIGNAL IS NOT BANDLIMITED. THEREFORE, IT CANNOT BE RECOVERED FROM ITS SAMPLES.

(c) Consider the following system for sampling and reconstruction of a continuous-time signal:



Suppose that the input to the C/D converter is

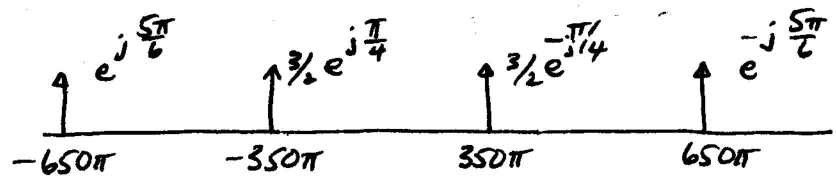
$$x(t) = 3\cos(350\pi t - \pi/4) + 2\sin(650\pi t - \pi/3)$$

What sampling frequency f_s will produce the signal

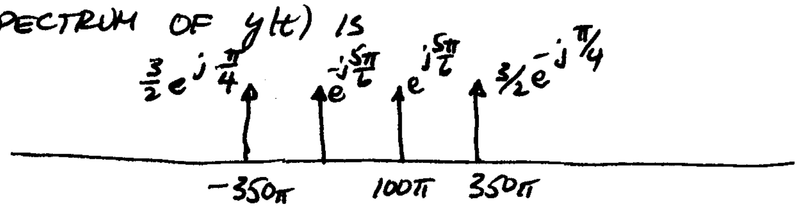
$$y(t) = 3\cos(350\pi t - \pi/4) - 2\sin(100\pi t + \pi/3)$$

at the output of the D/C converter? If none exist, write **NONE** and explain why.

THE SPECTRUM OF $x(t)$ IS



THE SPECTRUM OF $y(t)$ IS



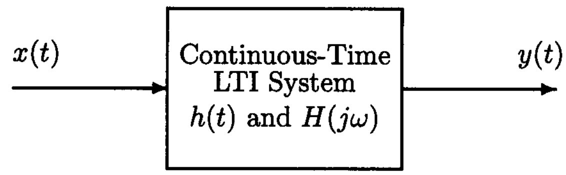
THE LINE AT 100π IS THE IMAGE OF THE LINE OF $x(t)$ LOCATED AT -650π.

$$\therefore\ 2\pi f_s = (650\pi - (-100\pi)) = 750\pi$$

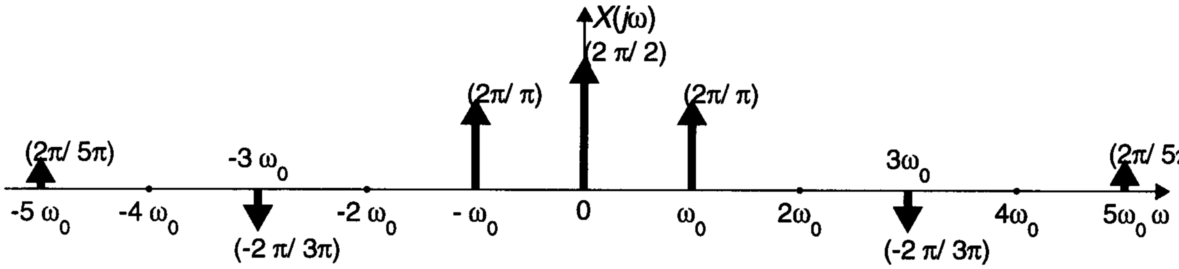
$$f_s = 375\ \text{Hz}.$$

SINCE THIS IS GREATER THAN 350 Hz., THERE WILL BE NO ALIASING OF THE COMPONENT WITH A FREQUENCY OF 175 Hz.

Problem F-02-Q.1.4:



The periodic input to the above LTI system has the Fourier transform $X(j\omega)$ drawn below:



where the dark arrows denote impulses. For the following outputs of the system, determine from the list below the frequency response of the system that could have produced that output when the input is the signal with the given Fourier transform, $X(j\omega)$. [*Circle the correct answer. There is only one correct answer in each case.*]

(a) $y(t) = \frac{1}{2} + \frac{4}{3\pi}\cos(\omega_0 t)$ (1) (2) (3) (4) (5) (6)

(b) $y(t) = x(t-1) - \frac{1}{2}$ (1) (2) (3) (4) (5) (6)

(c) $y(t) = x(t-1)$ (1) (2) (3) (4) (5) (6)

(d) $y(t) = \frac{1}{\pi}\cos(\omega_0 t)$ (1) (2) (3) (4) (5) (6)

(e) $y(t) = 2(\omega_0/\pi)\cos(\omega_0 t + \pi/2)$ (1) (2) (3) (4) (5) (6)

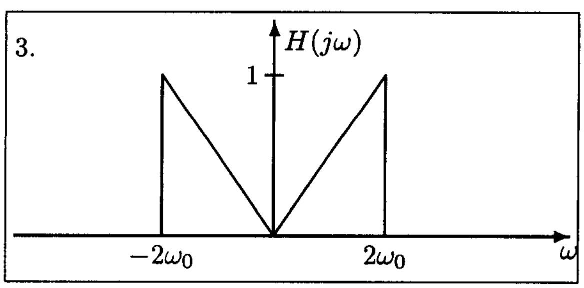
The possible filters are described by the following equations and graphs.[1]

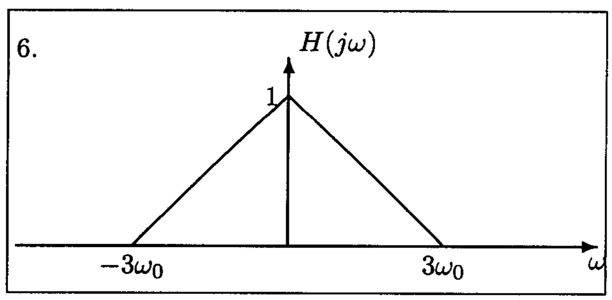
1. $H(j\omega) = \begin{cases} e^{-j\omega} & |\omega| < \omega_0/2 \\ 0 & |\omega| > \omega_0/2 \end{cases}$

2. $H(j\omega) = e^{-j\omega}$

4. $H(j\omega) = \begin{cases} j\omega & |\omega| < 3\omega_0/2 \\ 0 & |\omega| > 3\omega_0/2 \end{cases}$

5. $H(j\omega) = \begin{cases} 0 & |\omega| < \omega_0/2 \\ e^{-j\omega} & |\omega| > \omega_0/2 \end{cases}$

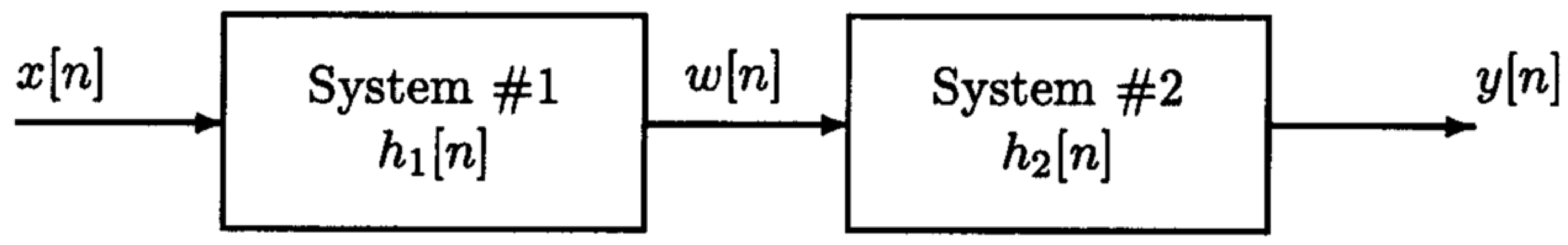




[1] *Some of these may not be needed as answers.*

Problem F-02-Q.1.5:

Shown in the figure below is a cascade of two linear time-invariant systems with impulse responses $h_1[n]$ and $h_2[n]$.



You are told that the unit sample impulse response of the first system is

$$h_1[n] = \delta[n] - \tfrac{1}{2}\delta[n-1]$$

(a) If $x[n] = \delta[n] + 2\delta[n-1] - \delta[n-3]$ find the output of the first filter, $w[n]$.

$$\begin{array}{ccccc} 1 & 2 & 0 & -1 & \\ 1 & -\frac{1}{2} & & & \\ \hline 1 & 2 & 0 & -1 & \\ & -\frac{1}{2} & -1 & 0 & \frac{1}{2} \\ \hline 1 & \frac{3}{2} & -1 & -1 & \frac{1}{2} \end{array}$$

$$w[n] = \delta[n] + \tfrac{3}{2}\delta[n-1] - \delta[n-2] - \delta[n-3] + \tfrac{1}{2}\delta[n-4]$$

(b) Suppose that the second system is designed so that when $x[n] = \delta[n]$, the output of the cascade is $y[n] = \delta[n]$. What is the unit sample response of the second system, $h_2[n]$?

WE MUST HAVE

$$h_1[n] * h_2[n] = \delta[n] \Longrightarrow H_1(z)H_2(z) = 1$$

$$H_1(z) = 1 - \tfrac{1}{2}z^{-1} \Longrightarrow H_2(z) = \frac{1}{1-\frac{1}{2}z^{-1}}$$

$$\therefore\ h_2[n] = \left(\tfrac{1}{2}\right)^n u[n]$$

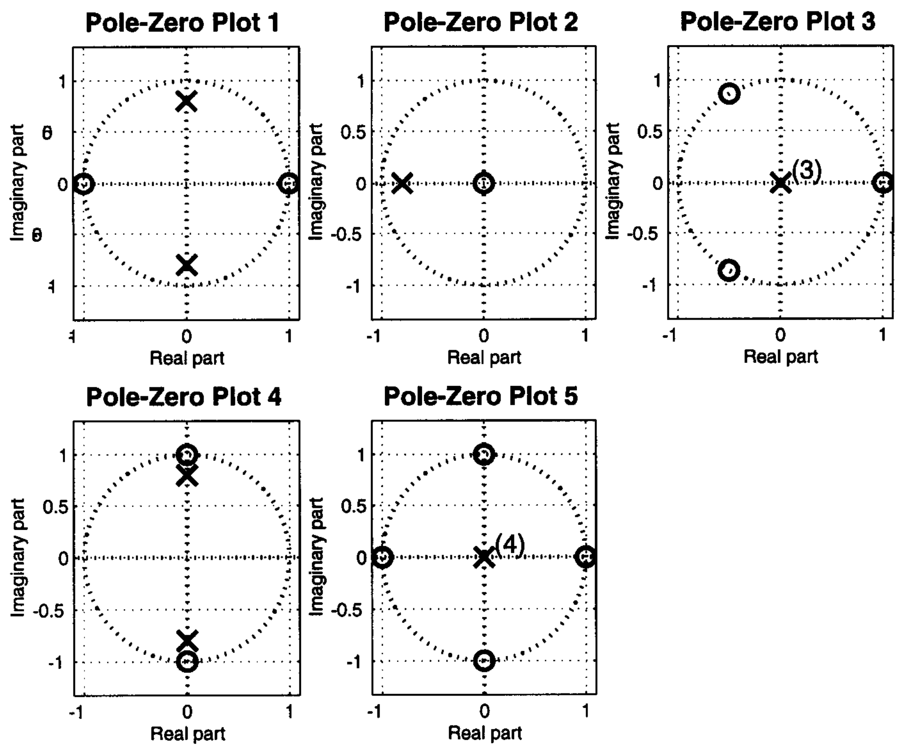
(c) If the unit sample response of the second system is $h_2[n] = \delta[n] + (0.8)^{n-1}u[n-1]$, what is the system function $H(z)$ of the cascade of the two systems? In other words, if $y[n] = x[n] * h[n]$, what is $H(z)$?

$$H(z) = H_1(z)H_2(z)$$

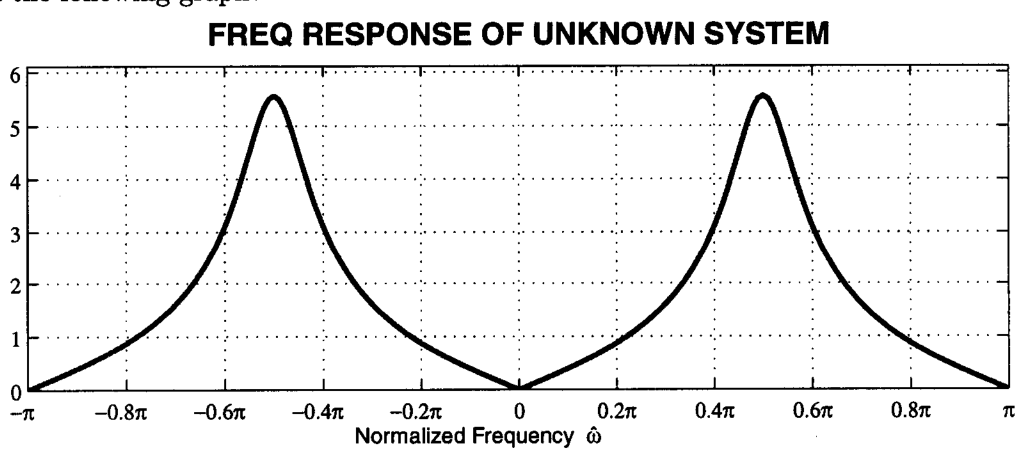
$$H_2(z) = 1 + \frac{z^{-1}}{1-0.8z^{-1}} = \frac{1+0.2z^{-1}}{1-0.8z^{-1}}$$

$$H(z) = \left(1 - \tfrac{1}{2}z^{-1}\right)\left[\frac{1+0.2z^{-1}}{1-0.8z^{-1}}\right]$$

Problem F-02-Q.1.6:



(a) Which of the above pole-zero plots represents the system whose frequency response is given in the following graph?



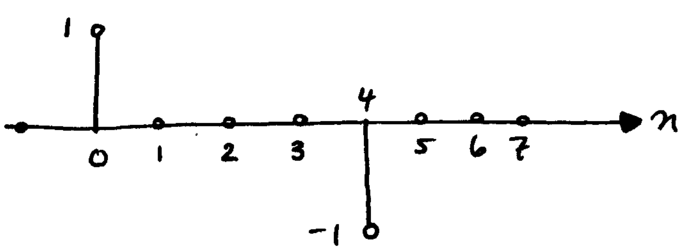
System Number = 1

(b) Now assume that each of the systems represented by the above pole-zero plots has impulse response $h_k[n]$, where k is the index shown in the title of the pole-zero plot. In the table below, indicate with an X which of the systems are FIR systems.

System #	1	2	3	4	5
FIR?			X		X

(c) Find the impulse response of a system that has a system function with a pole-zero plot as shown in **Pole-Zero Plot 5**, and make a **carefully labeled plot** of $h_5[n]$ versus n.

$$H_5(z) = (1-z^{-1})(1+z^{-1})(1+z^{-2})$$
$$= (1-z^{-2})(1+z^{-2}) = 1-z^{-4}$$



(d) Which of the above pole-zero plots represent a system for which

$$h_k[n] * \left[1+\cos(\pi n)\right] = 0$$

If more than one, write down all that apply. If none, write **NONE**.

System Number(s) = 1, 5

$H(j\omega)=0$ @ $\omega=0$ $\Rightarrow$ $H(z)$ HAS A ZERO AT $z=1$

$H(j\omega)=0$ @ $\omega=\pi$ $\quad$ $H(z)$ HAS A ZERO AT $z=-1$

Problem F-02-Q.1.7:

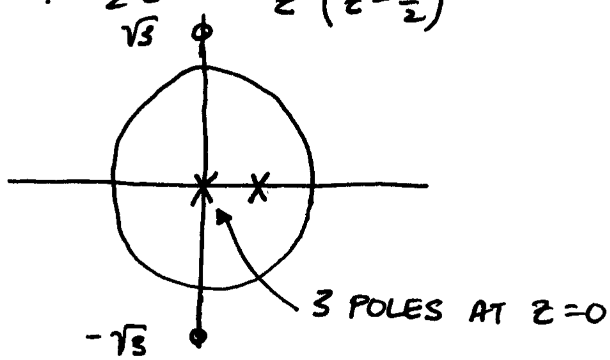
In each of the following Fourier transform problems, give your answer as a **simple formula** or a **carefully labeled plot.** $\boxed{\textbf{Explain}}$ the method used for each answer, e.g., for the transforms, state which property and transform pair you used.

(a) Make a carefully labeled plot of the poles and zeros of $H(z)$ when $h[n] = \Big[\delta[n] + 3\delta[n-2]\Big] * \Big[\left(\frac{1}{2}\right)^{n-2} u[n-2]\Big]$.

$$H(z) = \left[1 + 3z^{-2}\right]\left[\frac{z^{-2}}{1-\frac{1}{2}z^{-1}}\right]$$

CONV., DELAY, & LINEARITY PROPERTIES

$$= \frac{z^{-2} + 3z^{-4}}{1-\frac{1}{2}z^{-1}} = \frac{z^{2}+3}{z^{3}(z-\frac{1}{2})}$$



(b) Find $h[n]$ when $H(e^{j\hat{\omega}}) = \Big[2 + \cos(12\hat{\omega})\Big]e^{-2j\hat{\omega}}$

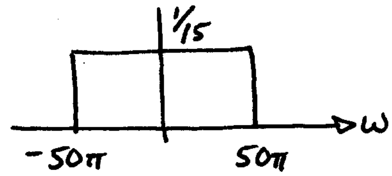
$$= \left[2 + \tfrac{1}{2}e^{+j12\hat{\omega}} + \tfrac{1}{2}e^{-j12\hat{\omega}}\right]e^{-j2\hat{\omega}}$$

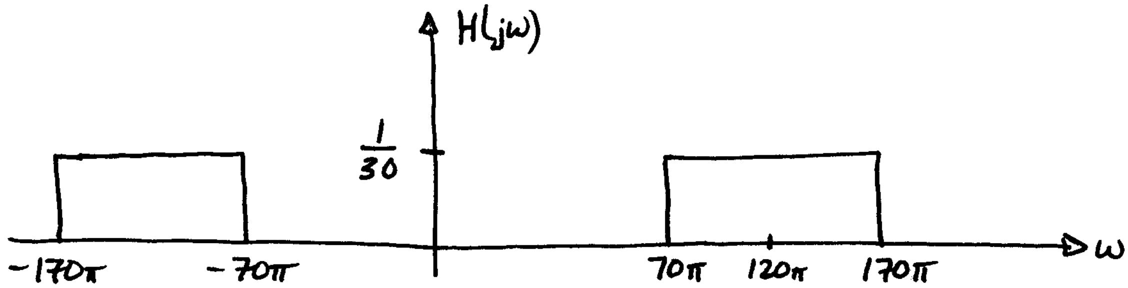
$$= \tfrac{1}{2}e^{j10\hat{\omega}} + 2e^{-j2\hat{\omega}} + \tfrac{1}{2}e^{-j14\hat{\omega}}$$

$$h[n] = \tfrac{1}{2}\delta[n+10] + 2\delta[n-2] + \tfrac{1}{2}\delta[n-14]$$

(c) Find $H(j\omega)$ when $h(t) = \dfrac{\sin(50\pi t)}{15\pi t}\cos(120\pi t)$.

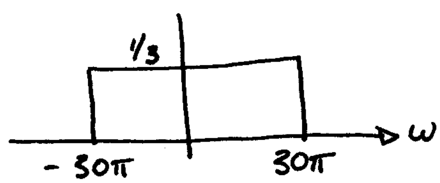
$$\frac{\sin(50\pi t)}{15\pi t} \longleftrightarrow$$





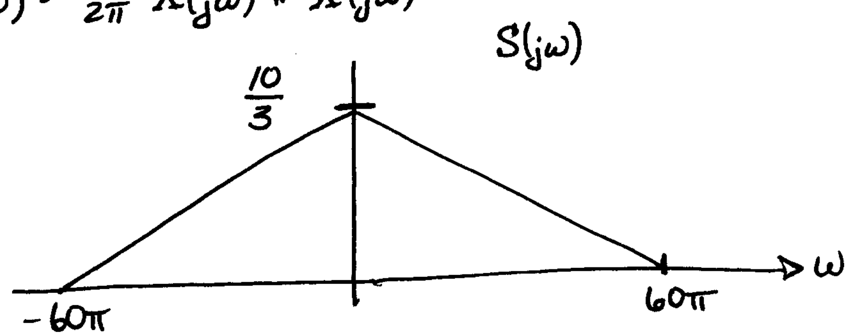
(d) Find $s(t)$ when $S(j\omega) = \left(\dfrac{\sin(30\pi t)}{3\pi t}\right)^2$.

$$\frac{\sin(30\pi t)}{3\pi t} \longleftrightarrow$$



$= X(j\omega)$

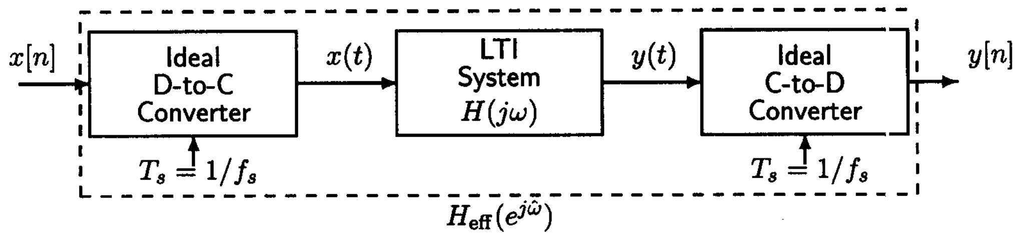
$$S(j\omega) = \frac{1}{2\pi} X(j\omega) * X(j\omega)$$



$$\frac{10}{3} = (60\pi)\left(\frac{1}{9}\right)\left(\frac{1}{2\pi}\right)$$

Problem F-02-Q.1.8:

A continuous-time linear time-invariant filter with an impulse response $h(t)$ is used to filter a discrete-time signal $x[n]$ as illustrated below.



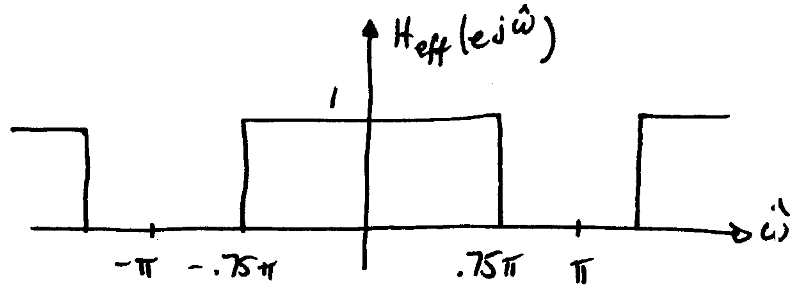
The continuous-time LTI system is an ideal lowpass filter with a cutoff frequency ω_c,

$$H(j\omega) = \begin{cases} 1 & ; \ |\omega| < \omega_c \\ 0 & ; \ \text{otherwise} \end{cases}$$

The overall system (dashed box) behaves as a discrete-time LTI system with an *effective* frequency response $H_{\text{eff}}(e^{j\hat{\omega}})$, i.e.,

$$Y(e^{j\hat{\omega}}) = H_{\text{eff}}(e^{j\hat{\omega}})X(e^{j\hat{\omega}})$$

(a) If $f_s = 1000$ Hz, and $\omega_c = 750\pi$, make a carefully labeled plot of the magnitude of the effective frequency response, $|H_{\text{eff}}(e^{j\hat{\omega}})|$. Make sure that you clearly label the x-axis and y-axis.



(b) What conditions, if any, must be imposed on $x[n]$, f_s, and ω_c in order for the output, $y[n]$, to be equal to the input, $x[n]$, for all n, i.e., $y[n] = x[n]$?

SINCE $x(t)$ WILL BE BANDLIMITED, $y(t)$ WILL BE APPROPRIATELY BANDLIMITED AND THERE ARE NO CONSTRAINTS ON f_s. THE FILTER, $H(j\omega)$, HOWEVER MUST PASS THE WHOLE SIGNAL.

$\therefore \quad \omega_c \geq \pi f_s$