GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

Final Exam

DATE: 10-Dec-01 COURSE: ECE 2025

NAME: ______________________________ gtxxx (login) # : ______________

LAST, FIRST

Recitation Section: **Circle the day & time** when your Recitation Section meets:

L01:Tues-9:30am (Bordelon)	L02:Thur-9:30am (Casinovi)	L03:Tues-12:00pm (Casinovi)
L04:Thur-12:00pm (Ji)	L05:Tues-1:30pm (Bordelon)	L06:Thur-1:30pm (Ji)
L07:Tues-3:00pm (Casinovi)	L08:Thur-3:00pm (Bordelon)	L09:Tues-4:30pm (Lanterman)
L10:Thur-4:30pm (Bordelon)	L11:Tues-6:00pm (Lanterman)	L13:Mon -3:00pm (Yezzi)
L14:Weds-3:00pm (Taylor)	L15:Mon -4:30pm (Verriest)	L16:Weds-4:30pm (Taylor)
L17:Mon -6:00pm (Verriest)	L20:Mon -1:30pm (McClellan)	RPK: (Abler)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted. However, one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes permitted. OK to write on both sides.
- JUSTIFY your reasoning clearly to receive any partial credit.
  Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
8	20	

Problem F-01-F.1:

(a) A continuous-time linear, time-invariant system has the impulse response

$$h(t) = \delta(t) + A\delta(t - \Delta).$$

Find the output of the system, $y(t)$, when the input is $x(t) = \sin(2000\pi t)$, $A = 1$, and $\Delta = 2.5 \times 10^{-4}$. Express your answer as a single sinusoid.

$y(t) =$

(b) Now assume that the input signal is $x(t) = \sin(2000\pi t)$, i.e., that the sinusoid is now zero for $t < 0$. Find values for A and Δ that will permit the new output $y(t)$ to be *exactly* three periods of a 1000Hz sine waveform and zero thereafter.

$A =$	$\Delta =$

Problem F-01-F.2:

A cascade of two FIR discrete-time systems is depicted by the following block diagram:

$x[n]$ → [LTI System #1 $H_1(z)$] → $w[n]$ → [LTI System #2 $H_2(z)$] → $y[n]$

The systems are defined by the following:

$$H_1(z) = (1 + z^{-2}) \quad \text{and} \quad h_2[n] = (-0.5)^{n-1}u[n-1].$$

(a) If the input to the first system is

$$x[n] = -\delta[n] + 2\delta[n-1] + \delta[n-2],$$

determine the output, $w[n]$, of the **first** system.

$w[n] =$

(b) Determine the system function $H(z)$ of the overall system.

$H(z) =$

(c) Determine the impulse response of the the overall system.

$h[n] =$

Problem F-01-F.3:

For each short question, pick a correct frequency[1] and enter its letter in the answer box[2]:

Frequency

(a) 8000 Hz

(b) 4000 Hz

(c) 2000 Hz

(d) 1600 Hz

(e) 1200 Hz

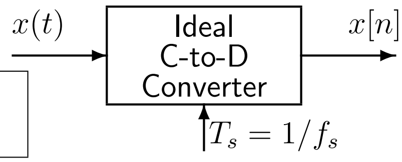
(f) 1000 Hz

(g) 800 Hz

(h) 500 Hz

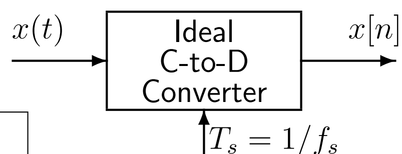
(i) 400 Hz

(a) If the output from an ideal C/D converter is $x[n] = 1000\cos(0.25\pi n)$, and the sampling rate is 8000 samples/sec, then determine one possible value of the input frequency of $x(t)$:



ANS =

(b) If the output from an ideal C/D converter is $x[n] = 1000\cos(0.25\pi n)$, and the input signal $x(t)$ defined by: $x(t) = 1000\cos(1800\pi t)$ then determine one possible value of the sampling frequency of the C-to-D converter:



ANS =

(c) Determine the Nyquist rate for sampling the signal $x(t)$ defined by: $x(t) = \Re e\{e^{j4000\pi t} + e^{j3000\pi t}\}$.

ANS =

[1] Some questions have more than one answer, but you only need to pick one correct answer from the list.
[2] It is possible to use an answer more than once.

Problem F-01-F.4:

For each of the following problems, **SIMPLIFY** your answer as much as possible.

(a) Evaluate $\Im m\{x[n]x^*[n-1]\}$ when $x[n] = e^{j(0.1)\pi n}$.

(b) Evaluate the following expression, $|e^{j\pi/6} - e^{-j\pi/6}| =$ ☐

(c) Evaluate the following integral, $\int_{-\infty}^{t} \delta(\tau - 7)\sin(\tau)e^{-j\pi\tau/2}\, d\tau$

(d) Evaluate the following integral, $\int_{-\infty}^{\infty} e^{-5t}u(t)e^{-j\omega t}dt.$

Problem F-01-F.5:

In each of the following problems, find the Fourier transform, or inverse Fourier transform. Give your answer as a simple formula or plot. (The symbol $*$ denotes covolution.)

(a) Find $Y(j\omega)$ when $y(t) = h(t) * x(t) = \cos(t) * \delta(t-2)$.

(b) Find $h(t)$ when $H(j\omega) = j\delta(\omega+1) * [\pi\delta(\omega) - \pi\delta(\omega-2)] + 2$.

(c) Find $v(t)$ when $V(j\omega) = \dfrac{j\omega}{9+3j\omega}$.

(d) Find $H(j\omega)$ when $h(t) = \dfrac{5\sin(2\pi(t-4))}{\pi(t-4)}$.

Problem F-01-F.6:

(a) The Fourier coefficients for the Fourier Series of a periodic signal $x(t)$ are defined using the following MATLAB code:

```
N = 2;
for k = -N:N
   if k == 0
      ak(k+N+1) = 0.5; % DC term
   else
      ak(k+N+1) = (j/(pi*k))*(1 - exp(j*k*pi));
   end
end
```

If the fundamental frequency is 20 Hz, sketch the Fourier transform of the signal, $X(j\omega)$.

(b) A cosine wave is mixed with a complex exponential function in a MATLAB simulation resulting in the output signal yy:

```
xx = cos(2*pi*1840*tt);
yy = xx.*exp(-j*2*pi*1160*tt);
```

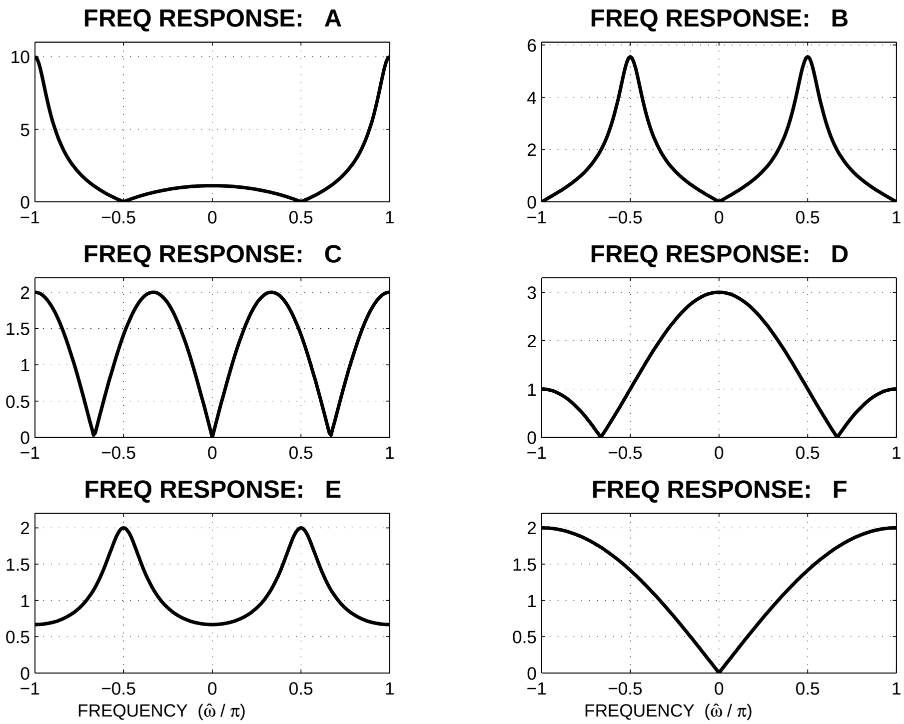
where tt is time in seconds. The result yy can be modeled as a continuous-time signal $y(t)$, which is then passed through a lowpass filter $H(j\omega)$ to produce the signal $s(t)$, where

$$H(j\omega) = \begin{cases} 1, & |\omega| < 2\pi(1000) \\ 0, & |\omega| > 2\pi(1000) \end{cases}$$

Write a formula for $s(t)$.

$s(t) =$

Problem F-01-F.7:



For each of the frequency response plots (A, B, C, D, E, F), determine which one of the following systems (specified by either an $H(z)$ or a difference equation) matches the frequency response (magnitude only). NOTE: the frequency axis is **<u>normalized</u>**; it is $\hat{\omega}/\pi$.

$\mathcal{S}_1: \quad H(z) = \dfrac{1 - z^{-2}}{1 + 0.64z^{-2}}$

$\mathcal{S}_2: \quad H(z) = 1 + z^{-1} + z^{-2}$

$\mathcal{S}_3: \quad H(z) = \dfrac{1 + z^{-1}}{1 - 0.9z^{-1}}$

$\mathcal{S}_4: \quad H(z) = z^{-1} - z^{-4}$

$\mathcal{S}_5: \quad y[n] = x[n] - x[n-1]$

$\mathcal{S}_6: \quad y[n] = -0.8y[n-1] + x[n] + x[n-2]$

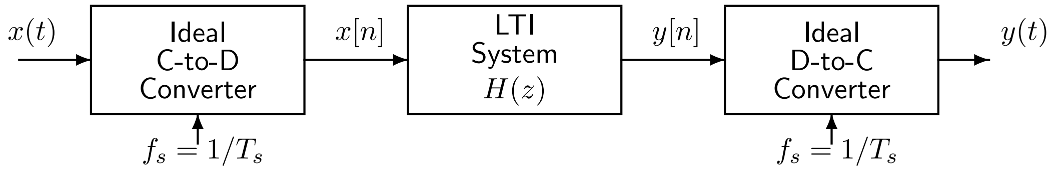
$\mathcal{S}_7: \quad y[n] = -0.5y[n-2] + x[n-1]$

$\mathcal{S}_8: \quad y[n] = 0.8y[n-1] + 0.5x[n]$

Mark your answers in the following table:

FREQUENCY RESPONSE	SYSTEM ($\mathcal{S}_{\#}$)	FREQUENCY RESPONSE	SYSTEM ($\mathcal{S}_{\#}$)
A		B	
C		D	
E		F	

Problem F-01-F.8:
Consider the following system for sampling, filtering, and reconstruction of a continuous-time signal:



where the LTI system function is $H(z) = 7z^{-2}$, and the continuous-time input signal is

$$x(t) = 3\cos(3000\pi t - 3\pi/5).$$

(a) Plot the complete frequency spectrum for $x[n]$ in the region $-\pi < \hat{\omega} \leq \pi$ for the case where $f_s = 2000$ samples/second.

(b) Determine an expression for the output $y(t)$ of this system for the input $x(t)$ indicated for the case where $f_s = 2000$ samples/second.

(c) Determine an expression for the output $y(t)$ for the same input signal if the sampling frequency is increased to $f_s = 4000$ samples/second.

Problem F-01-F.1:

(a) A continuous-time linear, time-invariant system has the impulse response

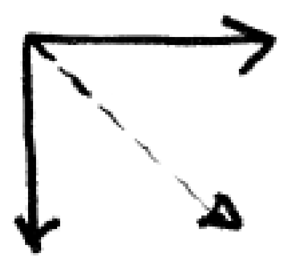
$$h(t) = \delta(t) + A\delta(t - \Delta).$$

Find the output of the system, $y(t)$, when the input is $x(t) = \sin(2000\pi t)$, $A = 1$, and $\Delta = 2.5 \times 10^{-4}$. Express your answer as a single sinusoid.

$$y(t) = \sin(2000\pi t) + \sin\left(2000\pi\left(t - \frac{1}{4000}\right)\right)$$

$$= \sin(2000\pi t) + \sin\left(2000\pi t - \frac{\pi}{2}\right)$$

$$= \sqrt{2}\sin\left(200\pi t - \frac{\pi}{4}\right)$$



$y(t) = \sqrt{2}\sin\left(2000\pi t - \frac{\pi}{4}\right)$

(b) Now assume that the input signal is $x(t) = \sin(2000\pi t)u(t)$, i.e., that the sinusoid is now zero for $t < 0$. Find values for A and Δ that will permit the new output $y(t)$ to be *exactly* three periods of a 1000 Hz sine waveform and zero thereafter.

3 PERIODS $\Rightarrow$ $\Delta = 3\left(\frac{1}{1000}\right) = 3 \times 10^{-3}$

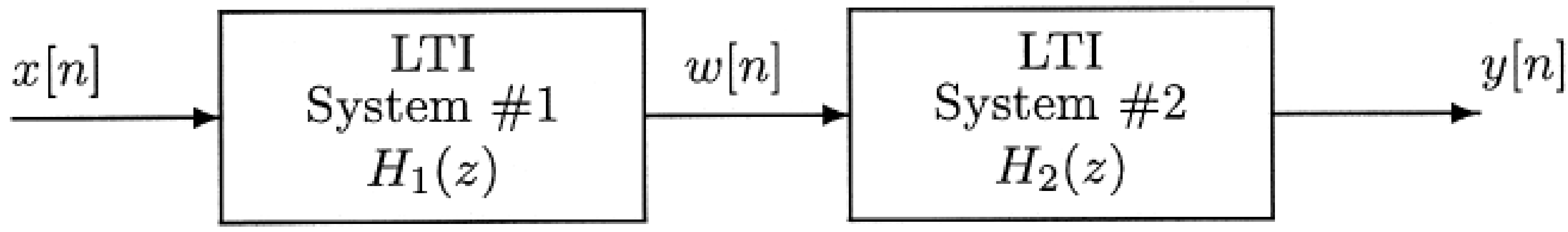
$A = -1$ WILL MAKE THE SUM ZERO FOR $t \geq \Delta$

$A = -1$

$\Delta = 3 \times 10^{-3}$

Problem F-01-F.2:

A cascade of two FIR discrete-time systems is depicted by the following block diagram:



The systems are defined by the following:

$$H_1(z) = (1 + z^{-2}) \quad \text{and} \quad h_2[n] = (-0.5)^{n-1}u[n-1].$$

(a) If the input to the first system is

$$x[n] = -\delta[n] + 2\delta[n-1] + \delta[n-2],$$

determine the output, $w[n]$, of the **first** system.

$$W(z) = (-1 + 2z^{-1} + z^{-2})(1 + z^{-2}) = -1 + 2z^{-1} + 2z^{-3} + z^{-4}$$

$w[n] = -\delta[n] + 2\delta[n-1] + 2\delta[n-3] + \delta[n-4]$

(b) Determine the system function $H(z)$ of the overall system.

$$H(z) = H_1(z)H_2(z)$$

$$= (1 + z^{-2})\left(\frac{z^{-1}}{1 + 0.5z^{-1}}\right) = \frac{z^{-1} + z^{-3}}{1 + 0.5z^{-1}}.$$

$H(z) = (z^{-1} + z^{-3})/(1 + 0.5z^{-1})$

(c) Determine the impulse response of the the overall system.

$h[n] = \left(-\frac{1}{2}\right)^{n-1}u[n-1] + \left(-\frac{1}{2}\right)^{n-3}u[n-3]$

Problem F-01-F.3:

For each short question, pick a correct frequency[1] and enter its letter in the answer box[2]:

Frequency

(a) 8000 Hz

(b) 4000 Hz

(c) 2000 Hz

(d) 1600 Hz

(e) 1200 Hz

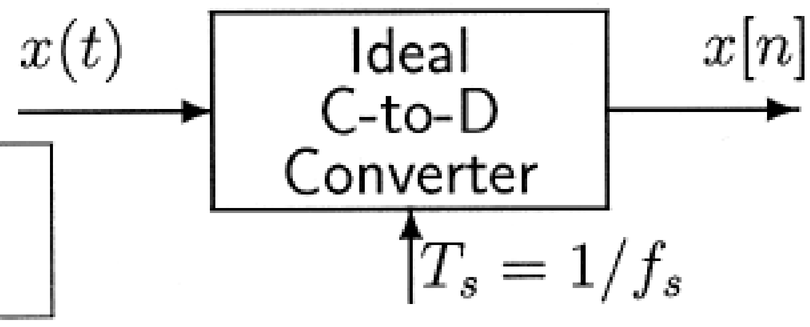
(f) 1000 Hz

(g) 800 Hz

(h) 500 Hz

(i) 400 Hz

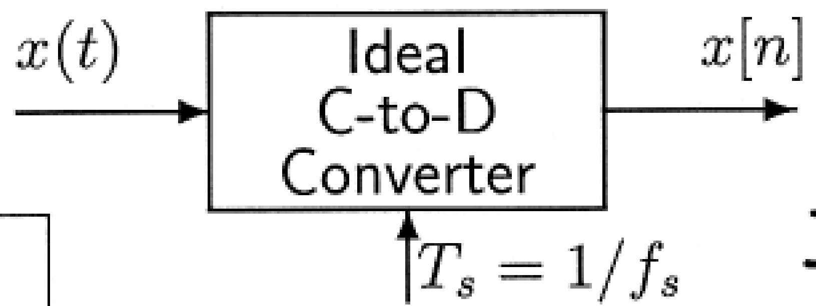
(a) If the output from an ideal C/D converter is $x[n] = 1000\cos(0.25\pi n)$, and the sampling rate is 8000 samples/sec, then determine one possible value of the input frequency of $x(t)$:



ANS = f

$$\frac{\pi}{4} = \frac{2\pi f}{8000} \Rightarrow f = \frac{\pi}{4} \cdot \frac{8000}{2\pi} = 1000$$

(b) If the output from an ideal C/D converter is $x[n] = 1000\cos(0.25\pi n)$, and the input signal $x(t)$ defined by: $x(t) = 1000\cos(1800\pi t)$ then determine one possible value of the sampling frequency of the C-to-D converter:



ANS = g

$$\pm\frac{\pi}{4} + 2\pi \ell = \frac{1800\pi}{f_s}$$

$$f_s = \frac{1800\pi}{\pi(2\ell \pm \frac{1}{4})} = \frac{7200}{8\ell+1}$$

(c) Determine the Nyquist rate for sampling the signal $x(t)$ defined by: $x(t) = \Re e\{e^{j4000\pi t} + e^{j3000\pi t}\}$.

ANS = b

$f_{max} = 2000$ Hz.

$f_s = 2f_{max}$

[1]Some questions have more than one answer, but you only need to pick one correct answer from the list.
[2]It is possible to use an answer more than once.

Problem F-01-F.4:

For each of the following problems, **SIMPLIFY** your answer as much as possible.

(a) Evaluate $\Im m\{x[n]x^*[n-1]\}$ when $x[n] = e^{j(0.1)\pi n}$.

$$= \Im m\{e^{j(0.1)\pi n} \cdot e^{-j(0.1)\pi(n-1)}\}$$

$$= \Im m\{e^{j(0.1)\pi}\} = \sin(0.1\pi)$$

(b) Evaluate the following expression, $|e^{j\pi/6} - e^{-j\pi/6}| =$ **1**

$$= 2j\sin\frac{\pi}{6} = j$$

$$|j| = 1$$

(c) Evaluate the following integral, $\int_{-\infty}^{t} \delta(\tau - 7)\sin(\tau)e^{-j\pi\tau/2}\,d\tau$

$$\sin(7)e^{-j\pi 7/2} = j\sin(7)$$

$$= j\sin(7)\int_{-\infty}^{t} \delta(\tau - 7)\,d\tau$$

$$= j\sin(7)\,u(t-7)$$

(d) Evaluate the following integral, $\int_{-\infty}^{\infty} e^{-5t}u(t)e^{-j\omega t}\,dt$.

THIS IS THE FOURIER TRANSFORM OF $x(t) = e^{-5t}u(t)$.

$\therefore$ $\text{ANS} = \dfrac{1}{5+j\omega}$

Problem F-01-F.5:

In each of the following problems, find the Fourier transform, or inverse Fourier transform. Give your answer as a simple formula or plot.

(a) Find $Y(j\omega)$ when $y(t) = h(t) * x(t) = \cos(t) * \delta(t-2)$.

← DELAY.

$$y(t) = \cos(t-2)$$

(b) Find $h(t)$ when $H(j\omega) = j\delta(\omega+1) * [\pi\delta(\omega) - \pi\delta(\omega-2)] + 2$.

$$= j\pi\delta(\omega+1) - j\pi\delta(\omega-1) + 2$$

$$h(t) = \sin t + 2\delta(t)$$

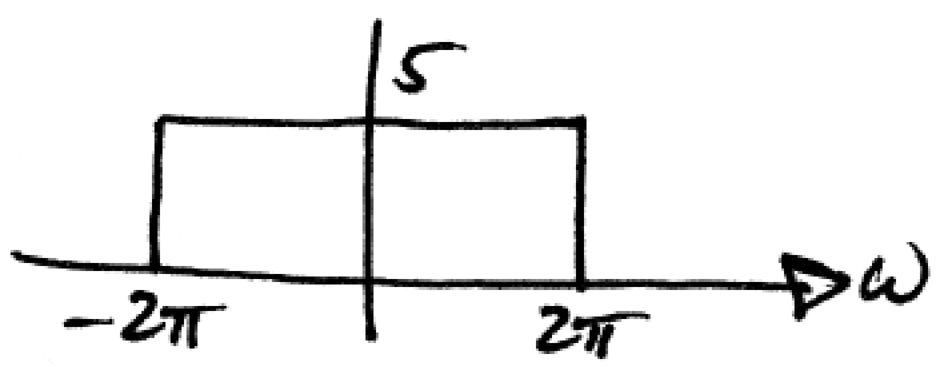
(c) Find $v(t)$ when $V(j\omega) = \frac{j\omega}{9+3j\omega}$. $= \frac{j\omega}{3}\left(\frac{1}{3+j\omega}\right)$

$$v(t) = \frac{1}{3}\frac{d}{dt}\left(e^{-3t}u(t)\right)$$

$$v(t) = -e^{-3t}u(t) + \frac{1}{3}\delta(t)$$

(d) Find $H(j\omega)$ when $h(t) = \frac{5\sin(2\pi(t-4))}{\pi(t-4)}$.

$$\frac{5\sin 2\pi t}{\pi t} \longleftrightarrow$$



$$H(j\omega) = \begin{cases} 5e^{-j4\omega}, & |\omega| < 2\pi \\ 0, & |\omega| > 2\pi \end{cases}$$

Problem F-01-F.6:

(a) The Fourier coefficients for the Fourier Series of a periodic signal $x(t)$ are defined using the following MATLAB code:

```
N = 2;
for k = -N:N
   if k == 0
      ak(k+N+1) = 0.5; % DC term
   else
      ak(k+N+1) = (j/(pi*k))*(1 - exp(j*k*pi));
   end
end
```

If the fundamental frequency is 20 Hz, sketch the Fourier transform of the signal, $X(j\omega)$.

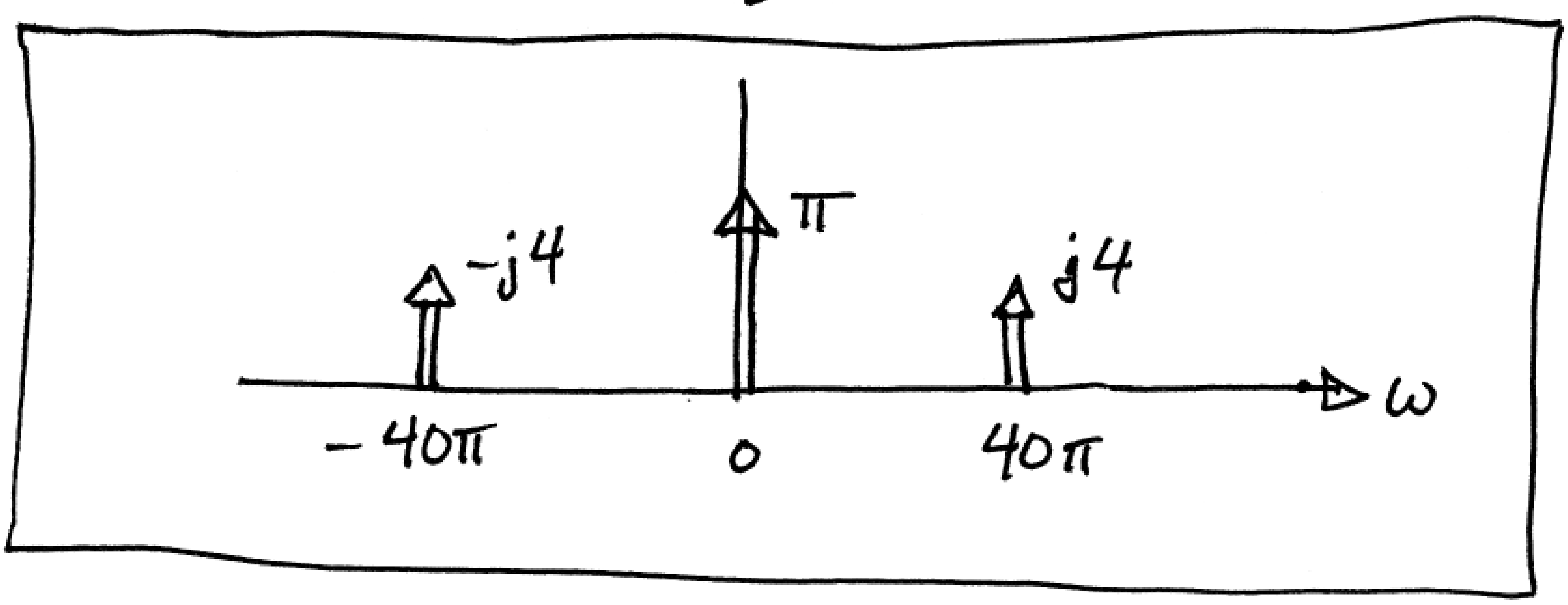
$$x(t) = \sum_k a_k e^{jk\omega_0 t} \qquad a_k = \begin{cases} \frac{1}{2}, & k=0 \\ j\frac{(1-e^{j\pi k})}{\pi k}, & k=\pm 1, \pm 2 \end{cases}$$

$$\omega_0 = 2\pi(20) = 40\pi$$

$$a_0 = \frac{1}{2}$$

$$a_1 = j\frac{2}{\pi} \qquad a_{-1} = -j\frac{2}{\pi}$$

$$a_2 = 0 \qquad a_{-2} = 0$$



(b) A cosine wave is mixed with a complex exponential function in a MATLAB simulation resulting in the output signal yy:

```
xx = cos(2*pi*1840*tt);
yy = xx.*exp(-j*2*pi*1160*tt);
```

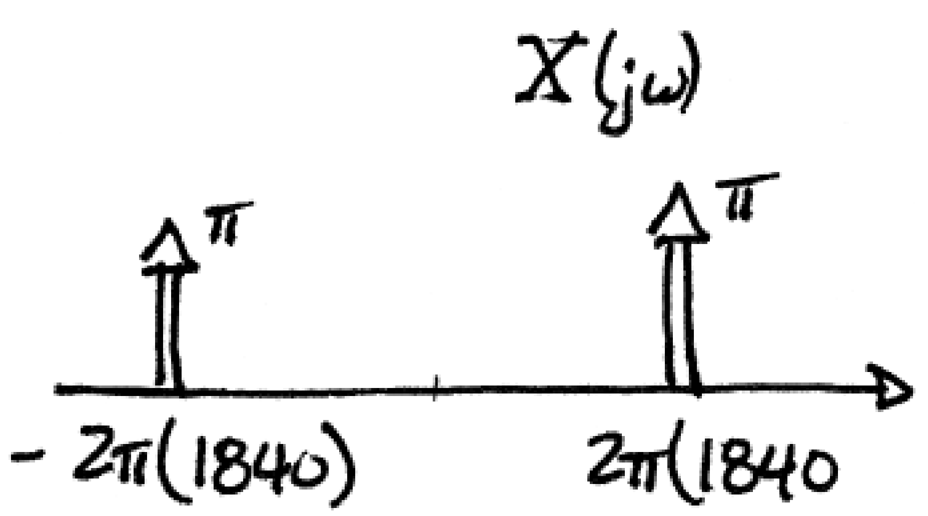
where tt is time in seconds. The result yy can be modeled as a continuous-time signal $y(t)$, which is then passed through a lowpass filter $H(j\omega)$ to produce the signal $s(t)$, where

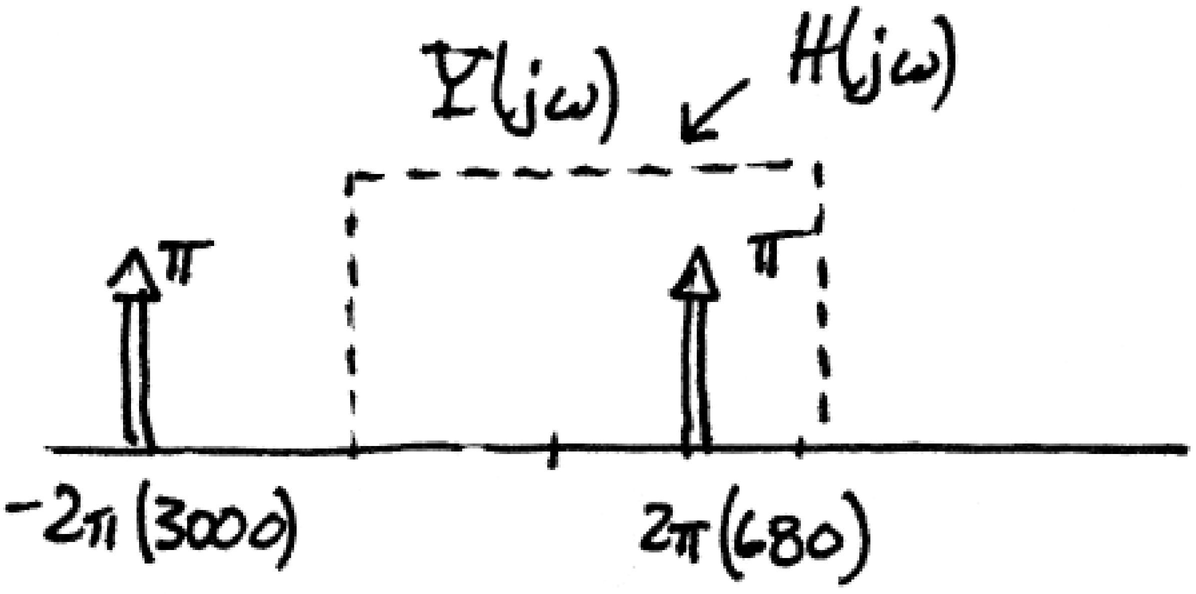
$$H(j\omega) = \begin{cases} 1, & |\omega| < 2\pi(1000) \\ 0, & |\omega| > 2\pi(1000) \end{cases}$$

Write a formula for $s(t)$.

$$y(t) = x(t)e^{-j2\pi(1160)t}$$

$$Y(j\omega) = X(j(\omega + 1160(2\pi)))$$

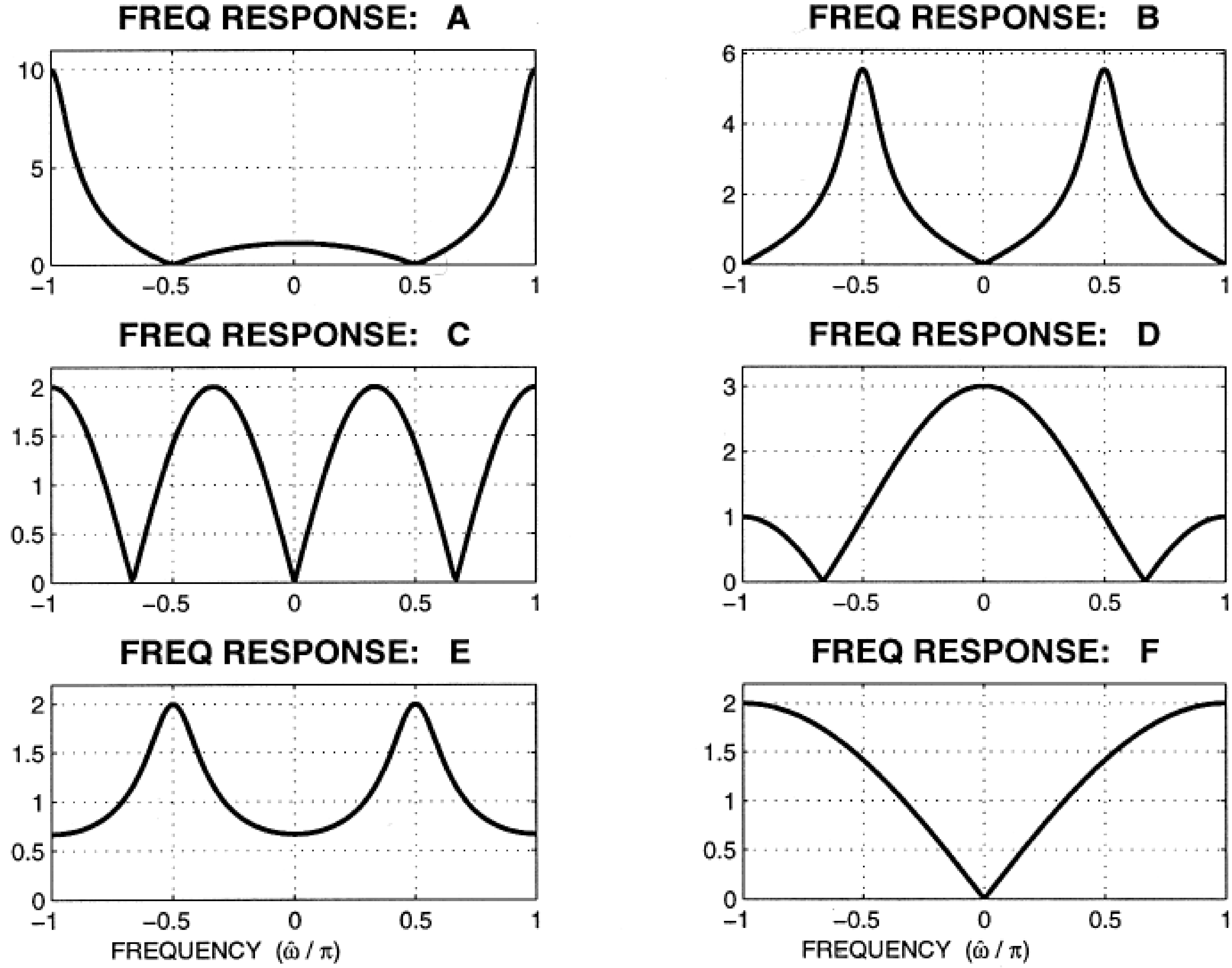




$$S(j\omega) = \pi\delta(\omega - 2\pi(680))$$

$s(t) = \frac{1}{2}e^{j2\pi(680)t}$

Problem F-01-F.7:



For each of the frequency response plots (A, B, C, D, E, F), determine which one of the following systems (specified by either an $H(z)$ or a difference equation) matches the frequency response (magnitude only). NOTE: the frequency axis is **normalized**; it is $\hat{\omega}/\pi$.

$\mathcal{S}_1$: $H(z) = \dfrac{1 - z^{-2}}{1 + 0.8z^{-2}}$

$\mathcal{S}_2$: $H(z) = 1 + z^{-1} + z^{-2}$

$\mathcal{S}_3$: $H(z) = \dfrac{1 + z^{-1}}{1 - 0.9z^{-1}}$

$\mathcal{S}_4$: $H(z) = z^{-1} - z^{-4}$

$\mathcal{S}_5$: $y[n] = x[n] - x[n-1]$

$\mathcal{S}_6$: $y[n] = -0.8y[n-1] + x[n] + x[n-2]$

$\mathcal{S}_7$: $y[n] = -0.5y[n-2] + x[n-1]$

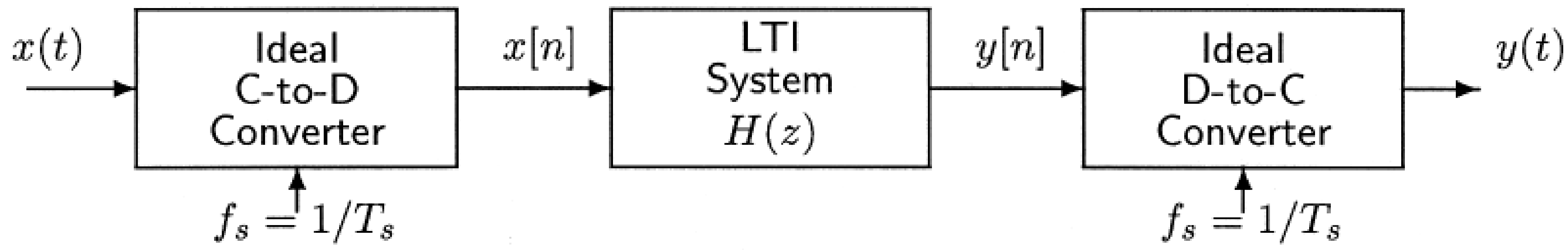
$\mathcal{S}_8$: $y[n] = 0.8y[n-1] + 0.5x[n]$

Mark your answers in the following table:

FREQUENCY RESPONSE	SYSTEM ($\mathcal{S}_{\#}$)	FREQUENCY RESPONSE	SYSTEM ($\mathcal{S}_{\#}$)
A	S_6	B	S_1
C	S_4	D	S_2
E	S_7	F	S_5

Problem F-01-F.8:

Consider the following system for sampling, filtering, and reconstruction of a continuous-time signal:



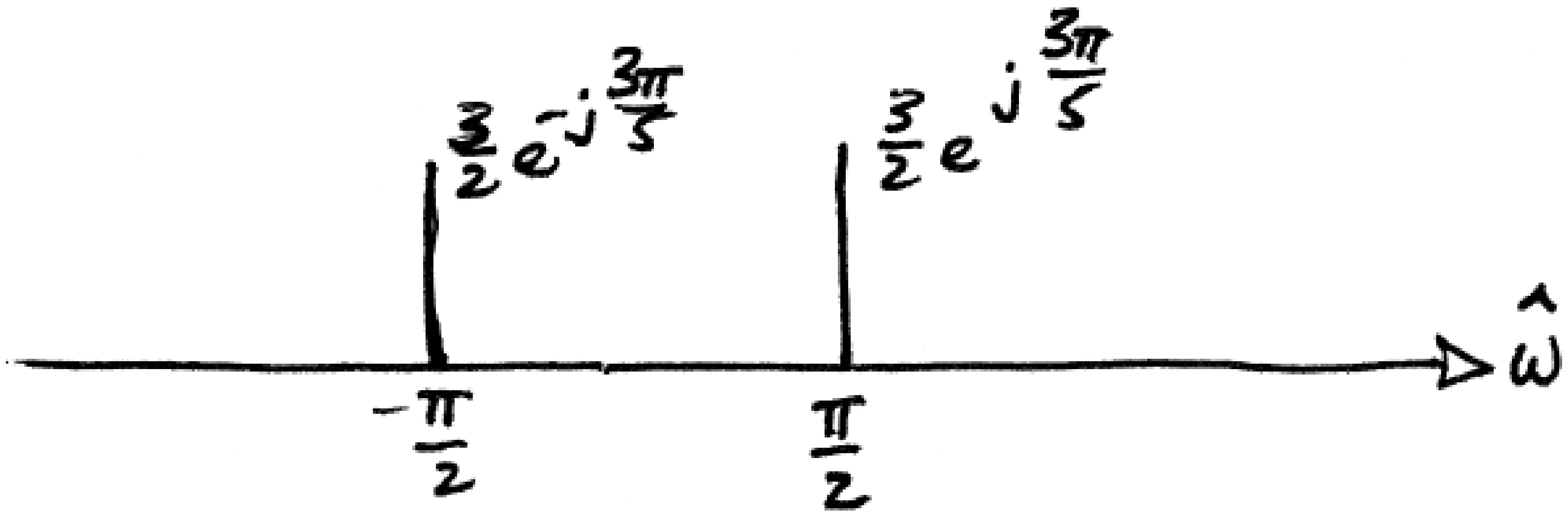
where the LTI system function is $H(z) = 7z^{-2}$, and the continuous-time input signal is

$$x(t) = 3\cos(3000\pi t - 3\pi/5).$$

(a) Plot the complete frequency spectrum for $x[n]$ in the region $-\pi < \hat{\omega} \leq \pi$ for the case where $f_s = 2000$ samples/second.

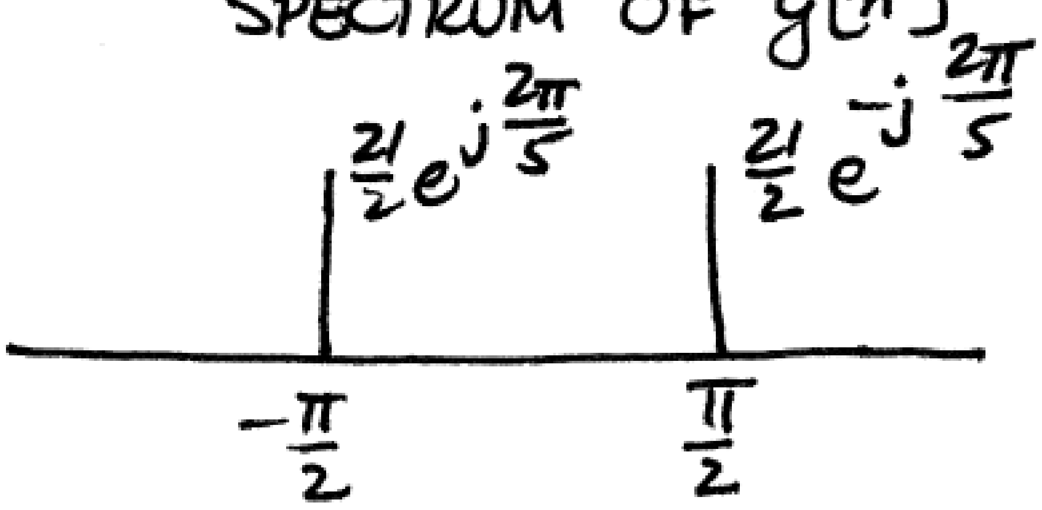
$$\hat{\omega} = \frac{\omega}{f_s} + 2\pi l$$

$$= \frac{3000\pi}{2000} = \frac{3\pi}{2} - 2\pi$$



(b) Determine an expression for the output $y(t)$ of this system for the input $x(t)$ indicated for the case where $f_s = 2000$ samples/second.

SPECTRUM OF $y[n]$



$$y(t) = 21\cos\left(1000\pi t - \frac{2\pi}{5}\right)$$

(c) Determine an expression for the outputs of the C/D, $x[n]$ and the D/C $y(t)$ for the same input signal if the sampling frequency is increased to $f_s = 4000$ samples/second.

$$\hat{\omega} = \frac{3000\pi}{4000} = \frac{3\pi}{4}$$

$$H(e^{j\hat{\omega}}) = 7e^{-j2\hat{\omega}}$$

$$= 7e^{-j\frac{3\pi}{2}} \quad @\ \hat{\omega} = \frac{3\pi}{4}$$

$$y(t) = 21\cos\left(3000\pi t - \frac{3\pi}{5} - \frac{3\pi}{2}\right)$$

$$y(t) = 21\cos\left(3000\pi t - \frac{\pi}{10}\right)$$