GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

FINAL EXAM

DATE: 30-Apr-04 COURSE: ECE-2025

NAME: ______________________________ GT #: ____________________

LAST, FIRST

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

	L03:Tues-Noon (Ji)		L04:Thur-Noon (Bordelon)
	L05:Tues-1:30pm (Ji)		L06:Thurs-1:30pm (Bordelon)
L11:M-3pm (McClellan)	L07:Tues-3pm (Fan)	L12:W-3pm (Bordelon)	
	L09:Tues-4:30pm (Fan)	L14:W-4:30pm (Bordelon)	GTREP: (Barnes)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning clearly to receive partial credit.
  Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	25	
2	25	
3	25	
4	25	
5	25	
6	25	
7	25	
8	25	

PROBLEM sp-04-F.1:

Circle the correct answer to each of the following short answer questions, and ***provide a short justification*** for your answer.

1. Pick the correct frequency response for the FIR filter: $y[n] = x[n-1] + x[n-3]$.
   (a) $2\cos(\hat{\omega})$
   (b) $2e^{-j1.5\hat{\omega}}\cos(1.5\hat{\omega})$
   (c) $2e^{-j1.5\hat{\omega}}\cos(0.5\hat{\omega})$
   (d) $2e^{-j2\hat{\omega}}\cos(\hat{\omega})$
   (e) none of the above

2. A sinusoidal signal $x(t)$ is defined by: $x(t) = \Re e\{(1+j)e^{j\pi t}\}$. When $x(t)$ is plotted versus time (t), its maximum value will be:
   (a) $A = 1$
   (b) $A = 1 + j$
   (c) $A = \sqrt{2}$
   (d) $A = 0$
   (e) none of the above

3. Determine the amplitude (A) and phase (ϕ) of the sinusoid that is the sum of the following three sinusoids: $10\cos(6t+\pi/2) + 7\cos(6t-\pi/6) + 7\cos(6t+7\pi/6)$,
   (a) $A = 10$ and $\phi = \pi/2$.
   (b) $A = 7$ and $\phi = \pi/2$.
   (c) $A = 0$ and $\phi = 0$.
   (d) $A = 3$ and $\phi = \pi/2$.
   (e) $A = 24$ and $\phi = \pi/2$.

4. Evaluate the complex number $z = \dfrac{j^{-1} - j^{-2}}{j^{-3} + j^{-4}}$.
   (a) $z = 0$
   (b) $z = j$
   (c) $z = -j$
   (d) $z = 1$
   (e) $z = -1$

PROBLEM sp-04-F.2:

For each of the following time-domain signals, select the correct match from the list of Fourier transforms below. ***Write each answer in the box provided.*** (The operator $*$ denotes convolution.)

(a) $x(t) = \dfrac{\sin(5\pi t)}{\pi t} * \delta(t-4)$

[ ]

(b) $x(t) = \dfrac{\sin(5\pi t)}{\pi t}$

[ ]

(c) $x(t) = 2\dfrac{\sin(\pi t)}{\pi t} * \cos(4\pi t)$

[ ]

(d) $x(t) = \dfrac{\sin(5\pi t)}{\pi t} * \dfrac{\sin(3\pi t)}{\pi t}$

[ ]

(e) $x(t) = \displaystyle\int_{-\infty}^{\infty} \delta(\lambda+3)\delta(t-\lambda-7)d\lambda$

[ ]

(f) $x(t) = 2\dfrac{\sin(\pi t)}{\pi t}\cos(4\pi t)$

[ ]

(g) $x(t) = \dfrac{d}{dt}\left\{\dfrac{\sin(5\pi t)}{\pi t}\right\}$

[ ]

Each of the time signals above has a Fourier transform that can be found in the list below.

[0] $X(j\omega) = u(\omega+5\pi) - u(\omega-5\pi)$

[1] $X(j\omega) = e^{-j4\omega}[u(\omega+5\pi) - u(\omega-5\pi)]$

[2] $X(j\omega) = u(\omega+3\pi) - u(\omega-3\pi)$

[3] $X(j\omega) = j\omega[u(\omega+5\pi) - u(\omega-5\pi)]$

[4] $X(j\omega) = e^{-j4\omega}$

[5] $X(j\omega) = 0$

[6] $X(j\omega) = e^{-j4\omega}[j\pi\delta(\omega+\pi) - j\pi\delta(\omega-\pi)]$

[7] $X(j\omega) = [j\pi u(\omega+\pi) - j\pi u(\omega-\pi)] * \delta(\omega-4\pi)$

[8] $X(j\omega) = 2u(\omega+\pi) - 2u(\omega-\pi) + \pi\delta(\omega+4\pi) + \pi\delta(\omega-4\pi)$

[9] $X(j\omega) = u(\omega+5\pi) - u(\omega+3\pi) + u(\omega-3\pi) - u(\omega-5\pi)$

PROBLEM sp-04-F.3:

The three subparts of this problem are completely independent of one another.

(a) When two finite-duration signals are convolved, the result is a finite-duration signal. In this subpart,

$$h(t) = t^2[u(t-17) - u(t-10)] \quad \text{and} \quad x(t) = 9[u(t-7) - u(t-30)]$$

Determine starting and ending times of output signal $y(t) = x(t) * h(t)$, i.e., find T_1 and T_2 so that $y(t) = 0$ for $t < T_1$ and for $t > T_2$.

$T_1 =$	sec.	$T_2 =$	sec.

(b) A system has the following impulse response:

$$h(t) = u(t-2)$$

Determine whether the system is stable and causal (circle your choice for both answers).

STABLE? YES NO	CAUSAL? YES NO

(c) For the convolution

$$y(t) = 3[u(t-9) - u(t-2)] * u(t-2)$$

determine $y(\infty) = \lim_{t\to\infty} y(t)$, i.e., the value of the signal $y(t)$ as $t \to \infty$.

$y(\infty) =$

PROBLEM sp-04-F.4:

Two questions that involve common operations done in the Lab:

(a) Suppose that a student enters the following MATLAB code:

```
nn = 0:3480099;
xx = (5/pi) * cos(2*pi*1.2*nn + 6.02);
soundsc(xx,20000)
```

Determine the analog frequency (in Hertz) that will be heard. (There might be folding or aliasing!)

FREQ =	Hz

(b) Suppose that a student writes the following MATLAB code to generate a sine wave:

```
tt = 0:1/12000:10000;
xx = sin(2*pi*1000*tt+pi/3);
soundsc(xx,fsamp);
```

Although the sinusoid was not written to have a frequency of 1900 Hz, it is possible to play out the vector `xx` so that it sounds like a 1900 Hz tone. Determine the value of `fsamp` that should be used to play the vector `xx` as a 1900 Hz tone. Write your answer as an integer.

`fsamp =`	Hz

(c) Consider the following piece of MATLAB code:

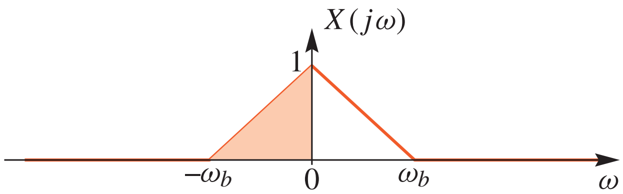
```
tt = 0:(1/16000):36;
xx = cos(2*pi*440*tt);
soundsc(xx,8000);
```

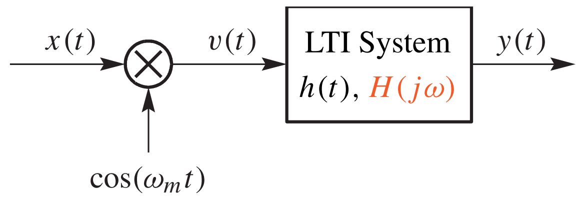
Determine the duration (in seconds) of the final played tone. (Assume that the computer has an infinite amount of memory so that we don't need to worry about running out.)

DURATION =	sec.

PROBLEM sp-04-F.5:

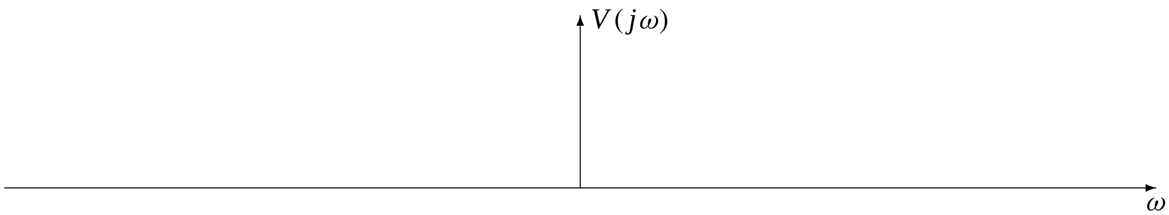
The system below involves a multiplier followed by a filter:





The Fourier transform of the input is also shown above. For all parts below, assume that $\omega_b = 20\pi$, and the frequency of the cosine multiplier is $\omega_m = 100\pi$ rad/s.

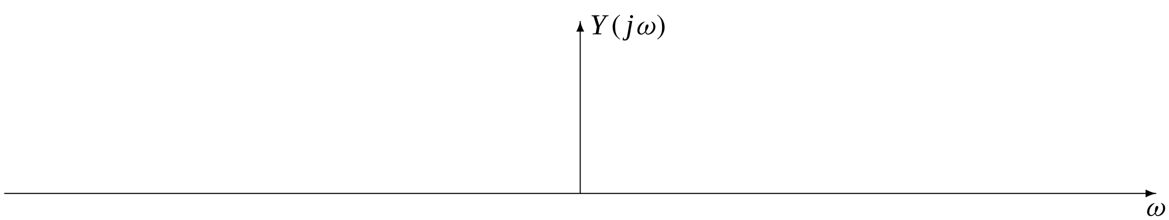
(a) Make a sketch of $V(j\omega)$, the Fourier transform of $v(t)$, the output of the multiplier, when the input is $X(j\omega)$ shown above with $\omega_b = 20\pi$ rad/s.



(b) If the filter is an ideal HPF defined by

$$H(j\omega) = \begin{cases} 0 & |\omega| < 100\pi \\ 1 & |\omega| \geq 100\pi \end{cases}$$

Make a sketch of $Y(j\omega)$, the Fourier transform of the output $y(t)$ when the input is $X(j\omega)$ shown above.



PROBLEM sp-04-F.6:

Suppose that two filters are cascaded. The system functions are

$$H_1(z) = \frac{3 + 3z^{-2}}{4 - z^{-2}} \qquad \text{and} \qquad H_2(z) = 8 - 4z^{-1}$$

(a) Determine the poles and zeros[1] of $H_1(z)$

POLES =
ZEROS =

(b) Determine the poles and zeros of $H_2(z)$

POLES =
ZEROS =

(c) The cascaded system can be combined into one overall system and then described by a single difference equation of the form:

$$y[n] = \alpha y[n-1] + \beta_0 x[n] + \beta_1 x[n-1] + \beta_2 x[n-2]$$

Determine the numerical values of α, β_0, β_1 and β_2.

$\alpha =$	$\beta_0 =$	$\beta_1 =$	$\beta_2 =$

[1] If necessary, include poles and zeros at $z = 0$ and at $z = \infty$, and indicate repeated poles or zeros.

PROBLEM sp-04-F.7:

A periodic signal $x(t)$ is represented as a Fourier series of the form

$$x(t) = \sum_{k=-\infty}^{\infty} \left(10\delta[k] + k^2 - 15(-1)^k\right) e^{j5\pi kt}$$

(a) Determine the fundamental period of the signal $x(t)$, i.e., the minimum period.

| $T_0 =$ | sec. (Give a numerical answer.)

(b) Determine the DC value of $x(t)$. Give your answer as a number.

| $DC =$ |

(c) Define a new signal by filtering $x(t)$ through the LTI system whose frequency response is

$$H(j\omega) = j\omega\, e^{-j0.1\omega}$$

The new signal, $y(t)$ can be expressed in the following Fourier Series with new coefficients $\{b_k\}$:

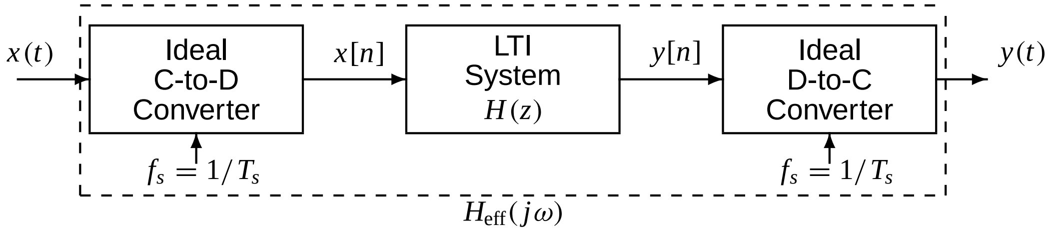
$$y(t) = \sum_{k=-\infty}^{\infty} b_k\, e^{j5\pi kt}$$

Fill in the following table, giving *numerical values* for each $\{b_k\}$ in polar form:.
Hint: Find a simple relationship between $\{b_k\}$ and $\{a_k\}$.

b_k	Magnitude	Phase
b_{-2}		
b_{-1}		
b_0		
b_1		
b_2		

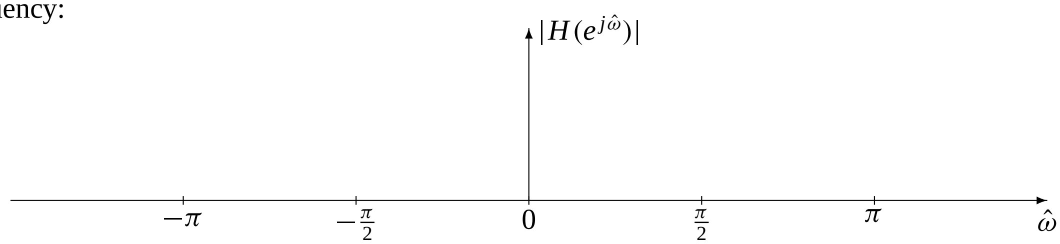
PROBLEM sp-04-F.8:

Consider the following system for discrete-time filtering of a continuous-time signal:



Assume that the discrete-time system has a system function $H(z)$ defined as: $H(z) = z^{-1} + z^{-3}$

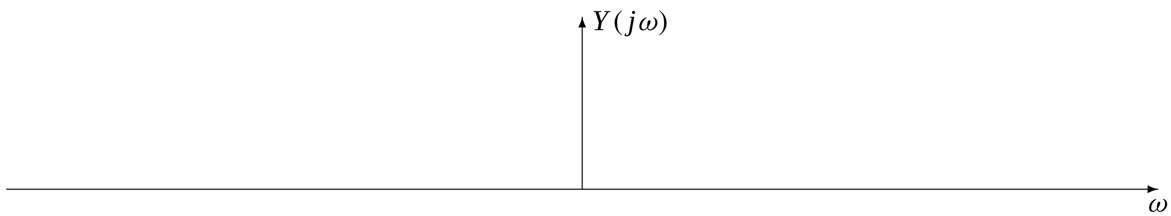
(a) Determine the frequency response of the discrete-time filter and plot its magnitude response versus frequency:



(b) Assume that the input signal $x(t)$ is a sum of cosines:

$$x(t) = 2\cos(40\pi t + \pi/4) + 7\cos(180\pi t + \pi/3)$$

For this input signal, determine the Fourier transform of the output signal $y(t)$ when the sampling rate is f_s **= 120 samples/sec.** Make a plot of $Y(j\omega)$, the Fourier transform of the output signal.



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SCHOOL of ELECTRICAL & COMPUTER ENGINEERING
FINAL EXAM

DATE: 30-Apr-04 COURSE: ECE-2025

NAME: Answer Key
LAST, FIRST

GT #: Version – 1

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PROBLEM sp-04-F.1:

Circle the correct answer to each of the following short answer questions, and ***provide a short justification*** for your answer.

1. Pick the correct frequency response for the FIR filter: $y[n] = x[n-1] + x[n-3]$.

   (a) $2\cos(\hat{\omega})$
   (b) $2e^{-j1.5\hat{\omega}}\cos(1.5\hat{\omega})$
   (c) $2e^{-j1.5\hat{\omega}}\cos(0.5\hat{\omega})$
   (d) $2e^{-j2\hat{\omega}}\cos(\hat{\omega})$
   (e) none of the above

   $b_k = \{0, 1, 0, 1\}$

   $H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}} + e^{-j3\hat{\omega}}$

   $= e^{-j2\hat{\omega}}(e^{j\hat{\omega}} + e^{-j\hat{\omega}})$

   $= e^{-j2\hat{\omega}}(2\cos\hat{\omega})$

2. A sinusoidal signal $x(t)$ is defined by: $x(t) = \Re e\{(1+j)e^{j\pi t}\}$. When $x(t)$ is plotted versus time (t), its maximum value will be:

   (a) $A = 1$
   (b) $A = 1 + j$
   (c) $A = \sqrt{2}$
   (d) $A = 0$
   (e) none of the above

   $1 + j = \sqrt{2}\, e^{j\pi/4}$

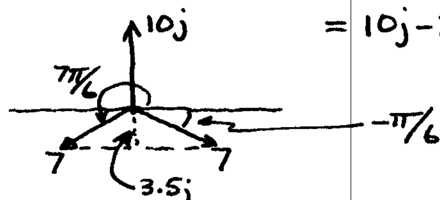
   $x(t) = \sqrt{2}\cos(\pi t + \pi/4)$

3. Determine the amplitude (A) and phase (ϕ) of the sinusoid that is the sum of the following three sinusoids: $10\cos(6t + \pi/2) + 7\cos(6t - \pi/6) + 7\cos(6t + 7\pi/6)$,

   (a) $A = 10$ and $\phi = \pi/2$.
   (b) $A = 7$ and $\phi = \pi/2$.
   (c) $A = 0$ and $\phi = 0$.
   (d) $A = 3$ and $\phi = \pi/2$.
   (e) $A = 24$ and $\phi = \pi/2$.

   $10e^{j\pi/2} + 7e^{-j\pi/6} + 7e^{j7\pi/6}$

   $= 10j - 3.5j - 3.5j + 6.06 - 6.06$



4. Evaluate the complex number $z = \dfrac{j^{-1} - j^{-2}}{j^{-3} + j^{-4}}$.

   (a) $z = 0$
   (b) $z = j$
   (c) $z = -j$
   (d) $z = 1$
   (e) $z = -1$

   $z = \dfrac{j^3 - j^2}{j + 1} = \dfrac{-j + 1}{j + 1} = \dfrac{-j(1 + j)}{j + 1} = -j$

PROBLEM sp-04-F.2:

For each of the following time-domain signals, select the correct match from the list of Fourier transforms below. ***Write each answer in the box provided.*** (The operator $*$ denotes convolution.)

(a) $x(t) = \dfrac{\sin(5\pi t)}{\pi t} * \delta(t-4)$ $\xrightarrow{\text{delay}}$ $[u(\omega+5\pi) - u(\omega-5\pi)]e^{-j4\omega}$

Answer: 1

(b) $x(t) = \dfrac{\sin(5\pi t)}{\pi t}$ $\xrightarrow{FT}$ $u(\omega+5\pi) - u(\omega+5\pi)$

Answer: 0

(c) $x(t) = 2\dfrac{\sin(\pi t)}{\pi t} * \cos(4\pi t)$ $[u(\omega+\pi) - u(\omega-\pi)][2\pi\delta(\omega+4\pi) + 2\pi\delta(\omega-4\pi)]$

Since the deltas are outside the passband, the answer is zero.

Answer: 5

(d) $x(t) = \dfrac{\sin(5\pi t)}{\pi t} * \dfrac{\sin(3\pi t)}{\pi t}$ $\rightarrow$ $[u(\omega+5\pi) - u(\omega-5\pi)] \cdot [u(\omega+3\pi) - u(\omega-3\pi)]$

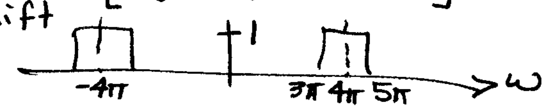
keep the shorter one

Answer: 2

(e) $x(t) = \displaystyle\int_{-\infty}^{\infty} \delta(\lambda+3)\delta(t-\lambda-7)d\lambda$ $= \delta(t+3) * \delta(t-7) = \delta(t-4) \xrightarrow{FT} e^{-j4\omega}$

Answer: 4

(f) $x(t) = 2\dfrac{\sin(\pi t)}{\pi t}\cos(4\pi t)$ $\xrightarrow[\text{shift}]{\text{freq}}$ $[u(\omega+\pi) - u(\omega-\pi)]$ shift up to 4π down to -4π



Answer: 9

(g) $x(t) = \dfrac{d}{dt}\left\{\dfrac{\sin(5\pi t)}{\pi t}\right\}$ $\xrightarrow[\text{property}]{\text{deriv}}$ $j\omega[u(\omega+5\pi) - u(\omega-5\pi)]$

Answer: 3

Each of the time signals above has a Fourier transform that can be found in the list below.

[0] $X(j\omega) = u(\omega+5\pi) - u(\omega-5\pi)$

[1] $X(j\omega) = e^{-j4\omega}[u(\omega+5\pi) - u(\omega-5\pi)]$

[2] $X(j\omega) = u(\omega+3\pi) - u(\omega-3\pi)$

[3] $X(j\omega) = j\omega[u(\omega+5\pi) - u(\omega-5\pi)]$

[4] $X(j\omega) = e^{-j4\omega}$

[5] $X(j\omega) = 0$

[6] $X(j\omega) = e^{-j4\omega}[j\pi\delta(\omega+\pi) - j\pi\delta(\omega-\pi)]$

[7] $X(j\omega) = [j\pi u(\omega+\pi) - j\pi u(\omega-\pi)] * \delta(\omega-4\pi)$

[8] $X(j\omega) = 2u(\omega+\pi) - 2u(\omega-\pi) + \pi\delta(\omega+4\pi) + \pi\delta(\omega-4\pi)$

[9] $X(j\omega) = u(\omega+5\pi) - u(\omega+3\pi) + u(\omega-3\pi) - u(\omega-5\pi)$

PROBLEM sp-04-F.3:

The three subparts of this problem are completely independent of one another.

(a) When two finite-duration signals are convolved, the result is a finite-duration signal. In this subpart,

$$h(t) = t^2[u(t-17) - u(t-10)] \quad \text{and} \quad x(t) = 9[u(t-7) - u(t-30)]$$

Determine starting and ending times of output signal $y(t) = x(t) * h(t)$, i.e., find T_1 and T_2 so that $y(t) = 0$ for $t < T_1$ and for $t > T_2$.

$T_1 = 17$	$T_2 = 47$

$h(t)$ starts at $t = 10$; ends at $t = 17$

$x(t)$ starts at $t = 7$; ends at $t = 30$

$\Rightarrow y(t)$ starts at $t = 17$; ends at $t = 47$

(b) A system has the following impulse response:

$$h(t) = u(t-2)$$

Determine whether the system is stable and causal (circle your choice for both answers).

STABLE? YES (NO)	CAUSAL? (YES) NO

$$\int_{-\infty}^{\infty} |h(t)|\,dt = \int_{2}^{\infty} 1\,dt \to \infty$$

$h(t) = 0$ for $t < 2$ $\Rightarrow$ causal

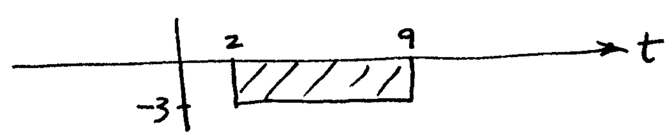
(c) For the convolution

$$y(t) = 3[u(t-9) - u(t-2)] * u(t-2)$$

determine $y(\infty) = \lim_{t\to\infty} y(t)$, i.e., the value of the signal $y(t)$ as $t \to \infty$.

$y(\infty) = -21$

plot $3[u(t-9) - u(t-2)]$



As $t \to \infty$, $y(\infty) = \text{area} = (-3)(7) = -21$

or, $y(t) = 3\,u(t-9) * u(t-2) - 3\,u(t-2) * u(t-2)$

$= 3(t-11)\,u(t-11) - 3(t-4)\,u(t-4)$

As $t \to \infty$, $y(t) \to 3(t-11) - 3(t-4) = -33 + 12 = -21$

PROBLEM sp-04-F.4:

Two questions that involve common operations done in the Lab:

(a) Suppose that a student enters the following MATLAB code:

```
nn = 0:3480099;
xx = (5/pi) * cos(2*pi*1.2*nn + 6.02);
soundsc(xx,20000)
```

Determine the analog frequency (in Hertz) that will be heard. (There might be folding or aliasing!)

FREQ = 4000	Hz

$\hat{\omega} = 2\pi(1.2)$ aliases to $2\pi(0.2)$

$\omega = \hat{\omega} f_s = 2\pi(0.2)(20{,}000)$

$f = \frac{\omega}{2\pi} = 0.2 \times 20{,}000 = 4{,}000$ Hz

(b) Suppose that a student writes the following MATLAB code to generate a sine wave:

```
tt = 0:1/12000:10000;
xx = sin(2*pi*1000*tt+pi/3);
soundsc(xx,fsamp);
```

Although the sinusoid was not written to have a frequency of 1900 Hz, it is possible to play out the vector xx so that it sounds like a 1900 Hz tone. Determine the value of `fsamp` that should be used to play the vector xx as a 1900 Hz tone. Write your answer as an integer.

fsamp = 22800	Hz

xx is $\sin(2\pi(1000)t + \pi/3)$ sampled at $f_s = 12{,}000$ Hz

$\Rightarrow \hat{\omega} = 2\pi(1000)/12000$

$\omega_{OUT} = \hat{\omega} f_s \Rightarrow 2\pi(1900) = \frac{2\pi(1000)}{12000} f_s$

$\Rightarrow f_s = \frac{1900}{1000} \times 12{,}000 = 22{,}800$ Hz

(c) Consider the following piece of MATLAB code:

```
tt = 0:(1/16000):36;
xx = cos(2*pi*440*tt);
soundsc(xx,8000);
```

Determine the duration (in seconds) of the final played tone. (Assume that the computer has an infinite amount of memory so that we don't need to worry about running out.)

DURATION = 72	sec.

xx is $\cos(2\pi(440)t)$ sampled at $f_s = 16{,}000$ Hz

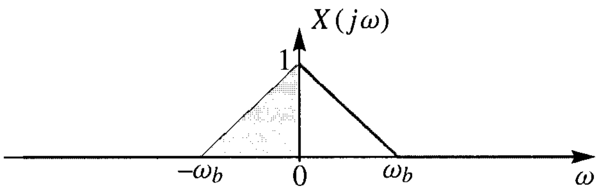
duration of xx = 36 sec.

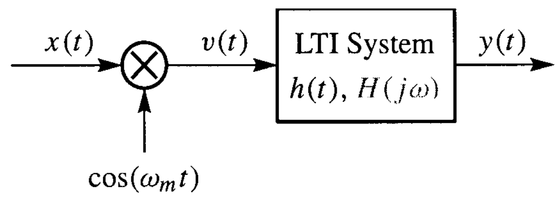
Playing out at 8000 Hz is playing slower

$\Rightarrow$ duration is longer: $\frac{16000}{8000} \times 36 = 72$ secs.

PROBLEM sp-04-F.5:

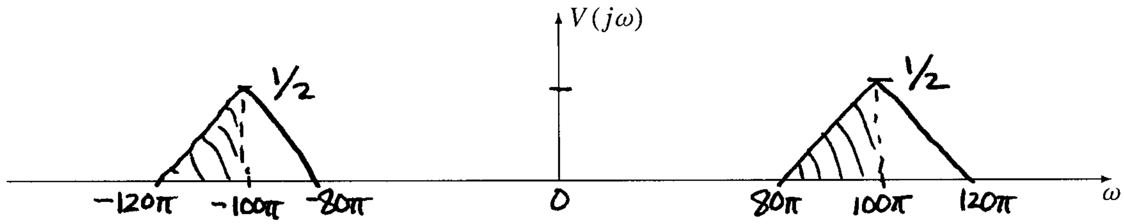
The system below involves a multiplier followed by a filter:





The Fourier transform of the input is also shown above. For all parts below, assume that $\omega_b = 20\pi$, and the frequency of the cosine multiplier is $\omega_m = 100\pi$ rad/s.

(a) Make a sketch of $V(j\omega)$, the Fourier transform of $v(t)$, the output of the multiplier, when the input is $X(j\omega)$ shown above with $\omega_b = 20\pi$ rad/s.

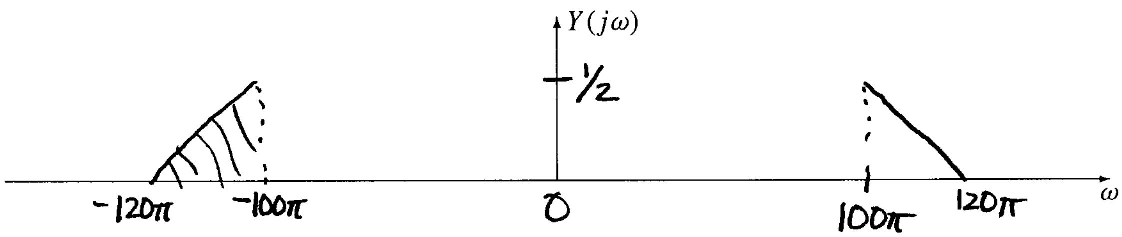


$$V(j\omega) = \frac{1}{2} X(j(\omega - 100\pi)) + \frac{1}{2} X(j(\omega + 100\pi))$$

(b) If the filter is an ideal HPF defined by

$$H(j\omega) = \begin{cases} 0 & |\omega| < 100\pi \\ 1 & |\omega| \geq 100\pi \end{cases}$$

Make a sketch of $Y(j\omega)$, the Fourier transform of the output $y(t)$ when the input is $X(j\omega)$ shown above.



PROBLEM sp-04-F.6:

Suppose that two filters are cascaded. The system functions are

$$H_1(z) = \frac{3 + 3z^{-2}}{4 - z^{-2}} \qquad \text{and} \qquad H_2(z) = 8 - 4z^{-1}$$

(a) Determine the poles and zeros[1] of $H_1(z)$

POLES = $\frac{1}{2}$, $-\frac{1}{2}$
ZEROS = $+j$, $-j$

$$H_1(z) = \frac{3(z^2+1)}{4(z^2 - \frac{1}{4})}$$

$$z^2 + 1 = 0 \Rightarrow z = \pm j$$

$$z^2 - \frac{1}{4} = 0 \Rightarrow z = \pm \frac{1}{2}$$

(b) Determine the poles and zeros of $H_2(z)$

POLES = 0
ZEROS = $\frac{1}{2}$

$$H_2(z) = \frac{8(z - \frac{1}{2})}{z}$$

(c) The cascaded system can be combined into one overall system and then described by a single difference equation of the form:

$$y[n] = \alpha y[n-1] + \beta_0 x[n] + \beta_1 x[n-1] + \beta_2 x[n-2]$$

Determine the numerical values of α, β_0, β_1 and β_2.

$\alpha = -\frac{1}{2}$	$\beta_0 = 6$	$\beta_1 = 0$	$\beta_2 = 6$

$$H_1(z)H_2(z) = \frac{3 + 3z^{-2}}{4(1 - \frac{1}{2}z^{-1})(1 + \frac{1}{2}z^{-1})} \cdot 8(1 - \tfrac{1}{2}z^{-1})$$

$$= \frac{(3 + 3z^{-2})8}{4(1 + \frac{1}{2}z^{-1})} = \frac{6 + 6z^{-2}}{1 + \frac{1}{2}z^{-1}}$$

[1] If necessary, include poles and zeros at $z = 0$ and at $z = \infty$, and indicate repeated poles or zeros.

PROBLEM sp-04-F.7:

A periodic signal $x(t)$ is represented as a Fourier series of the form

$$x(t) = \sum_{k=-\infty}^{\infty} \left(10\delta[k] + k^2 - 15(-1)^k\right) e^{j5\pi kt}$$

(a) Determine the fundamental period of the signal $x(t)$, i.e., the minimum period.

$T_0 = 2/5$ sec. (Give a numerical answer.) $\omega_0 = 5\pi \Rightarrow T_0 = \frac{2\pi}{\omega_0} = \frac{2}{5}$

(b) Determine the DC value of $x(t)$. Give your answer as a number.

$DC = -5$ $\quad a_0 = 10\delta[0] + 0^2 - 15(-1)^0 = 10 - 15 = -5$

(c) Define a new signal by filtering $x(t)$ through the LTI system whose frequency response is

$$H(j\omega) = j\omega e^{-j0.1\omega}$$

The new signal, $y(t)$ can be expressed in the following Fourier Series with new coefficients $\{b_k\}$:

$$y(t) = \sum_{k=-\infty}^{\infty} b_k e^{j5\pi kt}$$

Fill in the following table, giving *numerical values* for each $\{b_k\}$ in polar form:.
Hint: Find a simple relationship between $\{b_k\}$ and $\{a_k\}$.

b_k	Magnitude	Phase
b_{-2}	110π	$-\pi/2$
b_{-1}	80π	0
b_0	0	0
b_1	80π	0
b_2	110π	$\pi/2$

$b_k = a_k H(jk\omega_0) = a_k\left(jk5\pi e^{-j0.5k\pi}\right)$

$a_{-2} = 10\delta[-2] + (-2)^2 - 15(-1)^{-2}$
$= 0 + 4 - 15 = -11$

$b_{-2} = (-11)(-j10\pi e^{j\pi}) = -j\,110\pi$

$b_2 = (b_{-2})^* = j\,110\pi$

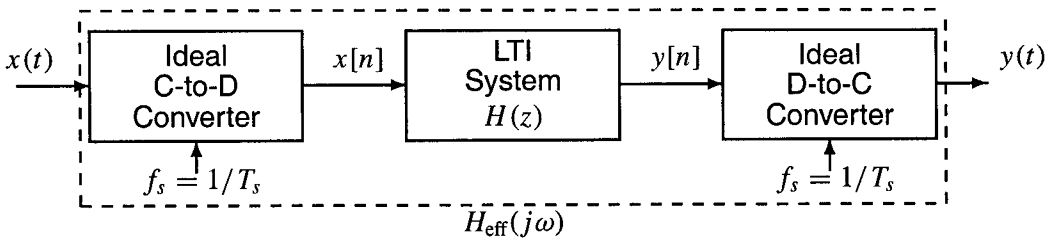
$b_0 = a_0 H(j0) = 0$

$a_1 = 10\delta[1] + 1^2 - 15(-1) = 1 + 15 = 16$

$b_1 = 16\left(j5\pi e^{-j\pi/2}\right) = 80\pi$

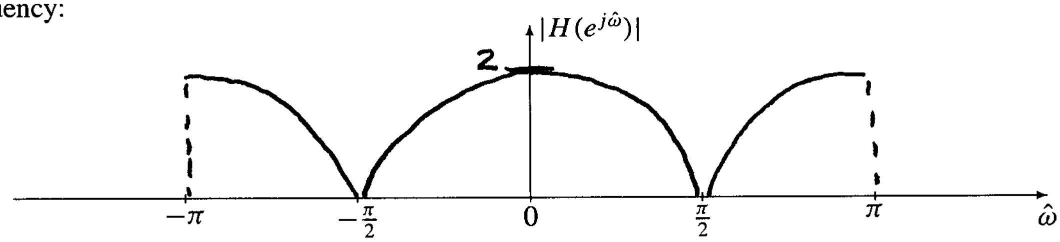
PROBLEM sp-04-F.8:

Consider the following system for discrete-time filtering of a continuous-time signal:



Assume that the discrete-time system has a system function $H(z)$ defined as: $H(z) = z^{-1} + z^{-3}$

(a) Determine the frequency response of the discrete-time filter and plot its magnitude response versus frequency:

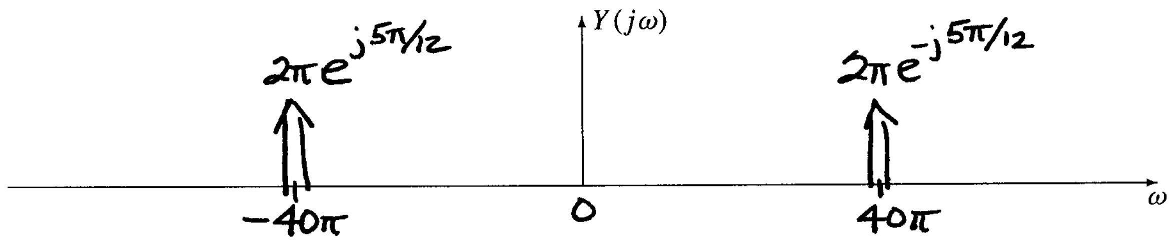


$$H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}} + e^{-j3\hat{\omega}}$$
$$= e^{-j2\hat{\omega}}(2\cos\hat{\omega})$$
$$\Rightarrow |H(e^{j\hat{\omega}})| = 2\,|\cos(\hat{\omega})|$$

(b) Assume that the input signal $x(t)$ is a sum of cosines:

$$x(t) = 2\cos(40\pi t + \pi/4) + 7\cos(180\pi t + \pi/3)$$

For this input signal, determine the Fourier transform of the output signal $y(t)$ when the sampling rate is f_s = **120 samples/sec.** Make a plot of $Y(j\omega)$, the Fourier transform of the output signal.



$$x[n] = 2\cos\left(\frac{40\pi n}{120} + \pi/4\right) + 7\cos\left(\frac{180\pi n}{120} + \pi/3\right)$$

$\hat{\omega} = \pi/3$

$\hat{\omega} = 1.5\pi$ aliases to $-\pi/2$

$H(e^{-j\pi/2}) = 0$ so this component is nulled out

$$H(e^{j\pi/3}) = e^{-j2\pi/3}\underbrace{2\cos(\pi/3)}_{2\times 1/2 = 1}$$
$$= 1e^{-j2\pi/3}$$

$$y[n] = 2\cos\left(\tfrac{\pi}{3}n + \pi/4 - 2\pi/3\right) = 2\cos\left(\tfrac{\pi}{3}n - \tfrac{5\pi}{12}\right)$$

convert $\hat{\omega}$ to ω: $\omega = \hat{\omega} f_s = \frac{\pi}{3} \times 120 = 40\pi$ rad/s