GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

FINAL EXAM

DATE: 05/01/02 COURSE: ECE 2025

NAME: ______________________________ GT #: ____________________

LAST, FIRST

Recitation Section: Circle the date & time when your Recitation Section meets (not Lab):

L02:Tues-9:30am (Bordelon)	L04:Tues-12:00pm (Yezzi)	L05:Thurs-1:30pm (Williams)
L06:Tues-1:30pm (Bordelon)	L07:Thur-3:00pm (Williams)	L08:Tues-3:00pm (Smith)
L11:Mon-3:00pm (Glytsis)	L14:Mon-4:00pm (McClellan)	RPK: (Abler) Valdosta (Fares)

- Write your name on the front page ONLY.
- Closed book, but one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes (original only) permitted. OK to write on both sides. A calculator is permitted. **CAREFULLY TEAR OFF THE LAST TWO PAGEs OF TRANSFORM TABLES.**
- JUSTIFY your reasoning CLEARLY to receive any partial credit. Explanations are also REQUIRED to receive FULL credit for any answer.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers, or write them in the boxes provided. If space is needed for scratch work, use the backs of pages.

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
8	20	
TOTAL	160	

Problem sp-02-F.1:

(a) Let $w(t) = \cos(100\pi t + \pi/4) + 2\cos(100\pi t - \pi/4) = A\cos(\omega_0 t + \phi)$.
Determine A, ω_0, and ϕ.

$A =$
$\omega_0 =$
$\phi =$

(b) A periodic signal $x(t)$ is given by

$$x(t) = 2 + 2\cos(1000\pi t + \theta) + \cos(1500\pi t + \psi).$$

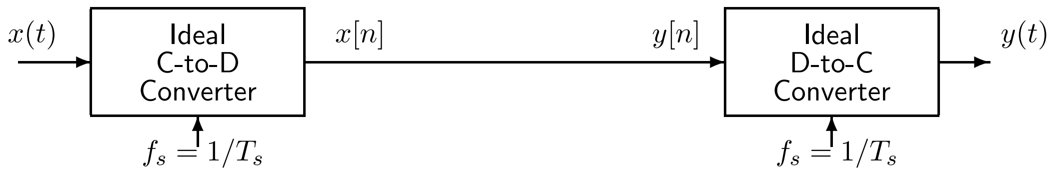
Determine the period T_0 of this signal.

$T_0 =$

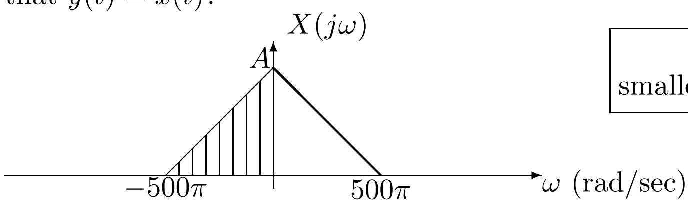
(c) If the Fourier series coefficients of the signal $x(t)$ in part (b) are $a_0 = 2$, $a_2 = e^{j\pi/2}$, $a_{-2} = e^{-j\pi/2}$, $a_3 = 0.5e^{-j\pi/6}$, and $a_{-3} = 0.5e^{j\pi/6}$, determine θ and ψ for the signal $x(t)$.

$\theta =$
$\psi =$

Problem sp-02-F.2:
Consider the following system for sampling and reconstruction of a continuous-time signal:



(a) Assume that the input signal $x(t)$ has a bandlimited Fourier transform $X(j\omega)$ as depicted below. For this input signal, what is the *smallest* value of the sampling frequency f_s such that $y(t) = x(t)$?



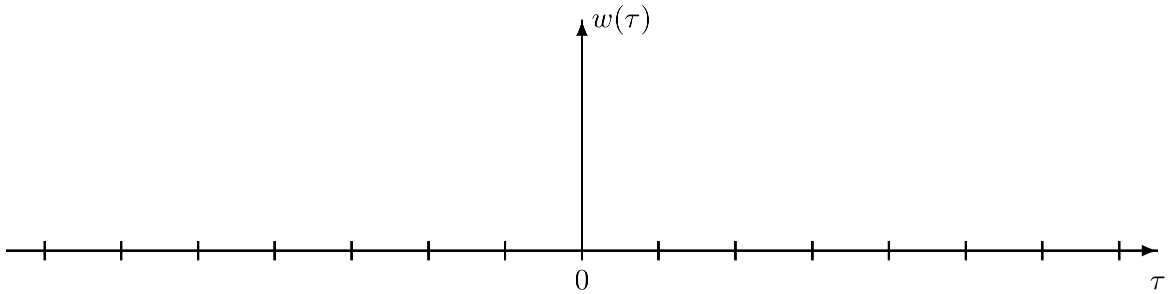
smallest f_s = ____________ samples/sec

(b) In this part, the input signal is $x(t) = 50 + 50\cos(50\pi t + \pi/3)$. If the sampling rate is $f_s = 30$ samples/sec, what is the corresponding output $y(t)$?

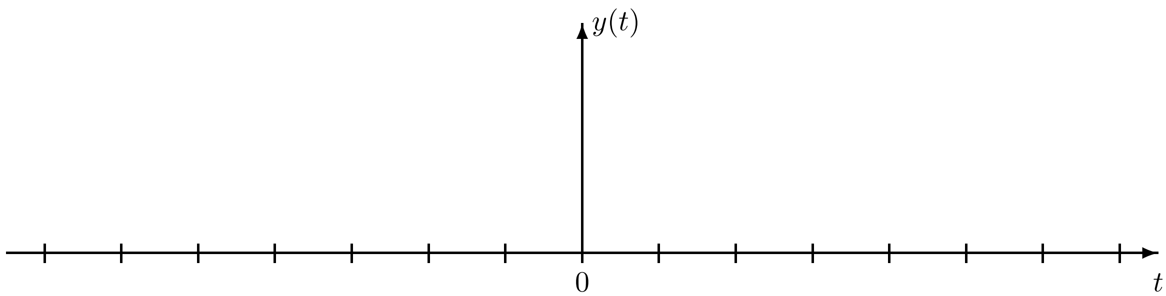
$y(t) =$

Problem sp-02-F.3:

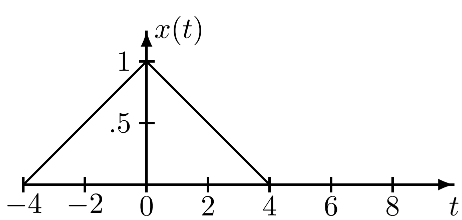
(a) Assume that $w(t) = u(t+2) - u(t-1) = \begin{cases} 1 & -2 < t < 1 \\ 0 & \text{otherwise} \end{cases}$. Plot $w(\tau)$ as a function of τ.

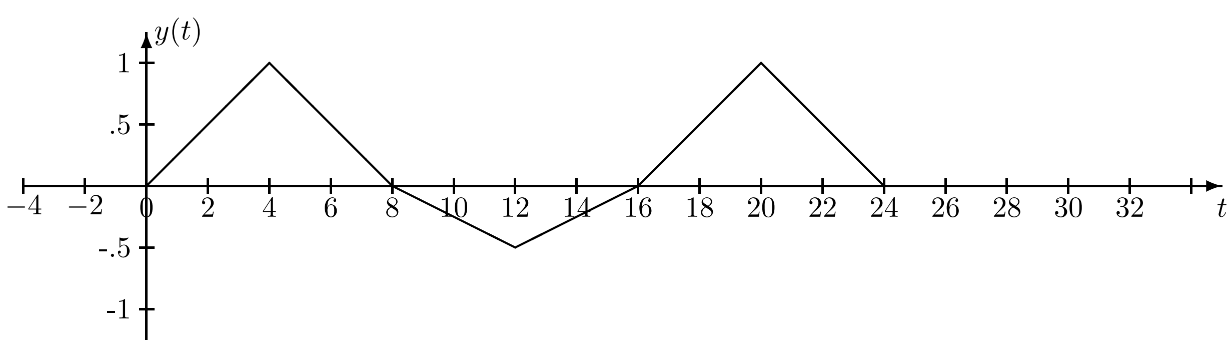


(b) Plot $y(t) = w(t) * w(t)$. Carefully label the amplitude and the time axis.



(c) When the input to an LTI system is $x(t)$, the output is the signal $y(t) = x(t) * h(t)$ plotted below. What is the impulse response $h(t)$ of the system?

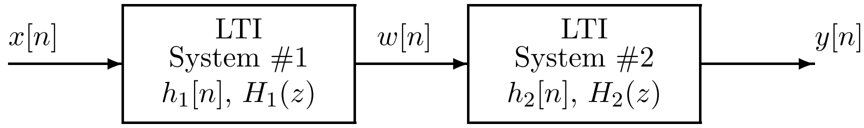




$h(t) =$

Problem sp-02-F.4:

A cascade of two discrete-time systems is depicted by the following block diagram:



System #1 is defined by the system function $H_1(z) = z^{-2}(1 + z^{-1})$ and System #2 is defined by the difference equation $y[n] = -0.8y[n-1] + 2w[n]$.

(a) If the input to the first system is $x[n] = \delta[n] - \delta[n-1]$, determine the output, $w[n]$, of the **first** system.

$w[n] =$

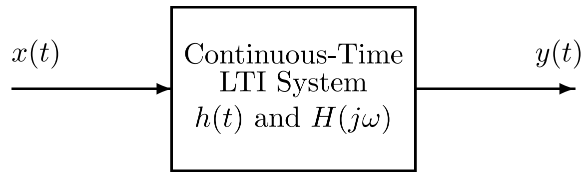
(b) Determine the system function $H(z)$ of the overall system.

$H(z) =$

(c) Determine the impulse response of the overall system.

$h[n] =$

Problem sp-02-F.5:

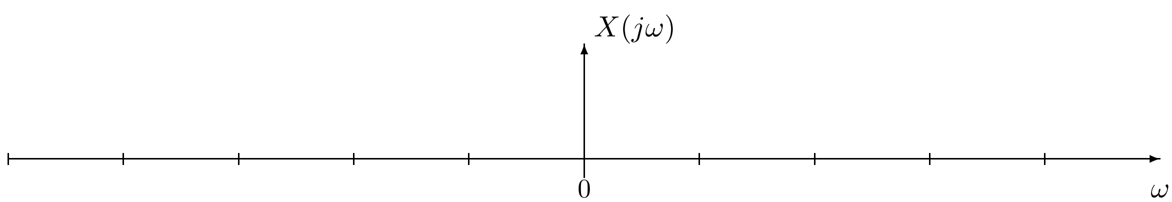


The periodic input to the above system is defined by the equation:

$$x(t) = \sum_{k=-2}^{2} a_k e^{j50\pi k t}, \quad \text{where} \quad a_k = \begin{cases} \dfrac{1/\pi}{|k|} & k \neq 0 \\ 2 & k = 0 \end{cases}.$$

(a) Determine the Fourier transform of the periodic signal $x(t)$. Give a formula and then plot it on the graph below. Label your plot carefully to receive full credit.

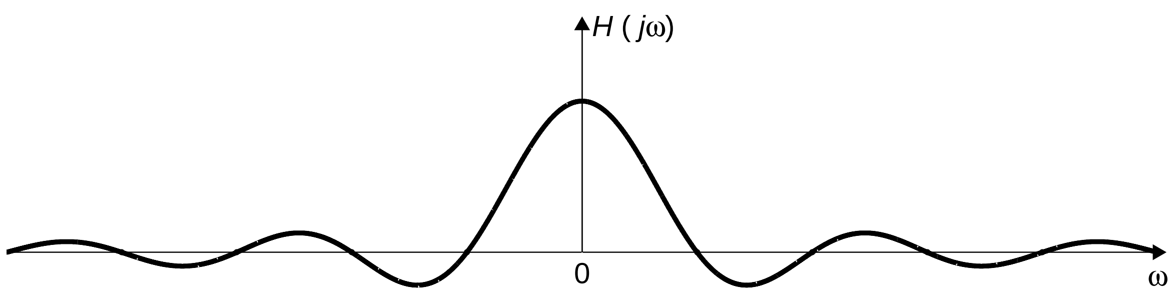
$X(j\omega) =$



(b) The frequency response of the LTI system is given by the following equation:

$$H(j\omega) = \frac{2\sin(\omega T/2)}{\omega T/2}.$$

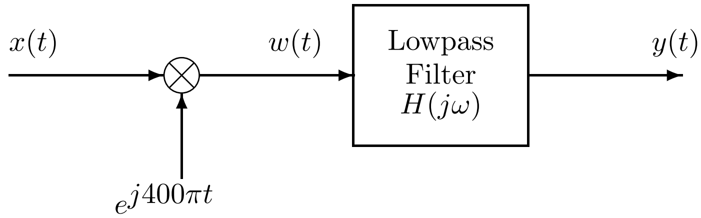
In the following plot, label the maximum amplitude and the frequencies at which $H(j\omega) = 0$ in terms of the parameter T.



(c) Determine the *smallest* value of the parameter T such that $y(t) = A$ for $-\infty < t < \infty$, where A is a constant. Determine the constant A.

$T =$	$A =$

Problem sp-02-F.6:



In the above modulation/filtering system, assume that the input signal $x(t)$ has a bandlimited Fourier transform $X(j\omega)$, as depicted in Figure A below.

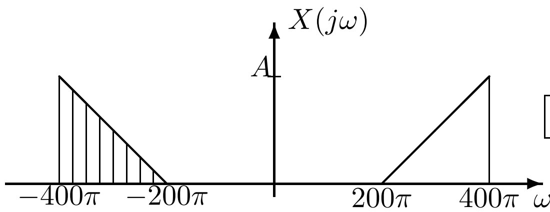
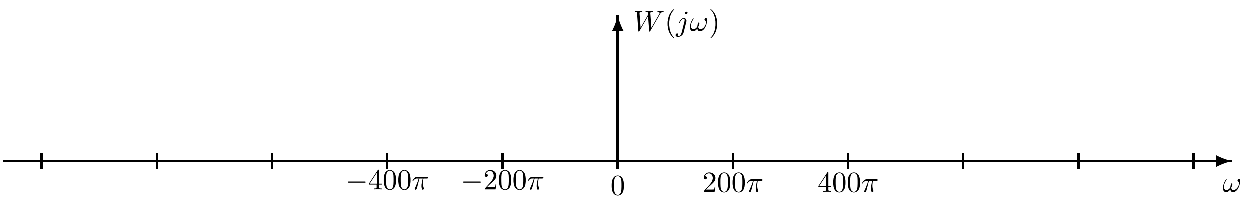


Figure A

(a) First give the general equation that expresses $W(j\omega)$, the Fourier transform of $w(t) = x(t)e^{j400\pi t}$, in terms of $X(j\omega)$.

$W(j\omega) =$ ________________________________

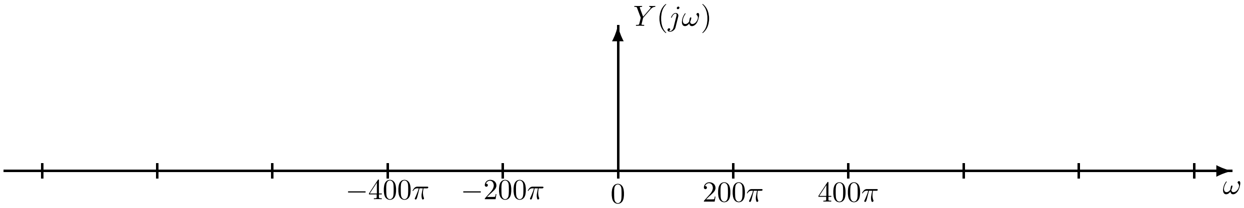
(b) Now **carefully** plot the Fourier transform $W(j\omega)$ for the specific input $x(t)$ whose Fourier transform $X(j\omega)$ is given above in Figure A. *Note that part of the Fourier transform $X(j\omega)$ is shaded. Mark the corresponding shaded region or regions in your plot of $W(j\omega)$, and be sure to carefully label both amplitudes and frequencies.*



(c) The frequency response of the lowpass filter is

$$H(j\omega) = \begin{cases} 2 & |\omega| \le 200\pi \\ 0 & |\omega| > 200\pi \end{cases}$$

Plot the Fourier transform $Y(j\omega)$ below for the $X(j\omega)$ given in Figure A above. *Be sure to carefully label both amplitudes and frequencies and be sure to shade the region corresponding to the original shaded region in the input spectrum.*



Problem sp-02-F.7:

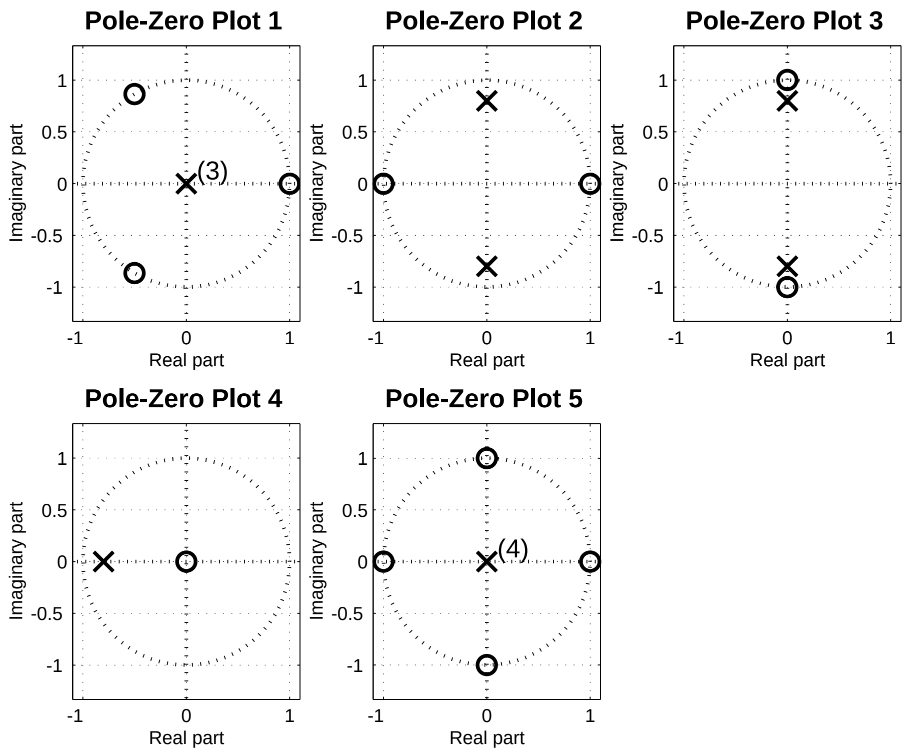
In each of the following problems, find the Fourier transform, or inverse Fourier transform. Give your answer as a simple formula or plot. (The symbol $*$ denotes covolution.)

(a) Find $Y(j\omega)$ when $y(t) = x(t) * h(t) = \frac{\sin 2t}{2\pi t} * [e^{-2t}u(t)]$.

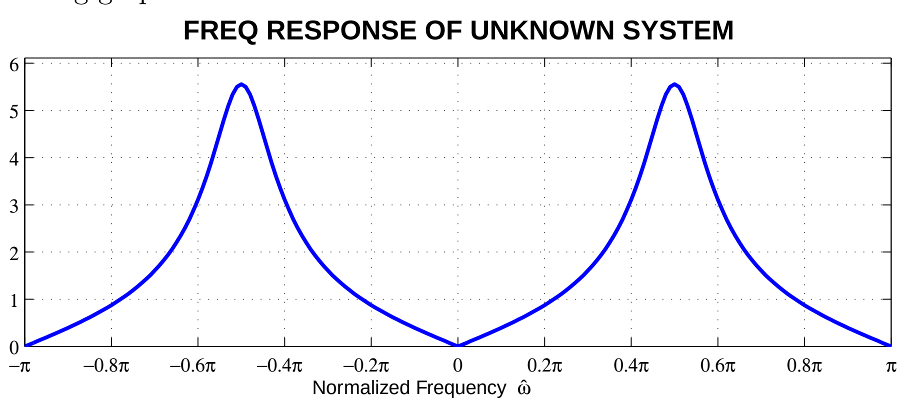
(b) Find $h(t)$ when $H(j\omega) = \delta(\omega + 4) * [u(\omega - 4) - u(\omega + 4)]$.

(c) Find $v(t)$ when $V(j\omega) = \frac{1 + e^{-j\omega}}{2 + j\omega}$.

Problem sp-02-F.8:



(a) Which of the above pole-zero plots represents the system whose frequency response is given in the following graph?



System #

(b) Now assume that each of the systems represented by the above pole-zero plots has impulse response $h_k[n]$, where k is the index shown in the title of the pole-zero plot.

(i) In the table below, indicate with an X which of the systems are FIR systems.

System #	1	2	3	4	5
FIR??					

(ii) In the table below, indicate with an X each of the systems for which $h_k[n] * e^{j\pi n} = 0$, where $*$ denotes convolution.

System #	1	2	3	4	5
$h_k[n] * e^{j\pi n} = 0??$					