

Problem Fall-Q.1.1:

Simplify the following complex-valued expressions. In each case reduce the answers to a **simple** numerical form. Let

$$V = -3 - j3\sqrt{3}.$$

(a) Express jV in polar form. In addition plot jV as a vector.

(b) Express the inverse of V in rectangular form. In addition plot $\frac{1}{V}$ as a vector.

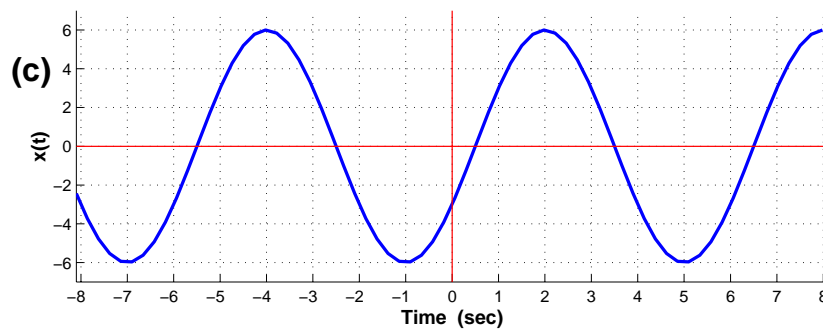
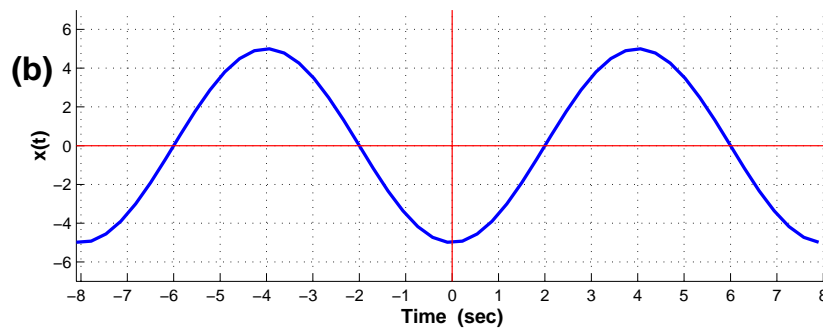
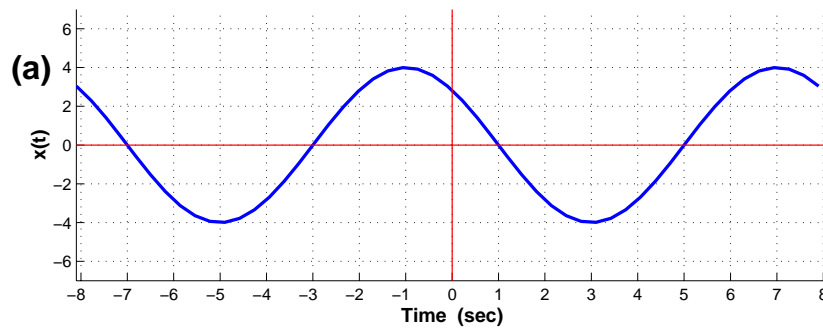
(c) If $Z = \frac{|V|}{V^*}$, express Z in polar form. In addition plot Z as a vector.

(d) Express $\Re\{j^3 V e^{j15t}\}$ in the standard “cosine” form.

Problem Fall-Q.1.2:

Several sinusoidal signals are plotted below. For each plot (a)–(c), determine the amplitude, phase (in radians) and frequency (in Hz). Write your answers in the following table:

PLOT	(a)	(b)	(c)
AMPLITUDE			
PHASE (in radians)			
FREQUENCY (in Hz)			



Problem Fall-Q.1.3:

The signal $x(t)$ is formed from the signal $v(t)$ by AM modulation. Assume that

$$v(t) = -2 + 2 \cos(6t - \pi/4)$$

and that

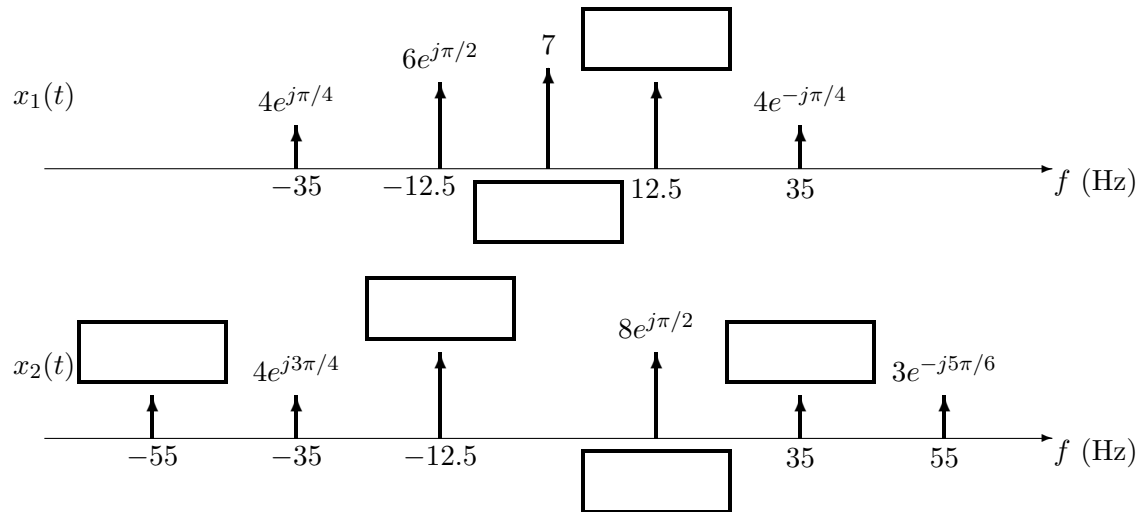
$$x(t) = v(t) \cos(30t).$$

- (a) Draw the spectrum for $v(t)$. Your sketch should be clearly labeled and all complex amplitudes should be indicated.

- (b) Draw the spectrum for $x(t)$. Your sketch should be clearly labeled and all complex amplitudes should be clearly indicated.

Problem Fall-Q.1.4:

- (a) The incomplete spectra for two *real* signals $x_1(t)$ and $x_2(t)$ are shown in the following figures. Fill in the empty boxes for the missing components.



- (b) Write an equation for $x_2(t)$ in terms of cosine functions.

Problem Fall-Q.1.5:

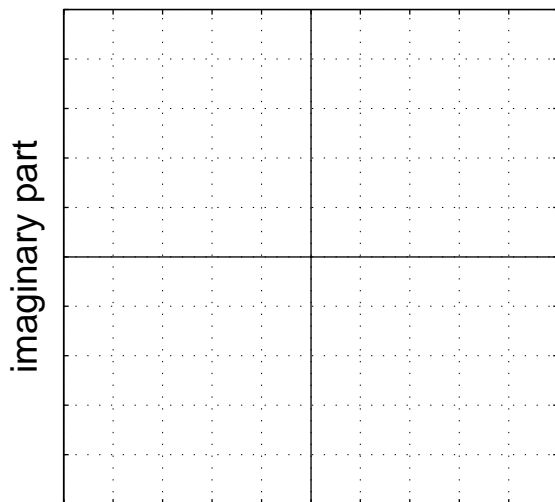
Define $x(t)$ as

$$x(t) = 2 \cos(5\pi(t - .6)) + \sqrt{2} \cos(5\pi t + \pi/4)$$

- (a) Use phasor addition to express $x(t)$ in the form $x(t) = A \cos(\omega_0 t + \phi)$ by finding the numerical values of A and ϕ , as well as ω_0 .

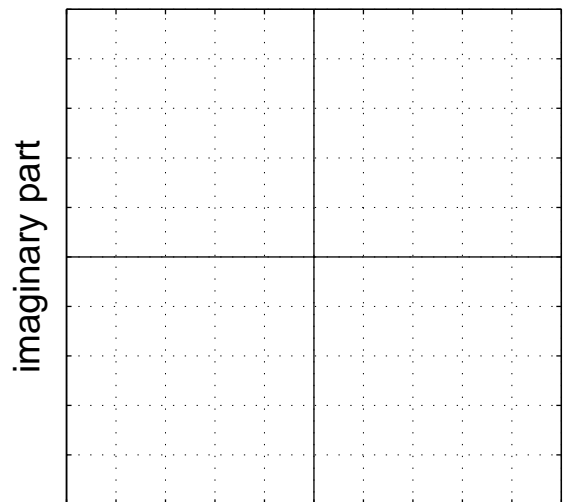
- (b) Make two complex plane plots to illustrate how complex amplitudes (phasors) were used to solve part (a). On the first plot, show the two complex amplitudes being added; on the second plot, show your solution as a vector and the addition of the two complex amplitudes as vectors (head-to-tail).

Two vectors here.



real part

Head-to-tail plot here.



real part

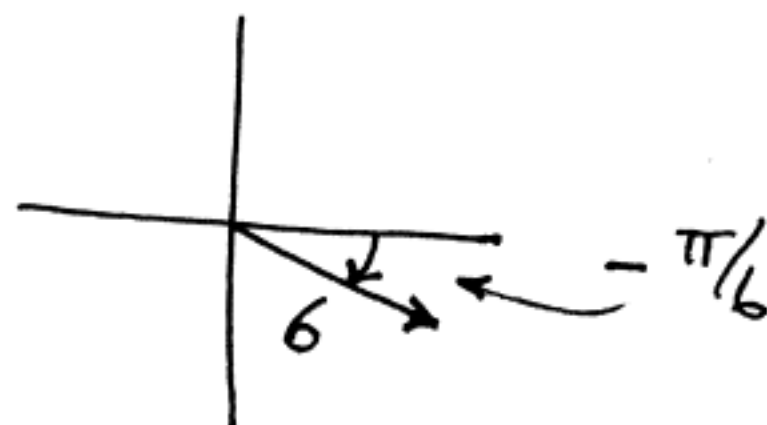
Problem Fall-Q.1.1:

Simplify the following complex-valued expressions. In each case reduce the answers to a **simple** numerical form. Let

$$V = -3 - j3\sqrt{3}.$$

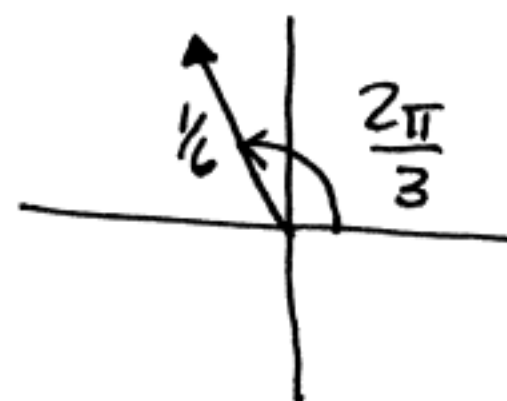
- (a) Express jV in polar form. In addition plot jV as a vector.

$$\begin{aligned} jV &= 3\sqrt{3} - j3 \\ &= 6e^{-j\pi/6} \end{aligned}$$



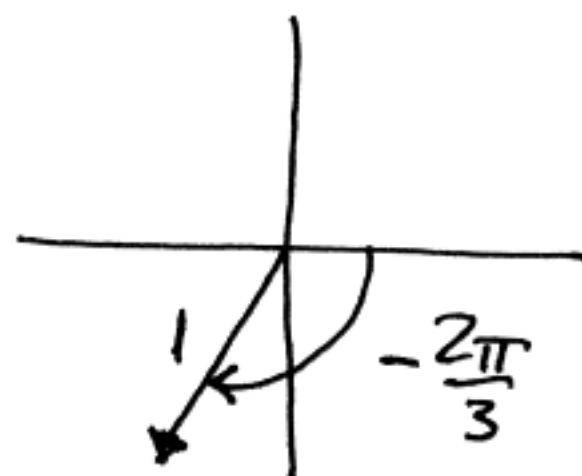
- (b) Express the inverse of V in rectangular form. In addition plot $\frac{1}{V}$ as a vector.

$$\begin{aligned} V &= 6e^{-j2\pi/3} \\ \frac{1}{V} &= \frac{1}{6}e^{j2\pi/3} \\ &= -\frac{1}{12} + j\frac{\sqrt{3}}{12} \end{aligned}$$



- (c) If $Z = \frac{|V|}{V^*}$, express Z in polar form. In addition plot Z as a vector.

$$Z = \frac{6}{6e^{j2\pi/3}} = e^{-j2\pi/3}$$



- (d) Express $\Re\{j^3 V e^{j15t}\}$ in the standard "cosine" form.

$$\Re\{j^3 V e^{j15t}\} = \Re\left\{e^{-j\frac{\pi}{2}} \cdot 6 \cdot e^{-j\frac{2\pi}{3}} \cdot e^{j15t}\right\} = 6 \cos\left(15t + \frac{5\pi}{6}\right)$$

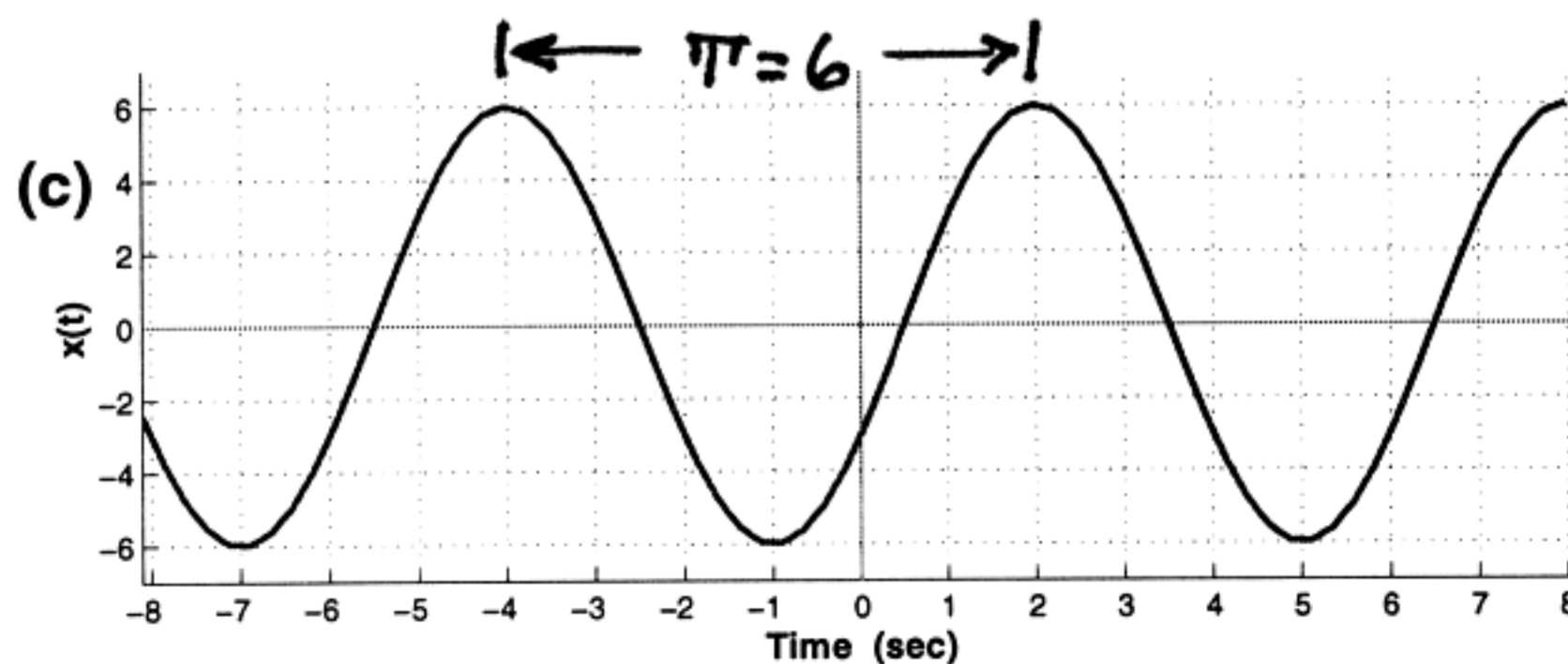
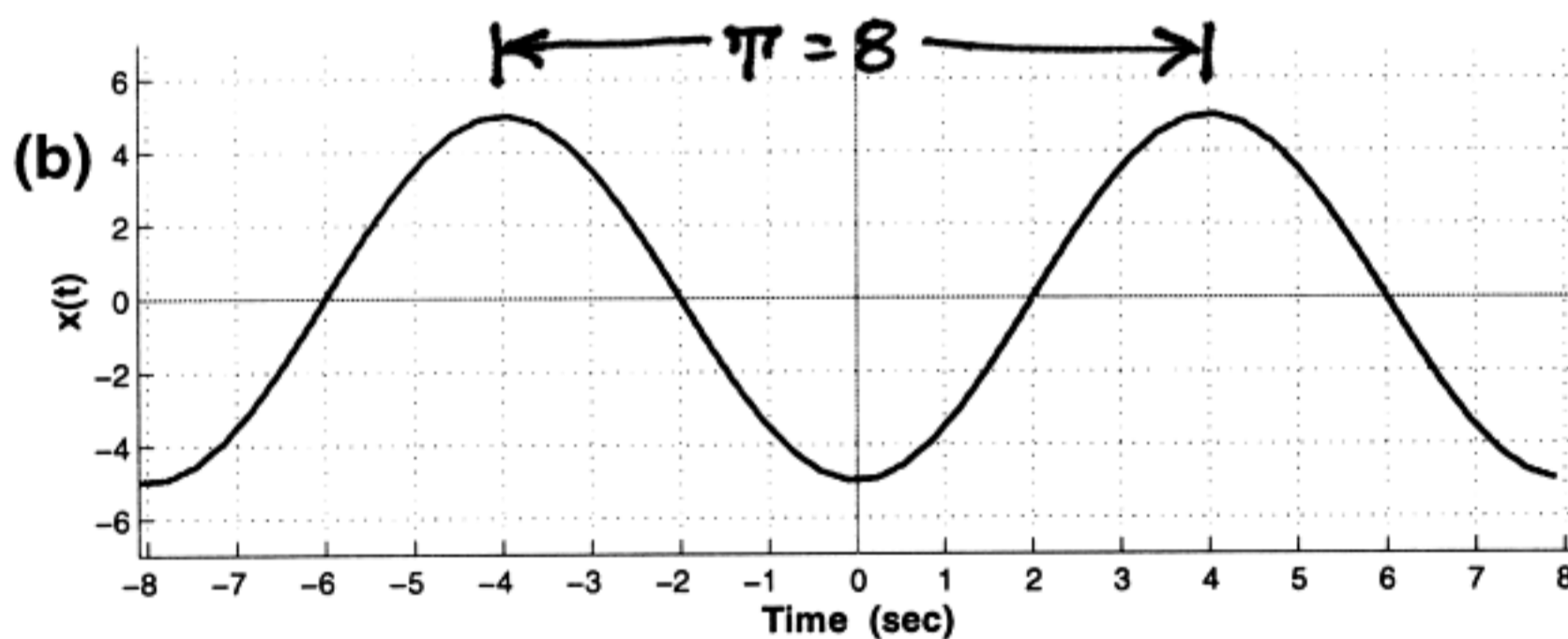
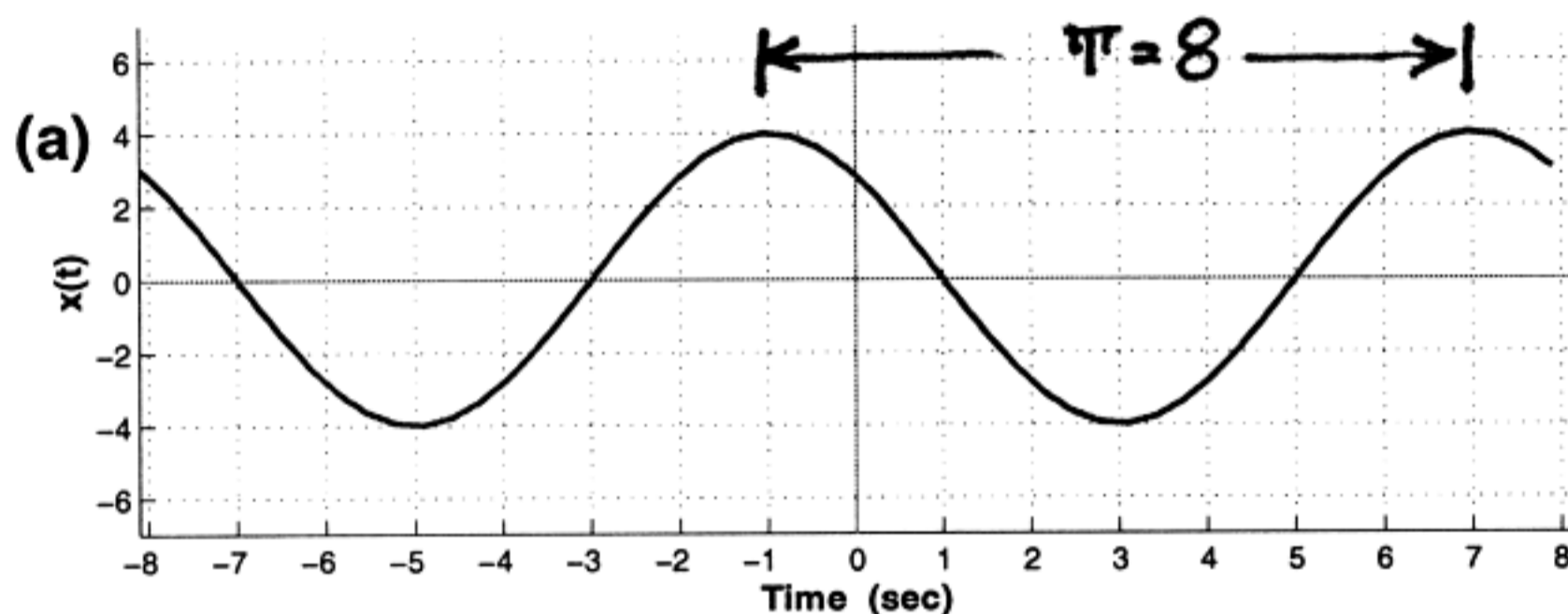
Problem Fall-Q.1.2:

Several sinusoidal signals are plotted below. For each plot (a)–(c), determine the amplitude, phase (in radians) and frequency (in Hz). Write your answers in the following table:

PLOT	(a)	(b)	(c)
AMPLITUDE	4	5	6
PHASE (in radians)	$\frac{\pi}{4}$	π	$-\frac{2\pi}{3}$
FREQUENCY (in Hz)	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{6}$

$$\phi = -\omega t_m = -2\pi f t_m$$

← ALLOW FOR EQUIVALENT PHASE



Problem Fall-Q.1.3:

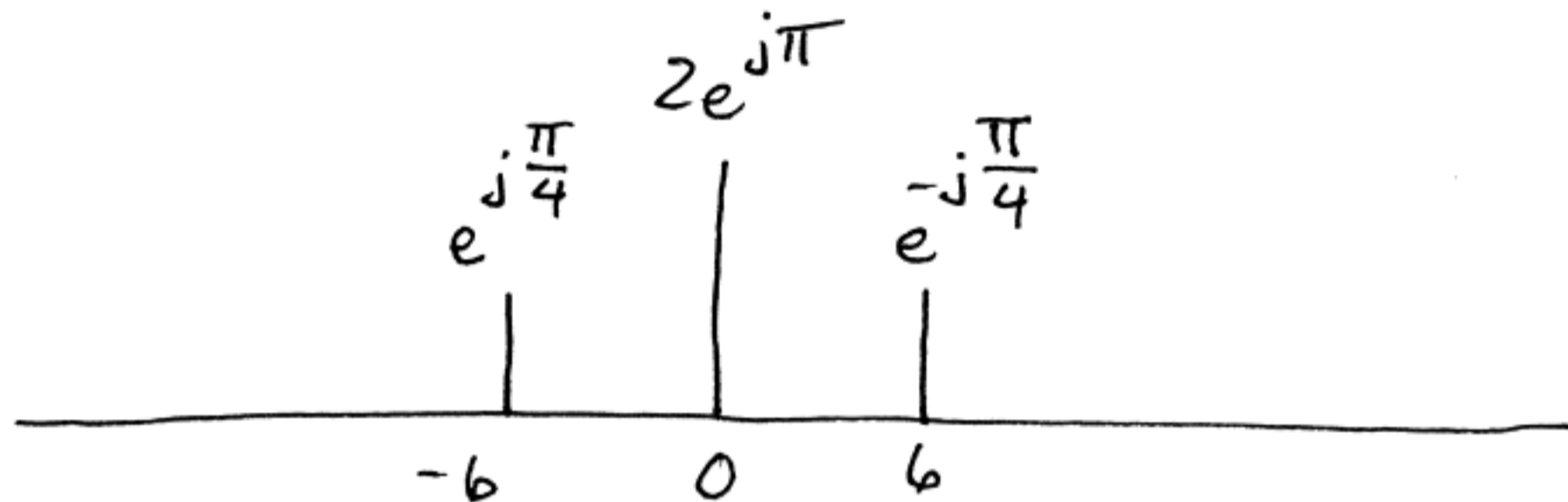
The signal $x(t)$ is formed from the signal $v(t)$ by AM modulation. Assume that

$$v(t) = -2 + 2 \cos(6t - \pi/4) = 2e^{j\pi} + e^{-j\pi/4} e^{j6t} + e^{j\pi/4} e^{-j6t}$$

and that

$$x(t) = v(t) \cos(30t).$$

- (a) Draw the spectrum for $v(t)$. Your sketch should be clearly labeled and all complex amplitudes should be indicated.



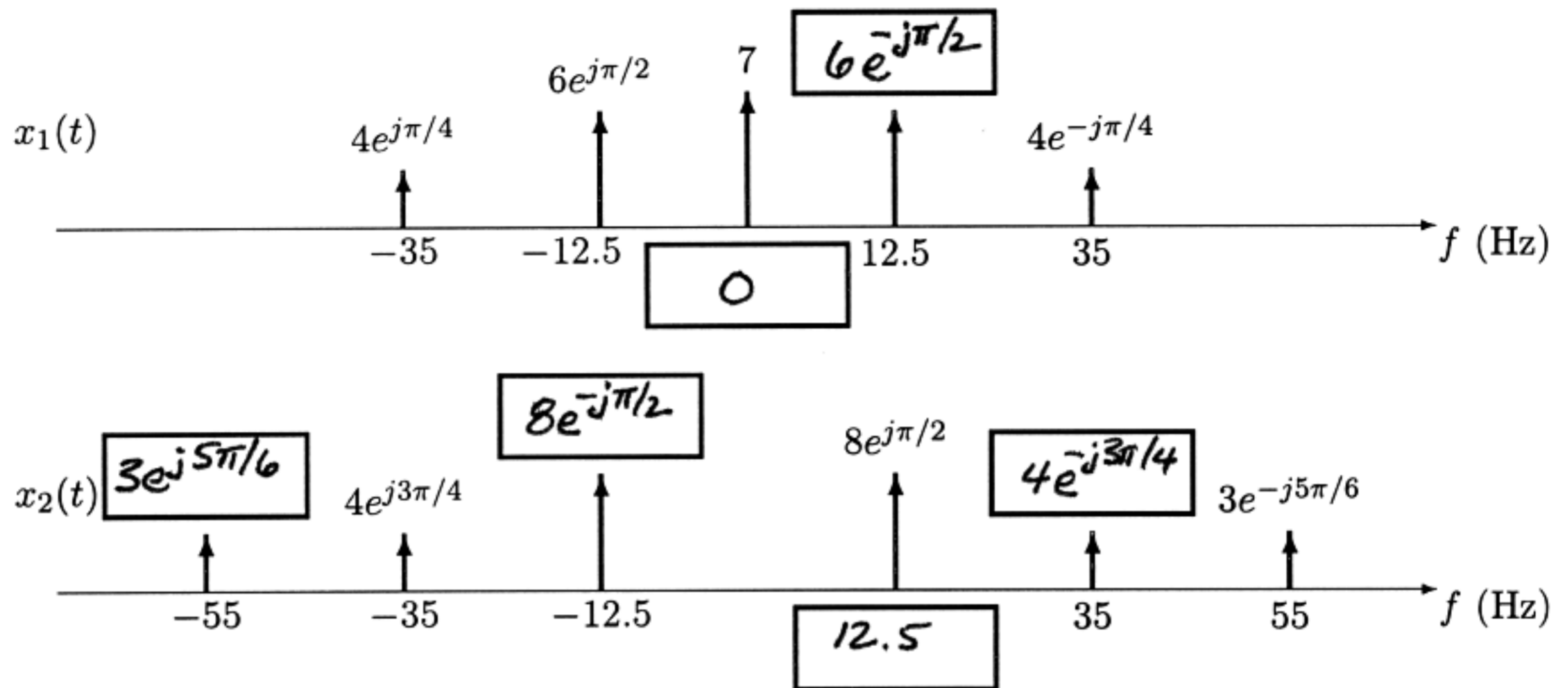
- (b) Draw the spectrum for $x(t)$. Your sketch should be clearly labeled and all complex amplitudes should be indicated.

$$\begin{aligned} X(t) &= v(t) \cos 30t = \left[2e^{j\pi} + e^{-j\pi/4} e^{j6t} + e^{j\pi/4} e^{-j6t} \right] \frac{1}{2} \left[e^{j30t} + e^{-j30t} \right] \\ &= e^{j\pi} e^{j30t} + \frac{1}{2} e^{-j\pi/4} e^{j36t} + \frac{1}{2} e^{j\pi/4} e^{j24t} + e^{j\pi} e^{-j30t} + \frac{1}{2} e^{-j\pi/4} e^{-j24t} \\ &\quad + \frac{1}{2} e^{j\pi/4} e^{-j36t} \end{aligned}$$



Problem Fall-Q.1.4:

- (a) The incomplete spectra for two *real* signals $x_1(t)$ and $x_2(t)$ are shown in the following figures. Fill in the empty boxes for the missing components.



- (b) Write an equation for $x_2(t)$ in terms of cosine functions.

$$x_2(t) = 16 \cos\left(2\pi(12.5)t + \frac{\pi}{2}\right) + 8 \cos\left(2\pi(35)t - \frac{3\pi}{4}\right) + 6 \cos\left(2\pi(55)t - \frac{5\pi}{6}\right)$$

Problem Fall-Q.1.5:Define $x(t)$ as

$$x(t) = 2 \cos(5\pi(t - .6)) + \sqrt{2} \cos(5\pi t + \pi/4)$$

- (a) Use phasor addition to express $x(t)$ in the form $x(t) = A \cos(\omega_0 t + \phi)$ by finding the numerical values of A and ϕ , as well as ω_0 .

$$x_1(t) = 2 \cos(5\pi t - 3\pi) = 2 \cos(5\pi t + \pi)$$

$$X_1 = 2e^{j\pi} = -2$$

$$X_2 = \sqrt{2} e^{j\frac{\pi}{4}} = 1 + j$$

$$X = -1 + j = \sqrt{2} e^{j\frac{3\pi}{4}}$$

$$x(t) = \sqrt{2} \cos(5\pi t + \frac{3\pi}{4})$$

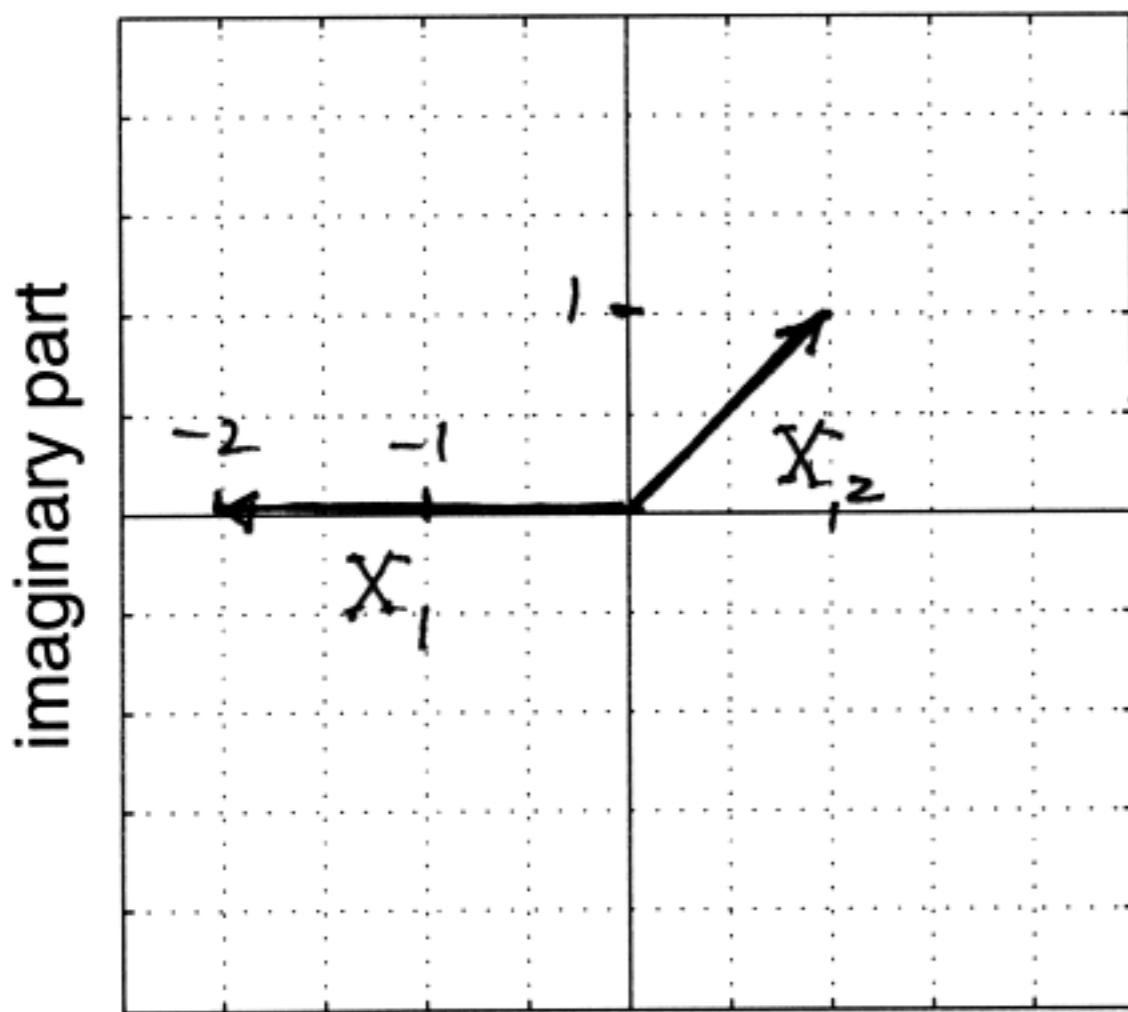
$$A = \sqrt{2}$$

$$\omega_0 = 5\pi$$

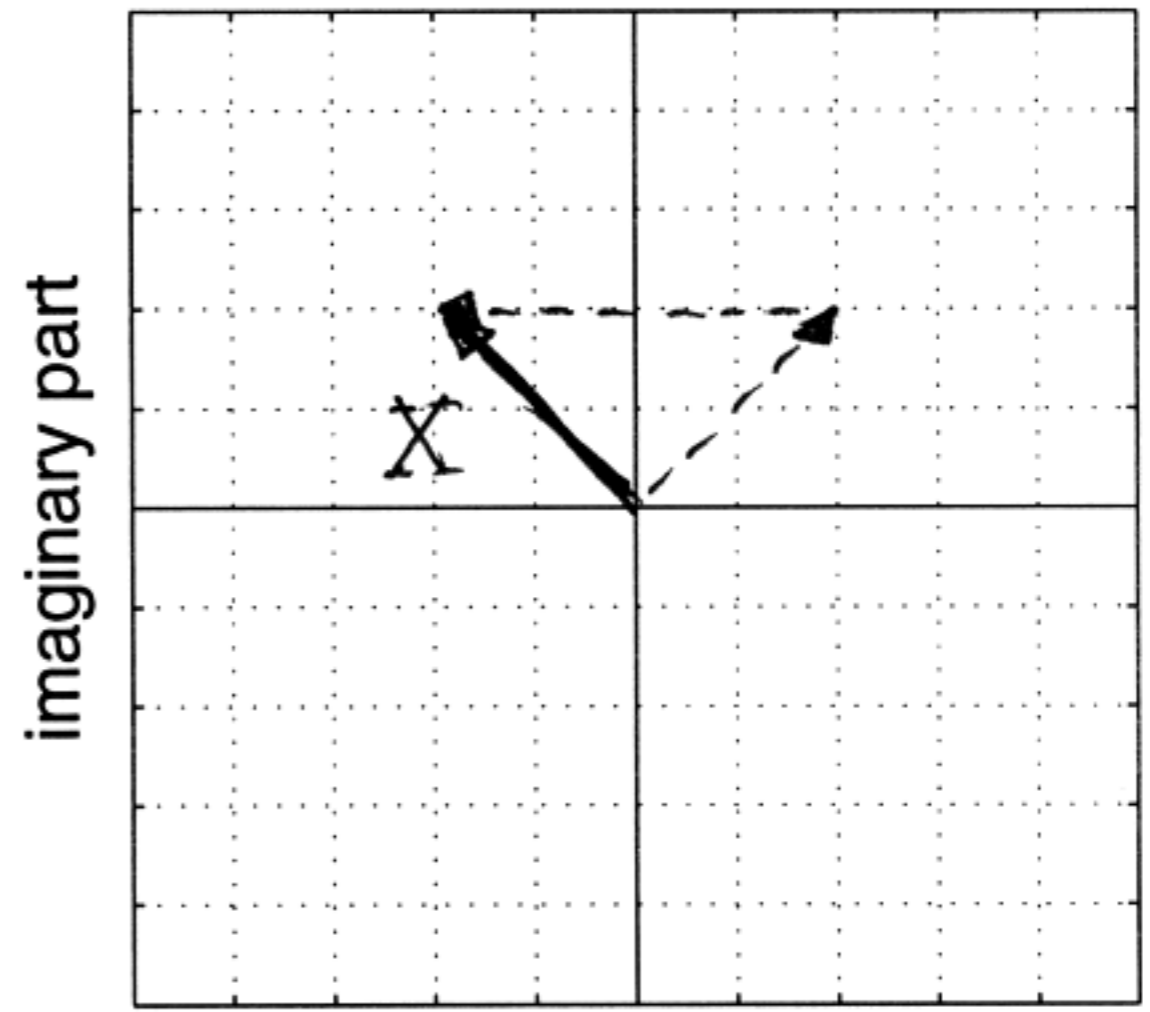
$$\phi = \frac{3\pi}{4}$$

- (b) Make two complex plane plots to illustrate how complex amplitudes (phasors) were used to solve part (a). On the first plot, show the two complex amplitudes being added; on the second plot, show your solution as a vector and the addition of the two complex amplitudes as vectors (head-to-tail).

Two vectors here.



Head-to-tail plot here.



real part

real part