**GEORGIA INSTITUTE OF TECHNOLOGY**

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

# QUIZ #1

DATE: 18-Sept-00 COURSE: ECE 2025

NAME: ______________________________ STUDENT #: ________________

LAST, FIRST

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Recitation Section: **Circle the day & time** when your Recitation Section meets:

| | | |
|---|---|---|
| L01:Tues-9:30am (Casinovi) | L02:Thur-9:30am (Bordelon) | L03:Tues-12:00pm (Casinovi) |
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| L13:Mon -3:00pm (Williams) | L14:Weds-3:00pm (Bordelon) | L15:Mon -4:30pm (Verriest) |
| L16:Weds-4:30pm (Dansereau) | L17:Mon -6:00pm (Verriest) | L18:Weds-6:00pm (Dansereau) |
| L19:Weds-1:30pm (Bordelon) | L20:Mon -1:30pm (Williams) | |

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- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted. However, one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes permitted. OK to write on both sides.
- Justify your reasoning clearly to receive any partial credit.
Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
Only these answers will be graded. Circle your answers, or write them in the boxes provided.
If space is needed for scratch work, use the backs of previous pages.

| *Problem* | *Value* | *Score* |
|---|---|---|
| 1 | 20 | |
| 2 | 20 | |
| 3 | 20 | |
| 4 | 20 | |
| 5 | 20 | |

## Problem F-00-Q.1.1:

This problem is concerned with operations on complex numbers. To receive full credit, you must show your work.

(a) Find the magnitude of the complex number $(1 + 3j)e^{j(0.4\pi)t}$.

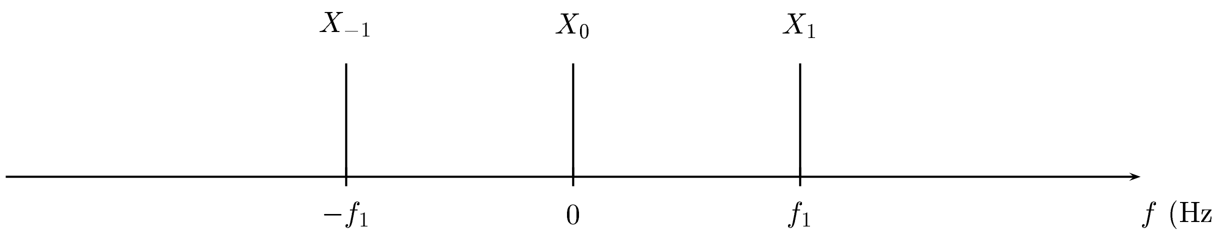
| |
|---|
| $\|(1 + 3j)e^{j(0.4\pi)t}\| =$ |

(b) Find ONE value for $\theta$ so that $\text{Re}\left\{(1 + j)e^{j\theta}\right\} = 0$.

| |
|---|
| $\theta =$ |

**Problem F-00-Q.1.2:**
In each of the following parts, two different representations for a signal are given. Find the values of the parameters in the second representation so that the two representations are equivalent.

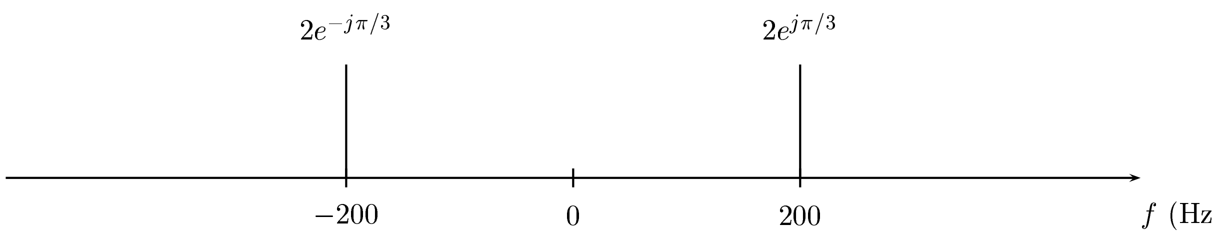
(a) A signal $x(t)$ is given by $x(t) = 3\cos(250\pi t - \pi/6)$, and its spectrum has the form




Determine the values for $f_1$, $X_0$, $X_1$, and $X_{-1}$. Note that the frequencies $f$ are given in Hertz.

| $f_1 =$ | $X_0 =$ | $X_1 =$ | $X_{-1} =$ |
|---|---|---|---|

(b) The spectrum of a signal $x(t)$ has the form



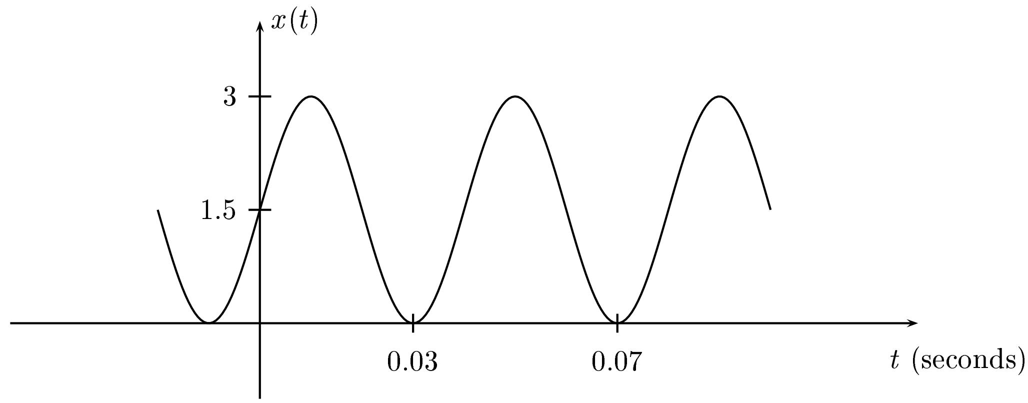

Therefore, the signal has the form

$$x(t) = A\cos(2\pi f_0(t - t_0))$$

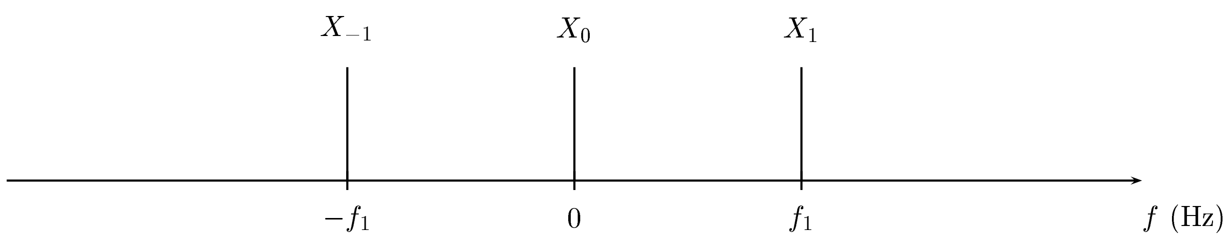
Determine the values for $A$, $f_0$, and $t_0$,

| $A =$ | $f_0 =$ | $t_0 =$ |
|---|---|---|

**Problem F-00-Q.1.3:**
A signal $x(t) = A\cos(2\pi f_1 t + \phi)$ is shown in the figure below,




The spectrum of $x(t)$ has the form




Determine the values for $f_1$, $X_0$, $X_1$, and $X_{-1}$. Note that the frequencies $f$ are given in Hertz.

| | | | |
|---|---|---|---|
| $f_1 =$ | $X_0 =$ | $X_1 =$ | $X_{-1} =$ |

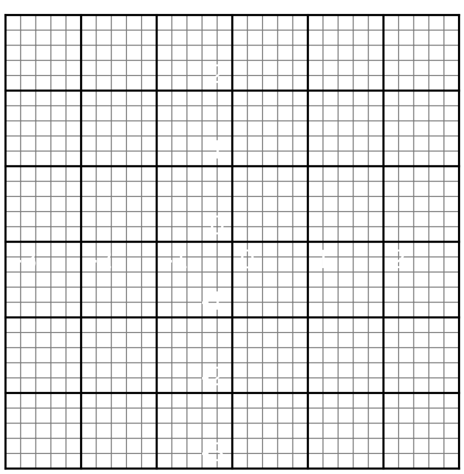
**Problem F-00-Q.1.4:**

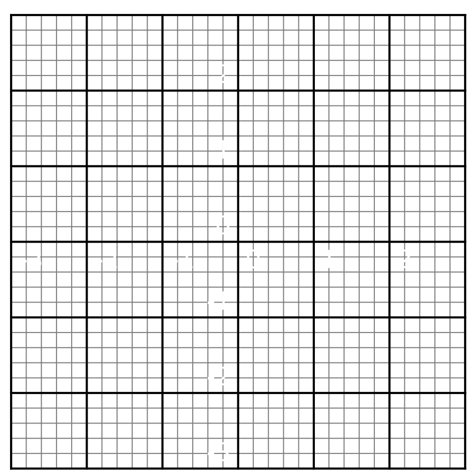
Define $x(t)$ as

$$x(t) = 7\cos(100\pi t - 3\pi/4) + 3\cos(100\pi(t + 0.005))$$

(a) Use phasor addition to express $x(t)$ in the form $x(t) = A\cos(\omega_0 t + \phi)$ by finding the numerical values of $A$ and $\phi$, as well as $\omega_0$.

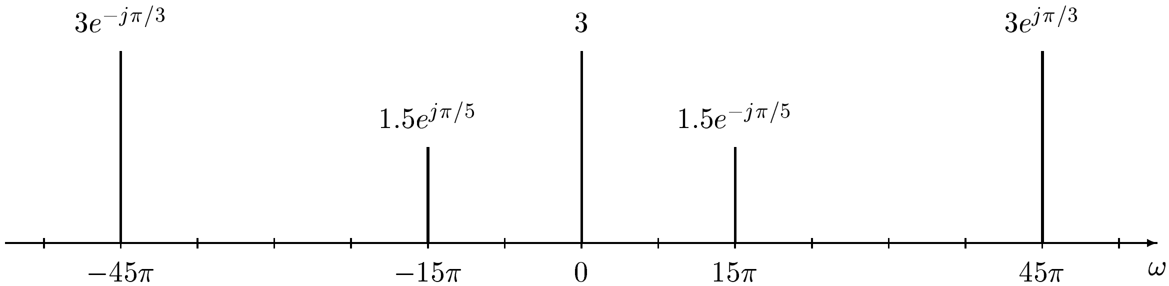
(b) Make two complex plane plots to illustrate how complex amplitudes (phasors) were used to solve part (a). On the first plot, show the two complex amplitudes being added; on the second plot, show your solution as a vector and the addition of the two complex amplitudes as vectors (head-to-tail).





**Problem F-00-Q.1.5:**

The spectrum of a signal $x(t)$ is shown in the following figure:




**Note that the frequency axis is radian frequency ($\omega$) *not* cyclic frequency ($f$).**

(a) Write an equation for $x(t)$ in terms of cosine functions.

(b) This signal is periodic. What is the fundamental frequency and the corresponding period of $x(t)$?

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| 5 | 20 | |

**Problem F-00-Q.1.1:**

This problem is concerned with operations on complex numbers. To receive full credit, you must show your work.

(a) Find the magnitude of the complex number $(1+3j)e^{j(0.4\pi)t}$.

$|(1+3j)e^{j(0.4\pi)t}| = \sqrt{10}$

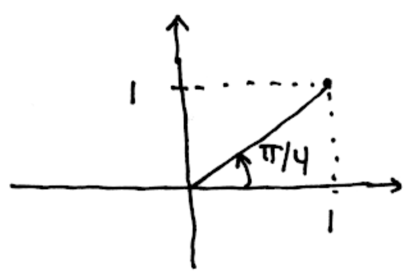
$$|(1+3j)\,e^{j(0.4\pi)t}| = |(1+3j)|\cdot\underbrace{|e^{j(0.4\pi)t}|}_{=1}$$

$$= |(1+3j)| = \sqrt{1^2+3^2} = \sqrt{10}$$

(b) Find ONE value for $\theta$ so that $\text{Re}\left\{(1+j)e^{j\theta}\right\} = 0$.

$\theta = \pi/4$

$$(1+j) = \sqrt{2}\,e^{j\pi/4}$$




$$\Rightarrow (1+j)e^{j\theta} = \sqrt{2}\,e^{j(\theta+\pi/4)} \text{ and}$$

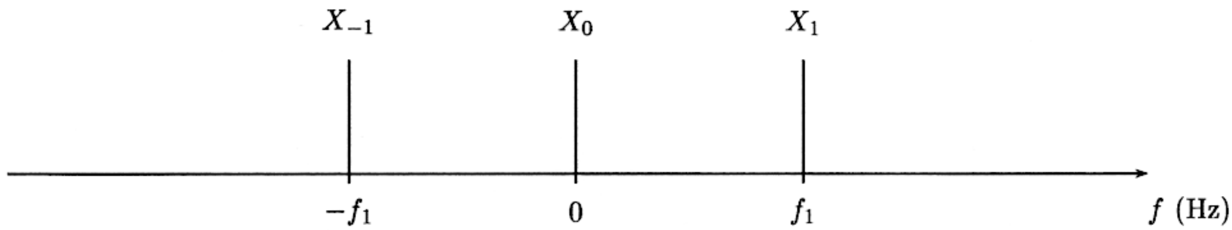
$$\text{Re}\left\{(1+j)e^{j\theta}\right\} = \sqrt{2}\cos(\theta+\pi/4)$$

So $\theta = \pi/4$ works. Another way is to draw a picture. Draw the vecter $(1+j)$ and note that multiplying by $e^{j\theta}$ rotates the vecter by $\theta$.

**Problem F-00-Q.1.2:**

In each of the following parts, two different representations for a signal are given. Find the values of the parameters in the second representation so that the two representations are equivalent.

(a) A signal $x(t)$ is given by $x(t) = 3\cos(250\pi t - \pi/6)$, and its spectrum has the form



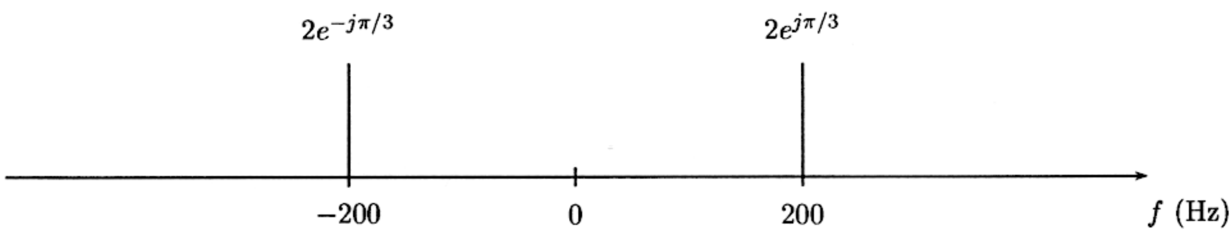

Determine the values for $f_1$, $X_0$, $X_1$, and $X_{-1}$. Note that the frequencies $f$ are given in Hertz.

| $f_1 = 125$ | $X_0 = 0$ | $X_1 = \frac{3}{2}e^{-j\pi/6}$ | $X_{-1} = \frac{3}{2}e^{j\pi/6}$ |
|---|---|---|---|

$$x(t) = 3\left\{\frac{1}{2}e^{j(250\pi t - \pi/6)} + \frac{1}{2}e^{-j(250\pi t - \pi/6)}\right\}$$

$$= \underbrace{\frac{3}{2}e^{-j\pi/6}}_{X_1} \cdot e^{j250\pi t} + \underbrace{\frac{3}{2}e^{j\pi/6}}_{X_{-1}} \cdot e^{-j250\pi t}$$

(b) The spectrum of a signal $x(t)$ has the form




Therefore, the signal has the form

$$x(t) = A\cos(2\pi f_0(t - t_0))$$

Determine the values for $A$, $f_0$, and $t_0$,

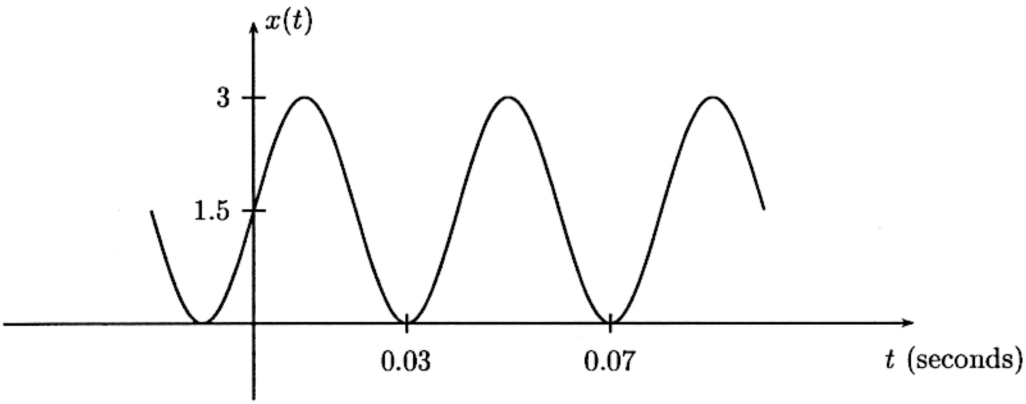
| $A = 4$ | $f_0 = 200$ | $t_0 = \frac{-1}{1200}$ |
|---|---|---|

$$x(t) = 2e^{j\pi/3} \cdot e^{j(200)2\pi t} + 2e^{-j\pi/3} e^{-j(200)2\pi t}$$

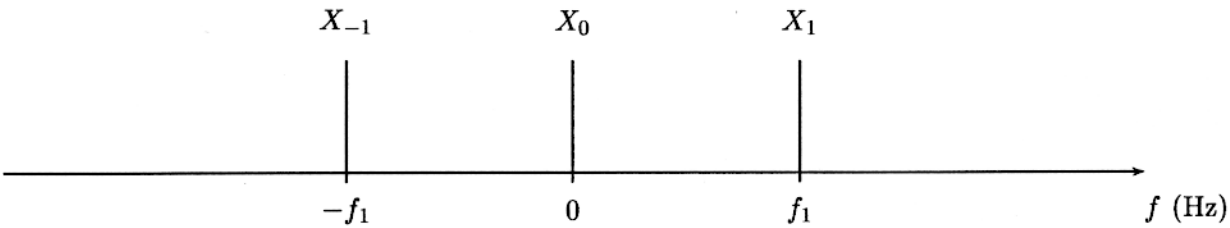
$$= 4\cos(2\pi(200)t + \pi/3) = 4\cos\left(400\pi\left(t + \frac{1}{1200}\right)\right)$$

↑ $f_0$ ↑ $A$ ↑ $t_0 = -\frac{1}{1200}$

**Problem F-00-Q.1.3:**
A signal $x(t) = A\cos(2\pi f_1 t + \phi)$ is shown in the figure below,




The spectrum of $x(t)$ has the form




Determine the values for $f_1$, $X_0$, $X_1$, and $X_{-1}$. Note that the frequencies $f$ are given in Hertz.

| | | | |
|---|---|---|---|
| $f_1 = 25$ | $X_0 = 1.5$ | $X_1 = \frac{3}{4}e^{-j\pi/2}$ | $X_{-1} = \frac{3}{4}e^{j\pi/2}$ |

Note that the period of $x(t)$ is $T = 0.04$ sec. Therefore, $f_1 = 1/T = 1/0.04 = 25$ Hz. Next observe that the dc (average) value of $x(t)$ is $X_0 = 1.5$. Finally, note that $x(t)$ is a cosine <u>delayed</u> by $t_0 = 0.01$ sec and has an amplitude of 1.5. Therefore,

$$x(t) = 1.5 + 1.5\cos\left(2\pi(25)(t - 0.01)\right)$$

$$= 1.5 + 1.5\cos\left(2\pi(25)t - 0.5\pi\right)$$

$\uparrow$ $X_0$ $\qquad$ $\uparrow$

$$X_1 = \frac{1.5}{2}e^{-j\pi/2} = \frac{3}{4}e^{-j\pi/2} = X_{-1}^*$$

**Problem F-00-Q.1.4:**

Define $x(t)$ as

$$x(t) = 7\cos(100\pi t - 3\pi/4) + 3\cos(100\pi(t + 0.005))$$

(a) Use phasor addition to express $x(t)$ in the form $x(t) = A\cos(\omega_0 t + \phi)$ by finding the numerical values of $A$ and $\phi$, as well as $\omega_0$.

$$x(t) = 7\cos(100\pi t - 3\pi/4) + 3\cos(100\pi t + \pi/2)$$

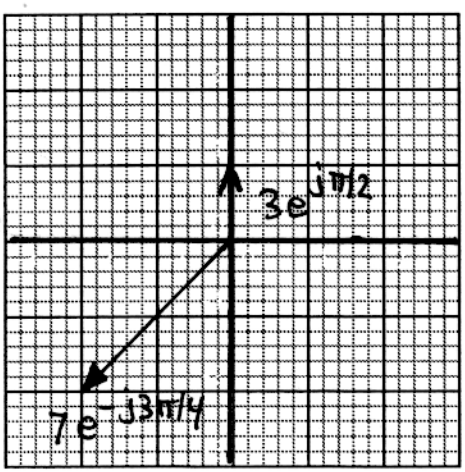
$$= \text{Re}\left\{7e^{-j3\pi/4}e^{j100\pi t} + 3e^{j\pi/2}e^{j100\pi t}\right\}$$

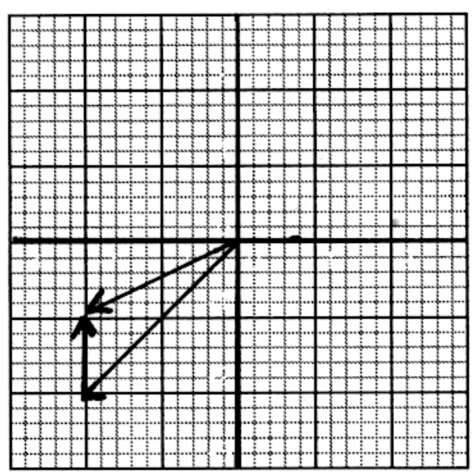
$$= \text{Re}\left\{\underbrace{\left(7e^{-j3\pi/4} + 3e^{j\pi/2}\right)}_{5.3199e^{-j0.8806\pi}}e^{j100\pi t}\right\}$$

$$= \text{Re}\left\{5.3199e^{-j0.8806\pi}\cdot e^{j100\pi t}\right\}$$

$$= 5.3199\cos(100\pi t - 0.8806\pi)$$

(b) Make two complex plane plots to illustrate how complex amplitudes (phasors) were used to solve part (a). On the first plot, show the two complex amplitudes being added; on the second plot, show your solution as a vector and the addition of the two complex amplitudes as vectors (head-to-tail).



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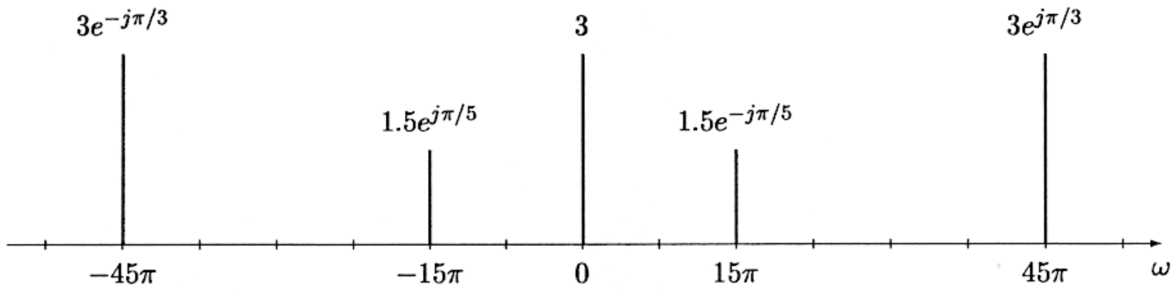
The spectrum of a signal $x(t)$ is shown in the following figure:




**Note that the frequency axis is radian frequency ($\omega$) *not* cyclic frequency ($f$).**

(a) Write an equation for $x(t)$ in terms of cosine functions.

$$x(t) = 3 + \underbrace{1.5e^{-j\pi/5}e^{j15\pi t} + 1.5e^{j\pi/5}e^{-j15\pi t}} + \underbrace{3e^{j\pi/3}e^{j45\pi t} + 3e^{-j\pi/3}e^{-j45\pi t}}$$

$$= 3 \quad + \quad 3\cos(15\pi t - \pi/5) \quad + \quad 6\cos(45\pi t + \pi/3)$$

(b) This signal is periodic. What is the fundamental frequency and the corresponding period of $x(t)$?

$$\omega_0 = 15\pi$$

The period is $T = \frac{1}{f_0} = \frac{2\pi}{\omega_0} = \frac{2}{15}$