GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #1

DATE: 13-Sept-04 COURSE: ECE-2025

NAME: ______________________________ GT #: ________________

LAST, FIRST

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

L02:Thurs-9:30 (Anderson)	L03:Tues-Noon (Williams)	L05:Tues-1:30 (Williams)
L06:Thurs-1:30 (Anderson)	L07:Tues-3:00 (Durey)	L08:Thurs-3:00 (Smith)
L09:Tues-4:30 (Durey)	L10:Thurs-4:30 (Smith)	L13:Mon-3:00 (McClellan)
L14:Wed-3:00 (Taylor)	L15:Mon-4:30 (Hayes)	L16:Wed-4:30 (Taylor)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- JUSTIFY your reasoning clearly to receive partial credit.
  Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	25	
2	25	
3	25	
4	25	

PROBLEM fa-04-Q.1.1:

Simplify the following complex-valued expressions. In each case reduce the answers to the **simple** numerical form requested. Let

$$U = -1 + j\sqrt{3}; \qquad V = \frac{1}{2}e^{-j\frac{3\pi}{4}}.$$

(a) Express $X = U + jV$ in rectangular form.

(b) Express $Y = UV^*$ in polar form.

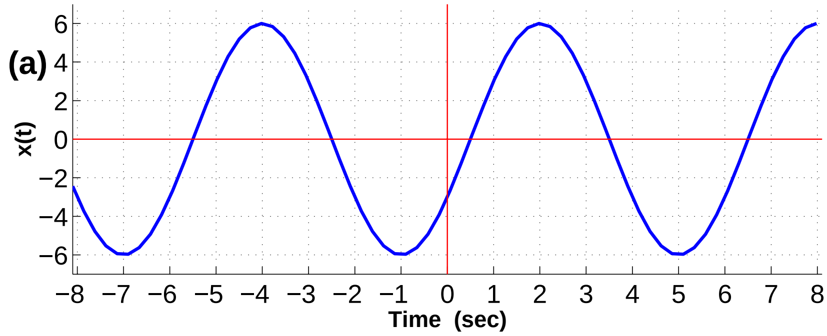
(c) Express $Z = V + V^*$ in rectangular form.

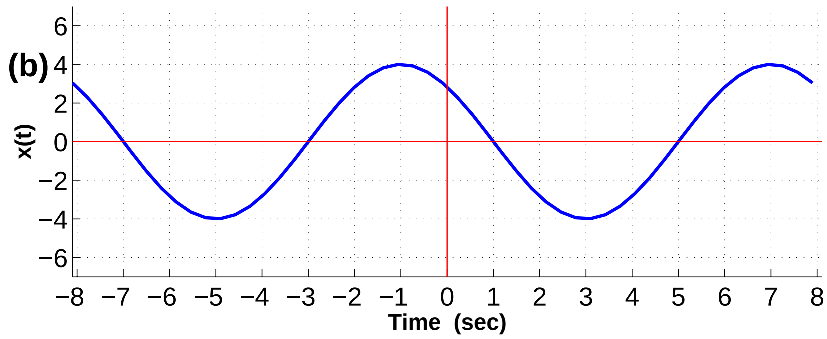
(d) Determine $\Im m\left\{\frac{V}{|U|}\right\}$.

(e) Express $\Re e\left\{jUe^{-j3t}\right\}$ in the standard "cosine" form.

PROBLEM fa-04-Q.1.2:

Two sinusoidal signals are plotted below. For each signal, determine the amplitude, phase (in radians) and frequency (in Hz).





(a) Determine the amplitude, A_a, frequency in Hz., f_a, and phase, ϕ_a, of the upper sinusoid.

(b) Determine the amplitude, A_b, frequency in Hz., f_b, and phase, ϕ_b, of the lower sinusoid.

(c) What is the fundamental frequency (in Hz.) of the sum of these two sinusoids?

PROBLEM fa-04-Q.1.3:

The following MATLAB code defines several signals that are then multiplied and summed:

```
tt = -10:0.001:10;
xxe = 4*cos(100*pi*tt);
xx1 = 5*cos( 3*pi*(tt + 1/4 ) );
xx2 = 5*cos( 3*pi*tt + 7*pi/2 );
xx = xxe.*(xx1 + xx2);
```

(a) If the signal $x_1(t)$ corresponds to the MATLAB vector `xx1`, determine the complex amplitude of $x_1(t)$.

(b) If the signal $x(t)$ corresponds to the MATLAB vector `xx`, then it is possible to express $x(t)$ in the form

$$x(t) = A\cos(\omega_1 t)\cos(\omega_2 t + \phi)$$

Determine the numerical values of A, ω_1, ω_2 and ϕ. *Hint:* Use phasor addition.

$A =$ ________

$\omega_1 =$ ________

$\omega_2 =$ ________

$\phi =$ ________

PROBLEM fa-04-Q.1.4:

Let

$$x_1(t) = 3\cos(5\pi t - \frac{\pi}{4}); \qquad x_2(t) = \Re e\{j2e^{j5\pi t}\}$$

(a) Plot the spectrum of $x_1(t)$. Be sure to label all of the frequencies and complex amplitudes.

(b) Plot the spectrum of $x_2(t)$. Be sure to label all of the frequencies and complex amplitudes.

(c) Plot the spectrum of $y(t) = x_1(t)x_2(t)$. Be sure to label all of the frequencies and complex amplitudes.

(d) Now let $z(t) = x_1(t)\cos(5\pi t + \phi)$. Determine **all** possible values of ϕ for which the DC value of $z(t)$ will be zero.

PROBLEM fa-04-Q.1.1:

Simplify the following complex-valued expressions. In each case reduce the answers to the **simple** numerical form requested. Let

$$U = -1 + j\sqrt{3}; \qquad V = \frac{1}{2}e^{-j\frac{3\pi}{4}}. = -\frac{1}{2\sqrt{2}} - j\frac{1}{2\sqrt{2}}$$

$$= 2e^{j2\pi/3}$$

(a) Express $X = U + jV$ in rectangular form.

$$= -1 + j\sqrt{3} + j\left(-\frac{1}{2\sqrt{2}} - j\frac{1}{2\sqrt{2}}\right)$$

$$= -1 + j\sqrt{3} - j\frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \boxed{\left(-1 + \frac{1}{2\sqrt{2}}\right) + j\left(\sqrt{3} - \frac{1}{2\sqrt{2}}\right)}$$

(b) Express $Y = UV^*$ in polar form.

$$= \left(2e^{j2\pi/3}\right)\left(\frac{1}{2}e^{j\frac{3\pi}{4}}\right) = e^{j\frac{17\pi}{12}} = \boxed{e^{-j\frac{7\pi}{12}}}$$

(c) Express $Z = V + V^*$ in rectangular form.

$$= -\frac{1}{2\sqrt{2}} - j\frac{1}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} + j\frac{1}{2\sqrt{2}}$$

$$= \boxed{-\frac{1}{\sqrt{2}}}$$

(d) Determine $\Im m\left\{\frac{V}{|U|}\right\}$. $= \Im m\left\{\frac{1}{2}\left(-\frac{1}{2\sqrt{2}} - j\frac{1}{2\sqrt{2}}\right)\right\}$

$$= \boxed{-\frac{1}{4\sqrt{2}}}$$

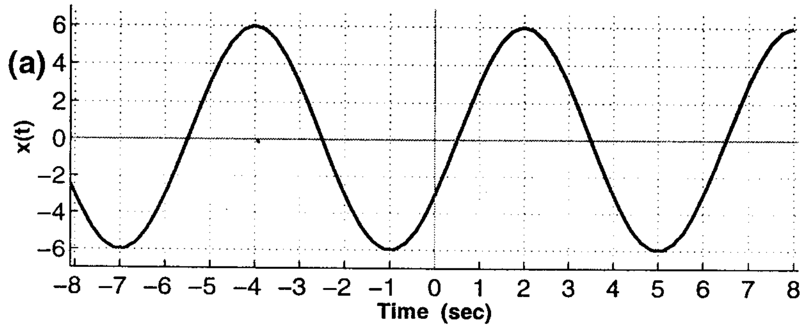
(e) Express $\Re e\left\{jUe^{-j3t}\right\}$ in the standard "cosine" form.

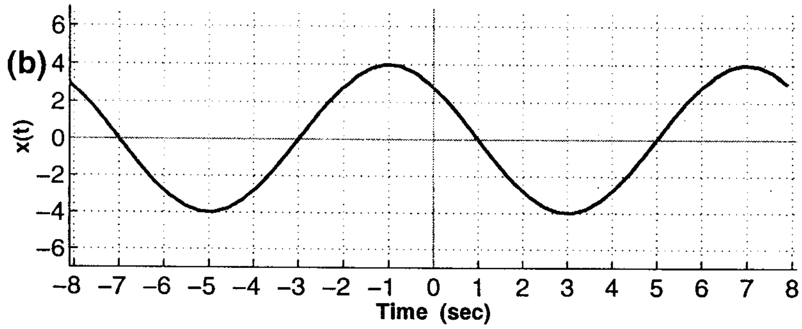
$$= \Re e\left\{e^{j\frac{\pi}{2}} \cdot 2e^{j\frac{2\pi}{3}} e^{-j3t}\right\}$$

$$= \Re e\left\{e^{-j\frac{5\pi}{6}} \cdot 2e^{-j3t}\right\} = \boxed{2\cos\left(3t + \frac{5\pi}{6}\right)}$$

PROBLEM fa-04-Q.1.2:

Two sinusoidal signals are plotted below. For each signal, determine the amplitude, phase (in radians) and frequency (in Hz).





(a) Determine the amplitude, A_a, frequency in Hz., f_a, and phase, ϕ_a, of the upper sinusoid.

$\boxed{A_a = 6}$ $\quad t_d = 2$ sec

$T_a = 6$ sec $\quad \phi_a = -\omega_a t_d = -\frac{2\pi}{6} \cdot 2 = \boxed{-\frac{2\pi}{3}}$ rad.

$\boxed{f_a = \frac{1}{6}\text{ Hz.}}$

(b) Determine the amplitude, A_b, frequency in Hz., f_b, and phase, ϕ_b, of the lower sinusoid.

$\boxed{A_b = 4}$ $\quad t_d = -1$ sec.

$T_b = 8$ sec. $\quad \phi_b = -\omega_b t_d = \left(\frac{2\pi}{8}\right) 1 = \boxed{\frac{\pi}{4}}$ rad.

$\boxed{f_b = \frac{1}{8}\text{ Hz.}}$

(c) What is the fundamental frequency (in Hz.) of the sum of these two sinusoids?

$f_a = \frac{4}{24} \quad f_b = \frac{3}{24} \Rightarrow \boxed{f_0 = \frac{1}{24}\text{ Hz.}}$

PROBLEM fa-04-Q.1.3:

The following MATLAB code defines several signals that are then multiplied and summed:

```
tt = -10:0.001:10;
xxe = 4*cos(100*pi*tt);
xx1 = 5*cos( 3*pi*(tt + 1/4 ) );
xx2 = 5*cos( 3*pi*tt + 7*pi/2 );
xx = xxe.*(xx1 + xx2);
```

(a) If the signal $x_1(t)$ corresponds to the MATLAB vector xx1, determine the complex amplitude of $x_1(t)$.

$$X_1 = 5e^{j\frac{3\pi}{4}}$$

(b) If the signal $x(t)$ corresponds to the MATLAB vector xx, then it is possible to express $x(t)$ in the form

$$x(t) = A\cos(\omega_1 t)\cos(\omega_2 t + \phi)$$

Determine the numerical values of A, ω_1, ω_2 and ϕ. *Hint:* Use phasor addition.

$A = (20)(0.7654) = 15.3073$

$\omega_1 = 100\pi$ rad/s

$\omega_2 = 3\pi$ rad/s

$\phi = -2.7489$ rad.

$$x(t) = 4\cos(100\pi t)\left[5\cos\left(3\pi t + \frac{3\pi}{4}\right) + 5\cos\left(3\pi t + \frac{7\pi}{2}\right)\right]$$

$$= 20\cos(100\pi t)\left[\cos\left(3\pi t + \frac{3\pi}{4}\right) + \cos\left(3\pi t + \frac{7\pi}{2}\right)\right]$$

$$X_1 = e^{j\frac{3\pi}{4}}$$

$$X_2 = e^{j\frac{7\pi}{2}}$$

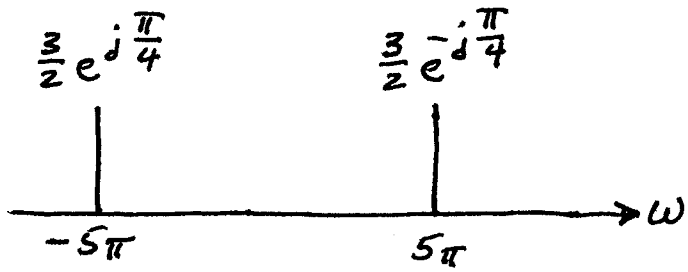
$$X_1 + X_2 = 0.7654\, e^{-j(2.7489)}$$

PROBLEM fa-04-Q.1.4:

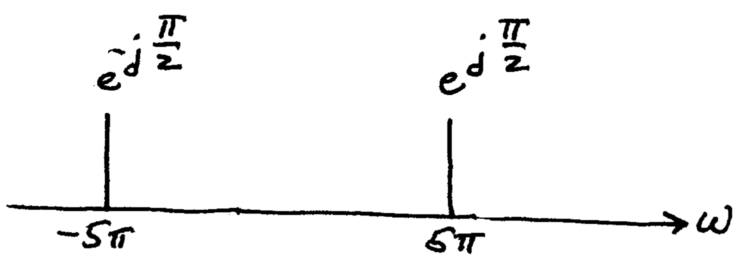
Let

$$x_1(t) = 3\cos(5\pi t - \frac{\pi}{4}); \qquad x_2(t) = \Re e\{j2e^{j5\pi t}\} = -2\sin(5\pi t)$$
$$= 2\cos(5\pi t + \pi/2)$$

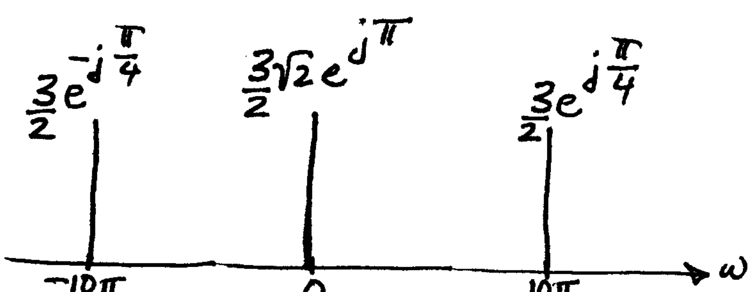
(a) Plot the spectrum of $x_1(t)$. Be sure to label all of the frequencies and complex amplitudes.



(b) Plot the spectrum of $x_2(t)$. Be sure to label all of the frequencies and complex amplitudes.



(c) Plot the spectrum of $y(t) = x_1(t)x_2(t)$. Be sure to label all of the frequencies and complex amplitudes.



(d) Now let $z(t) = x_1(t)\cos(5\pi t + \phi)$. Determine **all** possible values of ϕ for which the DC value of $z(t)$ will be zero.

$$z(t) = 3\cos(5\pi t - \tfrac{\pi}{4})\cos(5\pi t + \phi)$$
$$= \frac{3}{4}\left(e^{-j\frac{\pi}{4}}e^{j5\pi t} + e^{j\frac{\pi}{4}}e^{-j5\pi t}\right)\left(e^{j\phi}e^{j5\pi t} + e^{-j\phi}e^{-j5\pi t}\right)$$
$$= \frac{3}{4}e^{j(\phi-\frac{\pi}{4})}e^{j10\pi t} + \frac{3}{4}e^{-j(\phi-\frac{\pi}{4})}e^{-j10\pi t} + \underbrace{\left[e^{j(\phi+\frac{\pi}{4})} + e^{-j(\phi+\frac{\pi}{4})}\right]}_{\text{DC Value}}$$

∴ WE WANT

$$\cos(\phi + \tfrac{\pi}{4}) = 0 \Rightarrow \boxed{\phi = \tfrac{\pi}{4} + \pi k, \ k = \text{integer}}$$