GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #2

DATE: 2-Mar-01 COURSE: EE 2025

NAME: ______________________________ STUDENT #: ____________

LAST, FIRST

Recitation Section: Circle the date & time when your Recitation Section meets (not Lab):

Mon-3p (L11:McClellan)	M-4:30p (L13:Frazier)			
Tues-9:30a (L01:Casinovi)	T-Noon (L03:Casinovi)	T-1:30p (L05:Bordelon)	T-3p (L07:Bordelon)	T-4:30p (L09:Casinovi)
Thur-9:30a (L02:Bordelon)	Th-Noon (L04:Bordelon)	Th-1:30p (L06:Smith)	Th-3p (L08:Smith)	Th-4:30p (L09:Casinovi)
Th-6p (L10:Casinovi)				

- Write your name on the front page ONLY.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes permitted. OK to write on both sides.
- JUSTIFY your reasoning CLEARLY to receive any partial credit.
  Explanations are also REQUIRED to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of pages.

Problem	*Value*	*Score*
1	25	
2	25	
3	25	
4	25	

Problem s-01-Q.2.1:

We have shown that an LTI system can be represented in several equivalent ways. In each part below, you are given one representation of an LTI system and you are to provide the other representations requested. (Frequency response formulas can be given in any convenient form. You do **NOT** have to simplify them.)

(a) Frequency response:

Impulse response:

Difference equation: $y[n] = x[n] + x[n-2]$

(b) Frequency response:

Impulse response: $h[n] = \delta[n-1] - \delta[n-3]$

Difference equation:

(c) Frequency response: $\mathcal{H}(\hat{\omega}) = e^{-j\hat{\omega}}(2 + 2\cos(\hat{\omega}))$

Impulse response:

Difference equation:

(d) Frequency response:

Impulse response:

MATLAB Implementation: `y = conv([0,2,0,2],x)`

Problem s-01-Q.2.2:

A signal $x(t)$ is periodic with period $T_0 = 8$. Therefore it can be represented as a Fourier series of the form

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j(2\pi/8)kt}.$$

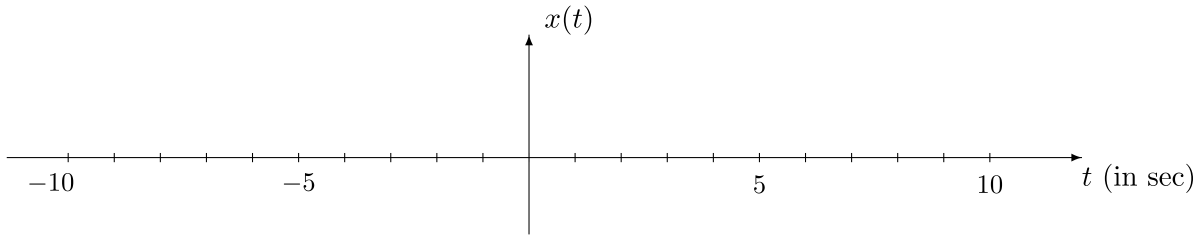
It is known that the Fourier series coefficients for this representation of a particular signal $x(t)$ are given by the integral

$$a_k = \frac{1}{8}\int_{-4}^{0} (4+t)e^{-j(2\pi/8)kt}\,dt. \qquad (1)$$

NOTE: Parts (c) and (d) can be worked independently of parts (a) and (b).

(a) In the expression for a_k in Equation (1) above, the integral and its limits define the signal $x(t)$. Determine an equation for $x(t)$ that is valid over one period.

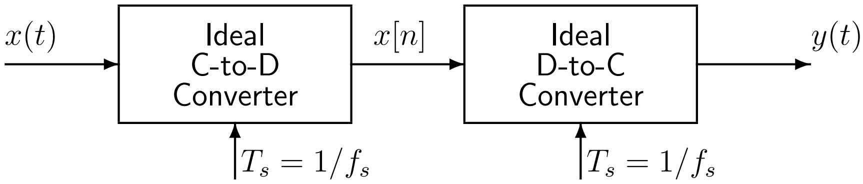
(b) Using your result from part (a), draw a plot of $x(t)$ over the range $-10 \le t \le 10$ seconds. Label it carefully.



(c) Which value of k in Equation (1) gives the DC (or average) value of $x(t)$? | $k =$ |

(d) Determine the DC value of $x(t)$.

Problem s-01-Q.2.3:



In all parts below, the sampling rates of the C/D and D/C converters are **equal,** and the input to the Ideal C/D converter is

$$x(t) = 2\cos(2\pi(50)t + \pi/2) + \cos(2\pi(150)t).$$

(a) If the output of the ideal D-to-C Converter is

$$y(t) = x(t) = 2\cos(2\pi(50)t + \pi/2) + \cos(2\pi(150)t),$$

what general statement can you make about the sampling frequency f_s in this case?

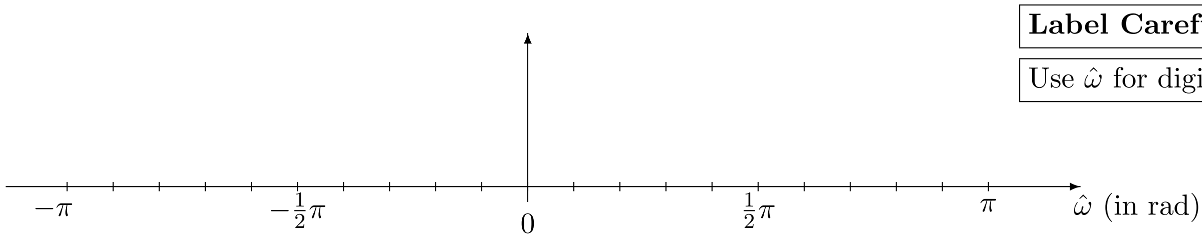
(b) If the sampling rate is $f_s = 250$ samples/sec., determine the discrete-time signal $x[n]$, and give an expression for $x[n]$ as a sum of cosines. *Make sure that all frequencies in your answer are positive and less than π radians.*

$x[n] =$

Plot the spectrum of this signal over the range of frequencies $-\pi \le \hat{\omega} \le \pi$. Make a plot for your answer, but label the frequency, amplitude and phase of each spectral component.

Label Carefully

Use $\hat{\omega}$ for digital freq.

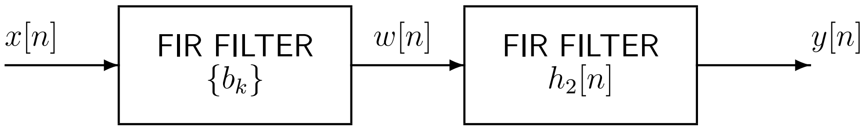


(c) If the output of the Ideal D-to-C Converter is

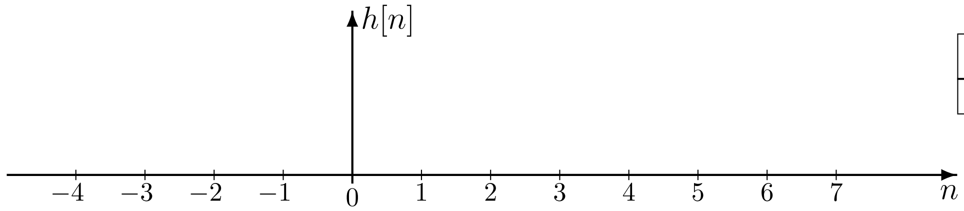
$$y(t) = 2\cos(2\pi(50)t + \pi/2) + 1,$$

determine the value of the sampling frequency f_s. (Remember that the input $x(t)$ is as defined above.)

Problem s-01-Q.2.4:



(a) If the filter coefficients of the first FIR filter are $\{b_k\} = \{0, 1, -2, 1\}$, and the impulse response of the second FIR filter is $h_2[n] = \delta[n] + 2\delta[n-2] + \delta[n-3]$, use convolution to determine the impulse response of the overall system, $h[n]$. Give your answer as a plot below.



Label Carefully

Plot zero values also

(b) Suppose that the overall frequency response of the cascade system (using different FIR filters from those in part (a)) is

$$\mathcal{H}(\hat{\omega}) = (2 + 2\cos(\hat{\omega}))e^{-j\hat{\omega}}$$

If the input signal is $x[n] = 10 + 6\cos(0.5\pi n + \pi/3)$ for $-\infty < n < \infty$, determine a simple mathematical expression for the overall output signal $y[n]$.

$y[n] =$

Problem s-01-Q.2.1:

We have shown that an LTI system can be represented in several equivalent ways. In each part below, you are given one representation of an LTI system and you are to provide the other representations requested. (Frequency response formulas can be given in any convenient form. You do **NOT** have to simplify them.)

(a) Frequency response: $\mathcal{H}(\hat{\omega}) = 1 + e^{-j2\hat{\omega}} = e^{-j\hat{\omega}}(2\cos\hat{\omega})$

Impulse response: $h[n] = \delta[n] + \delta[n-2]$

Difference equation: $y[n] = x[n] + x[n-2]$ $\{b_k\} = \{1, 0, 1\}$

(b) Frequency response: $\mathcal{H}(\hat{\omega}) = e^{-j\hat{\omega}} - e^{-j3\hat{\omega}} = e^{-j2\hat{\omega}}(2j\sin\hat{\omega})$

Impulse response: $h[n] = \delta[n-1] - \delta[n-3]$ $\{b_k\} = \{0, 1, 0, -1\}$

Difference equation: $y[n] = x[n-1] - x[n-3]$

(c) Frequency response: $\mathcal{H}(\hat{\omega}) = e^{-j\hat{\omega}}(2 + 2\cos(\hat{\omega})) = e^{-j\hat{\omega}}(2 + e^{j\hat{\omega}} + e^{-j\hat{\omega}})$

Impulse response: $h[n] = \delta[n] + 2\delta[n-1] + \delta[n-2]$ $\{b_k\} = \{1, 2, 1\}$

Difference equation: $y[n] = x[n] + 2x[n-1] + x[n-2]$

(d) Frequency response: $\mathcal{H}(\hat{\omega}) = 2e^{-j\hat{\omega}} + 2e^{-j3\hat{\omega}} = e^{-j2\hat{\omega}} \cdot 4\cos\hat{\omega}$

Impulse response: $h[n] = 2\delta[n-1] + 2\delta[n-3]$

MATLAB Implementation: `y = conv([0,2,0,2],x)` $\{b_k\} = \{0, 2, 0, 2\}$

Problem s-01-Q.2.2:

A signal $x(t)$ is periodic with period $T_0 = 8$. Therefore it can be represented as a Fourier series of the form

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j(2\pi/8)kt}.$$

It is known that the Fourier series coefficients for this representation of a particular signal $x(t)$ are given by the integral

$$a_k = \frac{1}{8}\int_{-4}^{0} (4+t)e^{-j(2\pi/8)kt}\,dt. \qquad (1)$$

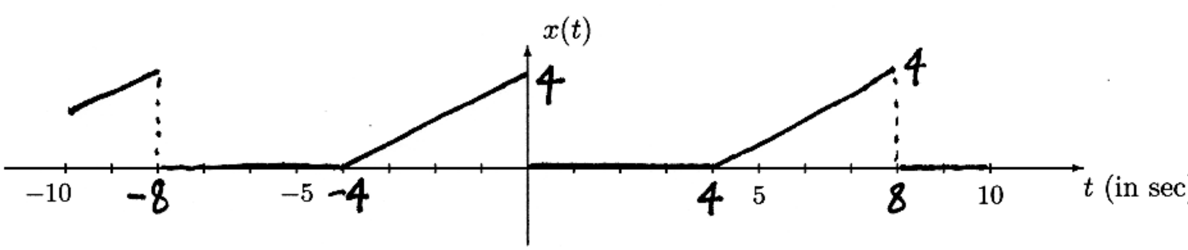
NOTE: Parts (c) and (d) can be worked independently of parts (a) and (b).

(a) In the expression for a_k in Equation (1) above, the integral and its limits define the signal $x(t)$. Determine an equation for $x(t)$ that is valid over one period.

↳ use the period $-4 \le t < 4$

$$x(t) = \begin{cases} 4+t & \text{for } -4 \le t \le 0 \\ 0 & \text{for } 0 < t < 4 \end{cases}$$

(b) Using your result from part (a), draw a plot of $x(t)$ over the range $-10 \le t \le 10$ seconds. Label it carefully.



(c) Which value of k in Equation (1) gives the DC (or average) value of $x(t)$? $k = 0$

(d) Determine the DC value of $x(t)$.

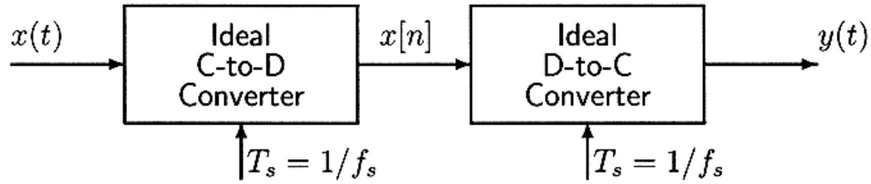
$$DC = \frac{1}{T_0}\cdot \text{Area} = \frac{1}{8}\cdot\frac{4\cdot 4}{2} = \frac{16}{16} = 1$$

or use the integral with $k=0$.

$$a_0 = \frac{1}{8}\int_{-4}^{0}(4+t)e^{-j0}\,dt = \frac{1}{8}\cdot 4t\Big|_{-4}^{0} + \frac{1}{8}\frac{t^2}{2}\Big|_{-4}^{0} = 0 - \frac{-16}{8} + 0 - \frac{16}{16}$$

$$= 2 - 1 = 1$$

Problem s-01-Q.2.3:



In all parts below, the sampling rates of the C/D and D/C converters are **equal,** and the input to the Ideal C/D converter is

$$x(t) = 2\cos(2\pi(50)t + \pi/2) + \cos(2\pi(150)t).$$

(a) If the output of the ideal D-to-C Converter is

$$y(t) = x(t) = 2\cos(2\pi(50)t + \pi/2) + \cos(2\pi(150)t),$$

what general statement can you make about the sampling frequency f_s in this case?

Use the Sampling Theorem:

$f_s \geq 2 f_{MAX}$ $\Rightarrow$ $f_s \geq 2 \cdot 150 = 300$ samples/sec

(b) If the sampling rate is $f_s = 250$ samples/sec., determine the discrete-time signal $x[n]$, and give an expression for $x[n]$ as a sum of cosines. *Make sure that all frequencies in your answer are positive and less than π radians.*

$x[n] = 2\cos(\frac{2\pi}{5}n + \pi/2) + \cos(\frac{4\pi}{5}n)$

$x[n] = 2\cos(2\pi(50)\frac{n}{250} + \pi/2) + \cos(2\pi(150)n/250)$ $\quad 2\pi(\frac{15}{25}) - 2\pi = -2\pi(\frac{2}{5})$

Plot the spectrum of this signal over the range of frequencies $-\pi \leq \hat{\omega} \leq \pi$. Make a plot for your answer, but label the frequency, amplitude and phase of each spectral component.

Label Carefully

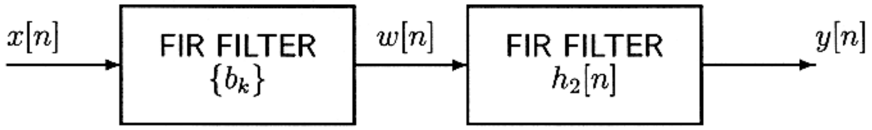
Use $\hat{\omega}$ for digital freq.

(c) If the output of the Ideal D-to-C Converter is

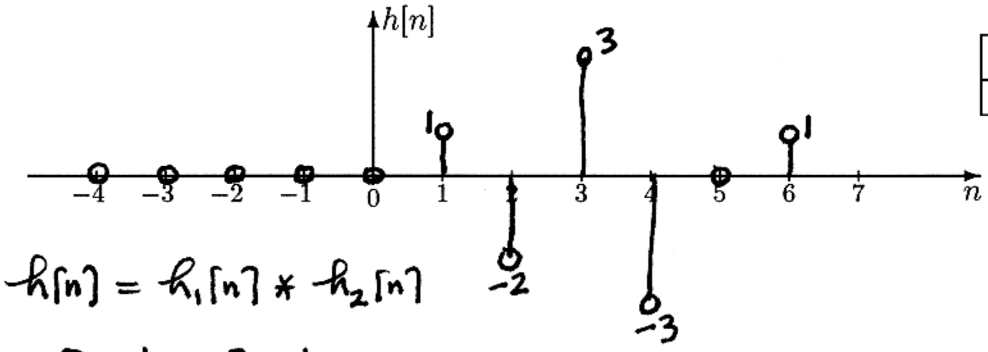
$$y(t) = 2\cos(2\pi(50)t + \pi/2) + 1,$$

determine the value of the sampling frequency f_s. (Remember that the input $x(t)$ is as defined above.) Need to alias 150 Hz to 0 Hz; in order to preserve 50 Hz, we need $f_s > 100$ Hz. $\Rightarrow f_s = 150$ samples/sec

Problem s-01-Q.2.4:



(a) If the filter coefficients of the first FIR filter are $\{b_k\} = \{0, 1, -2, 1\}$, and the impulse response of the second FIR filter is $h_2[n] = \delta[n] + 2\delta[n-2] + \delta[n-3]$, use convolution to determine the impulse response of the overall system, $h[n]$. Give your answer as a plot below.



Label Carefully

Plot zero values also

$h[n] = h_1[n] * h_2[n]$

```
  0   1  -2   1
  1   0   2   1
-----------------
  0   1  -2   1
      0   0   0   0
          0   2  -4   2
              0   1  -2   1
-----------------------------
  0   1  -2   3  -3   0   1
  ↑           ↑           ↑
 n=0         n=3         n=6
```

(b) Suppose that the overall frequency response of the cascade system (using different FIR filters from those in part (a)) is

$$\mathcal{H}(\hat{\omega}) = (2 + 2\cos(\hat{\omega}))e^{-j\hat{\omega}}$$

If the input signal is $x[n] = 10 + 6\cos(0.5\pi n + \pi/3)$ for $-\infty < n < \infty$, determine a simple mathematical expression for the overall output signal $y[n]$.

$y[n] = 40 + 12\cos(\frac{\pi}{2}n - \pi/6)$

The input signal has freqs: $\hat{\omega} = 0$ & $\hat{\omega} = 0.5\pi = \pi/2$

$\mathcal{H}(\hat{\omega})\big|_{\hat{\omega}=0} = (2 + 2\cos(0))e^{-j0} = 4$

$\mathcal{H}(\hat{\omega})\big|_{\hat{\omega}=\pi/2} = (2 + 2\cos(\pi/2))e^{-j\pi/2} = 2e^{-j\pi/2}$

$y[n] = 10 \cdot 4 + 6 \cdot 2\cos(\frac{\pi}{2}n + \underbrace{\frac{\pi}{3} - \frac{\pi}{2}}_{-\pi/6})$