

PROBLEM F-03-Q.2.1:

Questions about discrete-time signals and systems:

(a) Make a stem plot of $y[n] = \delta[n - 1] * (u[n + 2] - u[n - 2])$, where $*$ represents convolution.

(b) Make a stem plot of $y[n] = \delta[n - 1] (u[n + 2] - u[n - 2])$ versus n .

(c) Suppose a system is specified by the input/output relationship $y[n] = (x[n^c - a])^b$ where a is any integer, but c and b are *positive* integers.

In answering each of the following questions, give the *minimum* set of numerical constraints, i.e., don't give any more constraints than necessary to achieve the desired property for each question.

For example, your answer might be $a = -7$, $c = 3$, and b is unconstrained.

On this subproblem, you do not need to give any explanations. Feel free to use your intuition!

(i) What numerical constraints must we put on a , b , and c to ensure the system is *linear*?

(ii) What numerical constraints must we put on a , b , and c to ensure the system is *time-invariant*?

(iii) What numerical constraints must we put on a , b , and c to ensure the system is *causal*?

PROBLEM F-03-Q.2.2:

Suppose an LTI system has a frequency response given by $H(e^{j\hat{\omega}}) = 5j \sin(5\hat{\omega})e^{-j7\hat{\omega}}$

(a) Find $h[n]$, the system's impulse response, in terms of a sum of delta functions.

(b) Plot the magnitude of the frequency response versus $\hat{\omega}$.

(c) Evaluate the phase of the frequency response at $\hat{\omega} = 0.1\pi$. *The answer should be a number.*

PROBLEM F-03-Q.2.3:

Consider the signal $x[n] = 2\delta[n] + 3\delta[n - 1] - \delta[n - 2]$

- (a) Suppose $x[n]$ is input to an LTI system described by the difference equation

$$y_a[n] = x[n] - x[n - 1] - 2x[n - 3]$$

Find the output $y_a[n]$. Express your answer as a sum of delta functions.

- (b) Now suppose we have another LTI system with impulse response $h_b[n]$ which is unknown; but if we input the $x[n]$ given above, we get the output

$$y_b[n] = 6\delta[n] + 9\delta[n - 1] - 3\delta[n - 2] - 2\delta[n - 3] - 3\delta[n - 4] + \delta[n - 5]$$

The impulse response $h_b[n]$ can be expressed in the form: $h_b[n] = A\delta[n - p] + B\delta[n - q]$.
Find numerical values for A , B , p , and q .

PROBLEM F-03-Q.2.4:

A few questions about sampling:

(a) Suppose the sinusoid $x(t) = \cos(200\pi t - \pi/4)$ is input to an ideal continuous-to-discrete (C-D) converter with sampling frequency f_s .

(i) Find an f_s which would result in a discrete-time signal $x[n] = \cos(0.4\pi n - \pi/4)$.

(ii) Find an f_s which would result in a discrete-time signal $x[n] = \cos(0.4\pi n + \pi/4)$.

(b) Suppose a disc with a spot painted at one point along the edge is rotating *clockwise* at a certain speed, and a movie camera operating at **30 frames per second** is filming the rotating disc. Give *two* different *nonzero* disc rotation speeds, in terms of revolutions per second, which would make it look like the spot is standing still.

PROBLEM F-03-Q.2.5:

Suppose a discrete-time LTI system has frequency response $H(e^{j\hat{\omega}}) = \frac{1}{8} \frac{\sin(6\hat{\omega})}{\sin(\hat{\omega}/2)} \exp\left(-j\frac{11}{2}\hat{\omega}\right)$

(a) If the input to this system is

$$x[n] = 2 + \cos\left(\frac{\pi}{6}n\right) + \frac{1}{4} \cos\left(\frac{2\pi}{6}n\right) + \frac{1}{9} \cos\left(\frac{3\pi}{6}n\right) + \frac{1}{16} \cos\left(\frac{4\pi}{6}n\right),$$

Find the output $y[n]$ as a very simple formula. **Explain your reasoning.** Be clever.

(b) Suppose we want to implement the system with the $H(e^{j\hat{\omega}})$ given above with the following fragment of MATLAB code, where the variable `yy` contains the output and the variable `xx` contains the input:

```
hh = ????  
yy = conv(hh, xx);
```

Specify the row vector `hh`, i.e., `hh = [stuff goes here]`.