GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #2

DATE: 20-Oct-00 COURSE: ECE 2025

NAME: ______________________________ STUDENT #: ______________

LAST, FIRST

Recitation Section: **Circle the day & time** when your Recitation Section meets:

L01:Tues-9:30am (Casinovi)	L02:Thur-9:30am (Bordelon)	L03:Tues-12:00pm (Casinovi)
L04:Thur-12:00pm (Bordelon)	L05:Tues-1:30pm (Bordelon)	L06:Thur-1:30pm (Cassinovi)
L07:Tues-3:00pm (Bordelon)	L08:Thur-3:00pm (Hayes)	L09:Tues-4:30pm (Fekri)
L10:Thur-4:30pm (Li)	L11:Tues-6:00pm (Fekri)	L12:Thur-6:00pm (Li)
L13:Mon -3:00pm (Williams)	L14:Weds-3:00pm (Bordelon)	L15:Mon -4:30pm (Verriest)
L16:Weds-4:30pm (Dansereau)	L17:Mon -6:00pm (Verriest)	L18:Weds-6:00pm (Dansereau)
L19:Weds-1:30pm (Bordelon)	L20:Mon -1:30pm (Williams)	

- Write your name on **EVERY** page, and **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted. However, one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes permitted. OK to write on both sides.
- Justify your reasoning clearly to receive any partial credit.
  Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
4	20	
5	20	

NAME: ____________________ STUDENT #: ____________

Problem F-00-Q.2.1:

A periodic signal, $x(t)$, is given by

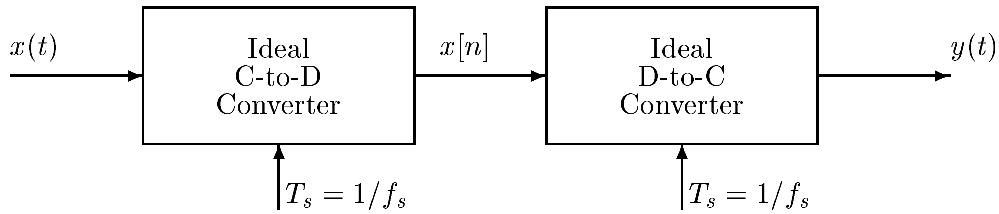
$$x(t) = 1 + 3\cos(300\pi t) + 2\sin(500\pi t - \pi/4)$$

(a) What is the period of $x(t)$?

(b) Find the Fourier series coefficients of $x(t)$.

NAME: ______________________ STUDENT #: ____________

Problem F-00-Q.2.2:



Suppose that the output of the D-to-C converter in the system above is found to be

$$y(t) = 2 + 10\cos(2\pi(150)t + \pi/3)$$

when the sampling rate is $f_s = 1/T_s = 400$ samples/second.

(a) Give an equation for $x[n]$ in terms of cosine functions. **Write your answer on the line below.**

Answer: $x[n] =$ ______________________________

(b) Determine two *different* input signals $x(t) = x_1(t)$ and $x(t) = x_2(t)$ that could have produced the given output of the D-to-C converter. **All of the frequencies in your answers must be positive and less than 400 Hz. Write your answers for both inputs on the lines below.**

Answer: $x_1(t) =$ ______________________________

Answer: $x_2(t) =$ ______________________________

NAME: ______________________________ STUDENT #: ______________

Problem F-00-Q.2.3:

For each of the following frequency responses on the left, pick one of the representations, S_1 through S_8 on the right, that defines *exactly* the same LTI system. Write your answer S_1, S_2, S_3, S_4, S_5, S_6, S_7, or S_8, in the box next to each frequency response.

ANS =	(a)	$1 + e^{j\hat{\omega}}$	S_1	$b_k = \{1, 0, 1\}$
ANS =	(b)	$2e^{-3j\hat{\omega}}$	S_2	$y[n] = \frac{1}{3}\{x[n] + x[n-1] + x[n-2]\}$
ANS =	(c)	$\dfrac{\sin(2\hat{\omega})}{\sin(\hat{\omega}/2)} e^{-3j\hat{\omega}/2}$	S_3	$h[n] = 0.5\delta[n] + 0.5\delta[n-2]$
ANS =	(d)	$e^{-j\hat{\omega}}\cos(\hat{\omega})$	S_4	$b_k = \{1, 1, 1, 1\}$
			S_5	$y[n] = x[n] + x[n-1]$
			S_6	$h[n] = \delta[n] - \delta[n-1]$
			S_7	$y[n] = x[n] + 2x[n-3]$
			S_8	$h[n] = 2\delta[n-3]$

NAME: ______________________ STUDENT #: ______________

Problem F-00-Q.2.4:

The frequency response of the filter above is

$$\mathcal{H}(e^{j\omega}) = \cos(\tfrac{1}{2}\hat{\omega})e^{-j\hat{\omega}}$$

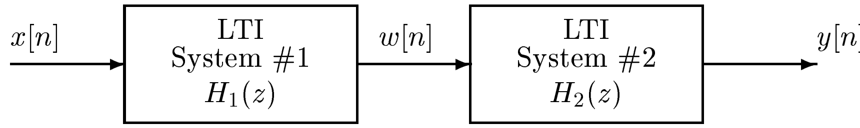
If the input signal is $x[n] = 7 + 2\cos(0.5\pi n + \pi)$ for $-\infty < n < \infty$,
determine a simple mathematical expression for the output signal $y[n]$.

$y[n] =$

NAME: ____________________ STUDENT #: ____________

Problem F-00-Q.2.5:

Consider the following cascade system:



where

$$H_1(z) = 2 - z^{-1} - z^{-2} \quad \text{and} \quad H_2(z) = 1 + \tfrac{1}{2}z^{-1}$$

(a) If the input $x[n]$ is a step,

$$x[n] = \begin{cases} 1 & ; \quad 0 \le n \\ 0 & ; \quad n < 0 \end{cases}$$

Find the output of the **first filter**, $w[n]$.

(b) Find and plot the impulse response $h[n]$ of the overall system.

Problem F-00-Q.2.1:

A periodic signal, $x(t)$, is given by

$$x(t) = 1 + 3\cos(300\pi t) + 2\sin(500\pi t - \pi/4)$$

(a) What is the period of $x(t)$?

The frequency of the first cosine in x(t) is 150 Hz, and the frequency of the second is 250 Hz. Therefore, the fundamental frequency (the greatest common divisor) is $f_s = 50$ Hz. Thus, the period is

$$T = 1/f_s = 1/50$$

(b) Find the Fourier series coefficients of $x(t)$.

Use Euler's formula, $\cos\theta = \frac{1}{2}e^{j\theta} + \frac{1}{2}e^{-j\theta}$, to express the cosines in terms of complex exponentials

$$x(t) = 1 + \frac{3}{2}e^{j300\pi t} + \frac{3}{2}e^{-j300\pi t} + e^{j(500\pi t - 3\pi/4)} + e^{-j(500\pi t - 3\pi/4)}$$

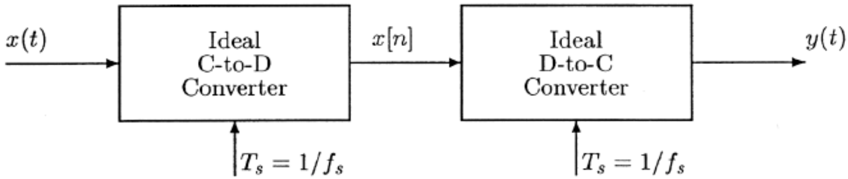
so we have

$$a_0 = 1$$

$$a_3 = a_{-3} = 3/2$$

$$a_5 = a_{-5}^* = e^{-j3\pi/4}$$

Problem F-00-Q.2.2:



Suppose that the output of the D-to-C converter in the system above is found to be

$$y(t) = 2 + 10\cos(2\pi(150)t + \pi/3)$$

when the sampling rate is $f_s = 1/T_s = 400$ samples/second.

(a) Give an equation for $x[n]$ in terms of cosine functions. **Write your answer on the line below.**

With no aliasing, going through a D/C converter is the inverse of going through the C/D. Therefore, we can get x[n] by passing y(t) through a C/D converter.

Answer: $x[n] =$ $2 + 10\cos(2\pi(150)n/400 + \pi/3)$

$= 2 + 10\cos(3n\pi/4 + \pi/3)$

(b) Determine two *different* input signals $x(t) = x_1(t)$ and $x(t) = x_2(t)$ that could have produced the given output of the D-to-C converter. **All of the frequencies in your answers must be positive and less than 400 Hz. Write your answers for both inputs on the lines below.**

Answer: $x_1(t) =$ $2 + 10\cos(2\pi(150)t + \pi/3)$

(no aliasing)

Answer: $x_2(t) =$ $2 + 10\cos(2\pi(250)t - \pi/3)$

We have folding in the second case.

Problem F-00-Q.2.3:

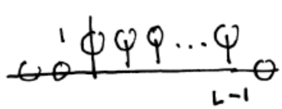
For each of the following frequency responses on the left, pick one of the representations, S_1 through S_8 on the right, that defines *exactly* the same LTI system. Write your answer S_1, S_2, S_3, S_4, S_5, S_6, S_7, or S_8, in the box next to each frequency response.

S_5 ANS =	(a)	$1 + e^{-j\hat{\omega}}$	S_1	$b_k = \{1, 0, 1\}$
S_8 ANS =	(b)	$2e^{-3j\hat{\omega}}$	S_2	$y[n] = \frac{1}{3}\{x[n] + x[n-1] + x[n-2]\}$
S_4 ANS =	(c)	$\frac{\sin(2\hat{\omega})}{\sin(\hat{\omega}/2)} e^{-3j\hat{\omega}/2}$	S_3	$h[n] = 0.5\delta[n] + 0.5\delta[n-2]$
S_3 ANS =	(d)	$e^{-j\hat{\omega}}\cos(\hat{\omega})$	S_4	$b_k = \{1, 1, 1, 1\}$
			S_5	$y[n] = x[n] + x[n-1]$
			S_6	$h[n] = \delta[n] - \delta[n-1]$
			S_7	$y[n] = x[n] + 2x[n-3]$
			S_8	$h[n] = 2\delta[n-3]$

a) $H(\hat{\omega}) = 1 + e^{-j\hat{\omega}} \Rightarrow h[n] = \delta[n] + \delta[n-1]$

$y[n] = x[n] + x[n-1] \Rightarrow S_5$

b) $H(\hat{\omega}) = 2e^{-3j\hat{\omega}} \Rightarrow h[n] = 2\delta[n-3] \Rightarrow S_8$

c) 

$\Rightarrow H(\hat{\omega}) = e^{-(\frac{L-1}{2})\hat{\omega}} \frac{\sin(L\hat{\omega}/2)}{\sin(\hat{\omega}/2)}$

$\Rightarrow L = 4 \Rightarrow S_4$

d) $e^{-j\hat{\omega}}\cos\hat{\omega} = \frac{1}{2} + \frac{1}{2}e^{-2j\hat{\omega}} \Rightarrow h[n] = \frac{1}{2}\delta[n] + \frac{1}{2}\delta[n-2]$

$\Rightarrow S_3$

Problem F-00-Q.2.4:

The frequency response of the filter above is

$$\mathcal{H}(e^{j\omega}) = \cos(\tfrac{1}{2}\hat{\omega})e^{-j\hat{\omega}}$$

If the input signal is $x[n] = 7 + 2\cos(0.5\pi n + \pi)$ for $-\infty < n < \infty$,
determine a simple mathematical expression for the output signal $y[n]$.

$y[n] = 7 + \sqrt{2}\cos(0.5\pi n + \pi/2)$

Recall that

$$x[n] = \cos(n\hat{\omega}) \Rightarrow y[n] = |H(\hat{\omega})| \cos(n\hat{\omega} + \angle H(\hat{\omega}))$$

Therefore, with

- $H(\hat{\omega})\big|_{w=0} = 1$
- $H(\hat{\omega})\big|_{\hat{\omega}=\pi/2} = \cos(\pi/4)\, e^{-j\pi/2} = \frac{1}{\sqrt{2}}\, e^{-j\pi/2}$

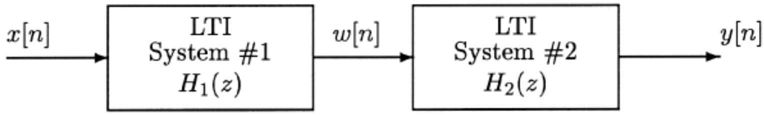
then

$$y[n] = 7 + \frac{1}{\sqrt{2}}\, 2\cos(0.5\pi n + \pi - \pi/2)$$

$$= 7 + \sqrt{2}\cos(0.5\pi n + \pi/2)$$

Problem F-00-Q.2.5:

Consider the following cascade system:



where

$$H_1(z) = 2 - z^{-1} - z^{-2} \quad \text{and} \quad H_2(z) = 1 + \tfrac{1}{2}z^{-1}$$

(a) If the input $x[n]$ is a step,

$$x[n] = \begin{cases} 1 & ; \quad 0 \le n \\ 0 & ; \quad n < 0 \end{cases}$$

Find the output of the **first filter**, $w[n]$.

Since $h_1[n] = 2\delta[n] - \delta[n-1] - \delta[n-2]$, then

$$w[n] = h_1[n] * x[n] = \{2\delta[n] - \delta[n-1] - \delta[n-2]\} * u[n]$$

$$= 2u[n] - u[n-1] - u[n-2]$$

$$= \begin{cases} 2 & n=0 \\ 1 & n=1 \\ 0 & \text{else} \end{cases}$$

(b) Find and plot the impulse response $h[n]$ of the overall system.

$$H(z) = H_1(z)H_2(z) = (2 - z^{-1} - z^{-2})(1 + \tfrac{1}{2}z^{-1})$$

$$= 2 + (1-1)z^{-1} + (-1-\tfrac{1}{2})z^{-2} - \tfrac{1}{2}z^{-3}$$

$$= 2 - \tfrac{3}{2}z^{-2} - \tfrac{1}{2}z^{-3}$$

so

$$h[n] = 2\delta[n] - \tfrac{3}{2}\delta[n-2] - \tfrac{1}{2}\delta[n-3]$$