GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #2

DATE: 22-Oct-01 COURSE: ECE 2025

NAME: ______________________________ STUDENT #: ________________

LAST, FIRST

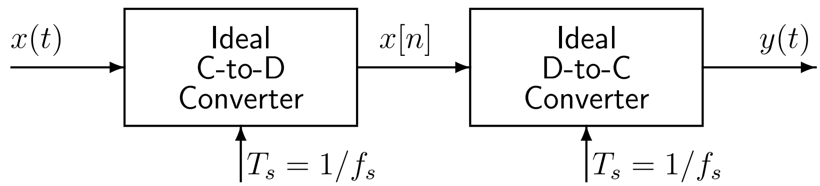
Recitation Section: **Circle the day & time** when your Recitation Section meets:

L01:Tues-9:30am (Bordelon)	L02:Thur-9:30am (Casinovi)	L03:Tues-12:00pm (Casinovi)
L04:Thur-12:00pm (Ji)	L05:Tues-1:30pm (Bordelon)	L06:Thur-1:30pm (Ji)
L07:Tues-3:00pm (Casinovi)	L08:Thur-3:00pm (Bordelon)	L09:Tues-4:30pm (Lanterman)
L10:Thur-4:30pm (Bordelon)	L11:Tues-6:00pm (Lanterman)	L13:Mon -3:00pm (Yezzi)
L14:Weds-3:00pm (Taylor)	L15:Mon -4:30pm (Verriest)	L16:Weds-4:30pm (Taylor)
L17:Mon -6:00pm (Verriest)	L20:Mon -1:30pm (McClellan)	RPK: (Abler)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted. However, one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes permitted. OK to write on both sides.
- JUSTIFY your reasoning clearly to receive any partial credit.
  Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
4	20	
5	20	

Problem F-01-Q.2.1:

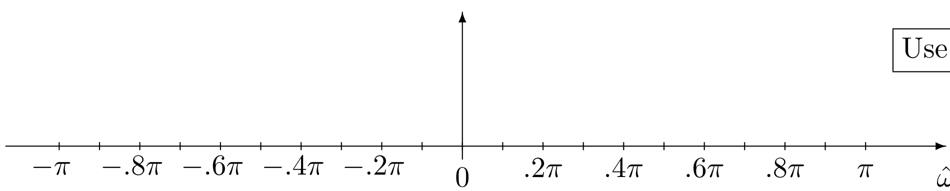


Suppose that the continuous-time input $x(t)$ to the above system is given as

$$x(t) = \cos(16000\pi t) + \cos(4000\pi t) + \cos(1000\pi t).$$

(a) What sampling rate is required such that no aliasing occurs for $x(t)$? $f_s =$

(b) Given that $f_s = 10,000$ samples/second, plot the frequency spectrum for $x[n]$.



Use $\hat{\omega}$ for digital freq.

(c) Given that $x(t) = \cos(26000\pi t)$ and $f_s = 10000$ samples/second, write a simplified expression for the output $y(t)$ in terms of cosine functions.

Problem F-01-Q.2.2:

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A periodic signal, $x(t)$, is given by

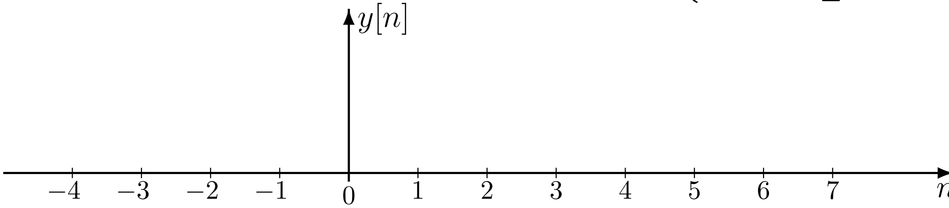
$$x(t) = 1 + 2\cos(300\pi t + \pi/4) + \sin(500\pi t)$$

(a) What is the period of $x(t)$?

(b) Find the Fourier series coefficients of $x(t)$ for $-6 \le k \le 6$.

Problem F-01-Q.2.3:

(a) If the filter coefficients of an FIR filter are $\{b_k\} = \{0, 1, -1, 1\}$, make a plot of the output when the input is the unit step signal: $x[n] = u[n] = \begin{cases} 0 & \text{for } n < 0 \\ 1 & \text{for } n \geq 0 \end{cases}$



Label Carefully

Plot zero values also

(b) Suppose that the frequency response of a different FIR filter is

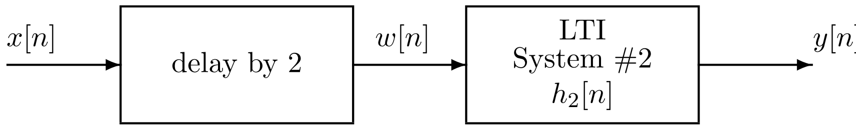
$$\mathcal{H}(\hat{\omega}) = \cos(\tfrac{1}{2}\hat{\omega})e^{-j\hat{\omega}}$$

If the input signal is $x[n] = 1 + 3\cos(\pi n + \pi)$ for $-\infty < n < \infty$, determine a simple mathematical expression for the output signal $y[n]$.

$y[n] =$

Problem F-01-Q.2.4:

Consider the following cascade system:



(a) Find and plot the magnitude of the frequency response of the first filter $|\mathcal{H}_1(\hat{\omega})|$.

(b) If the overall impulse response of the cascade is

$$h_{eq}[n] = \delta[n-3] + \frac{1}{2}\delta[n-4]$$

determine the impulse response of the second filter $h_2[n]$.

Problem F-01-Q.2.5:

We have shown that an LTI system can be represented in several equivalent ways. In each part below, you are given one representation of an LTI system and you are to provide the other representations requested. (Frequency response formulas can be given in any convenient form. You do **NOT** have to simplify them.)

(a) Frequency response:

Impulse response:

Difference equation: $y[n] = x[n] + 2x[n-1] + x[n-2]$

(b) Frequency response: $\mathcal{H}(\hat{\omega}) = e^{-j\hat{\omega}}(2\cos(\hat{\omega}))$

Impulse response:

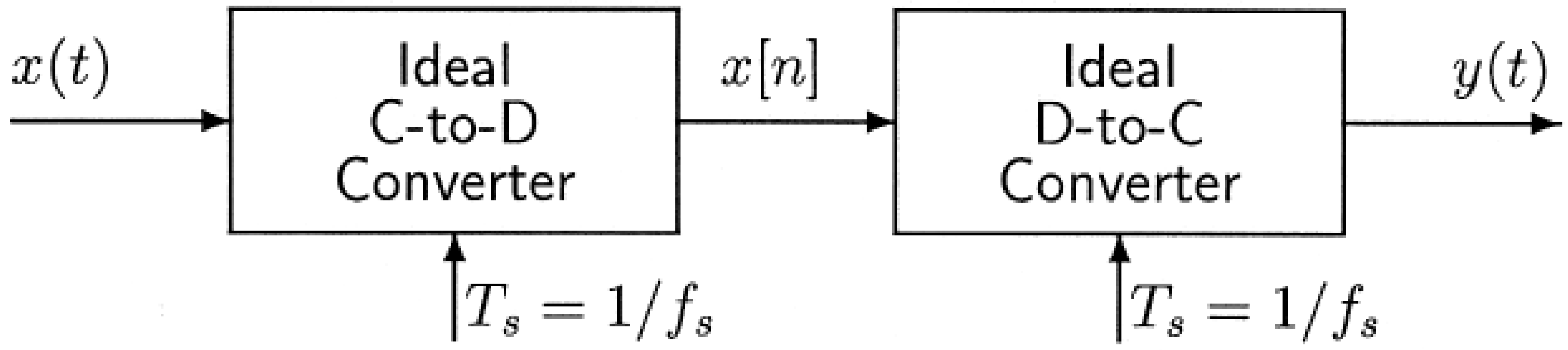
Difference equation:

(c) Frequency response:

Impulse response:

MATLAB Implementation: `y = conv([0,1,0,-1],x)`

Problem F-01-Q.2.1:



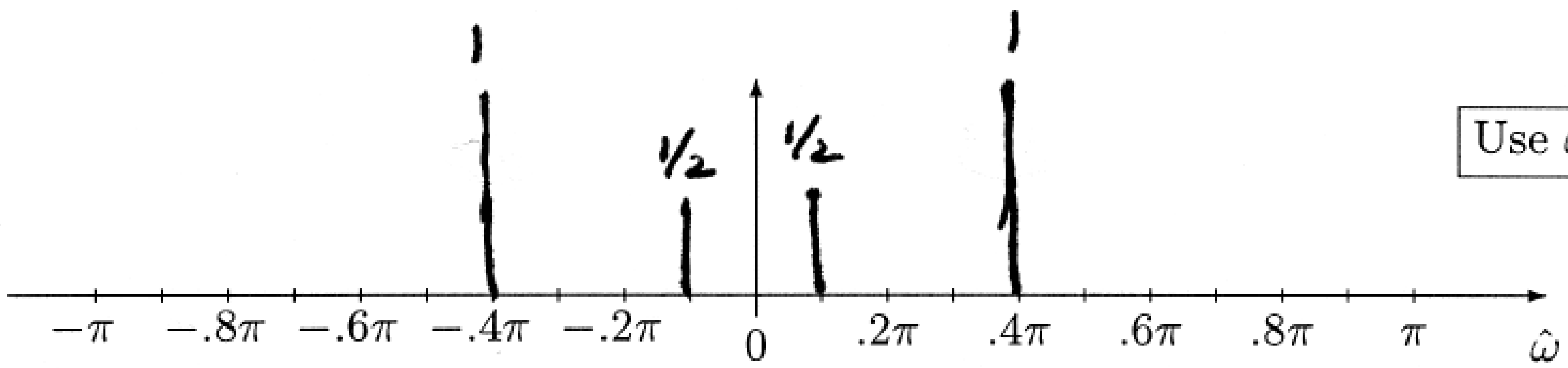
Suppose that the continuous-time input $x(t)$ to the above system is given as

$$x(t) = \cos(16000\pi t) + \cos(4000\pi t) + \cos(1000\pi t).$$

$f_{max} = 8{,}000$ Hz.

(a) What sampling rate is required such that no aliasing occurs for $x(t)$? $f_s =$ 16,000 Hz.

(b) Given that $f_s = 10,000$ samples/second, plot the frequency spectrum for $x[n]$.



Use $\hat{\omega}$ for digital freq.

(c) Given that $x(t) = \cos(26000\pi t)$ and $f_s = 10000$ samples/second, write a simplified expression for the output $y(t)$ in terms of cosine functions.

$y(t) = \cos(6000\pi t)$

NOTE THAT ALIASING OCCURS.

Problem F-01-Q.2.2:

A periodic signal, $x(t)$, is given by

$$x(t) = 1 + 2\cos(300\pi t + \pi/4) + \sin(500\pi t)$$

(a) What is the period of $x(t)$?

FUNDAMENTAL FREQ: $\omega_0 = \frac{2\pi}{T} = 100\pi$

$\Rightarrow T = \frac{1}{50}$ SEC. $= 20$ mSEC.

(b) Find the Fourier series coefficients of $x(t)$ for $-6 \leq k \leq 6$.

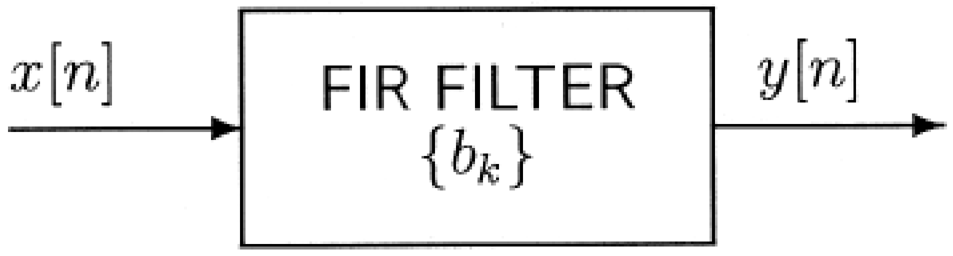
USING EULER'S RELATION

$$x(t) = 1 + e^{j\frac{\pi}{4}} \cdot e^{j3(100\pi)t} + e^{-j\frac{\pi}{4}} \cdot e^{-j3(100\pi)t} + \frac{1}{2} e^{-j\frac{\pi}{2}} \cdot e^{j5(100\pi)t} + \frac{1}{2} e^{j\frac{\pi}{2}} e^{-j5(100\pi)t}$$

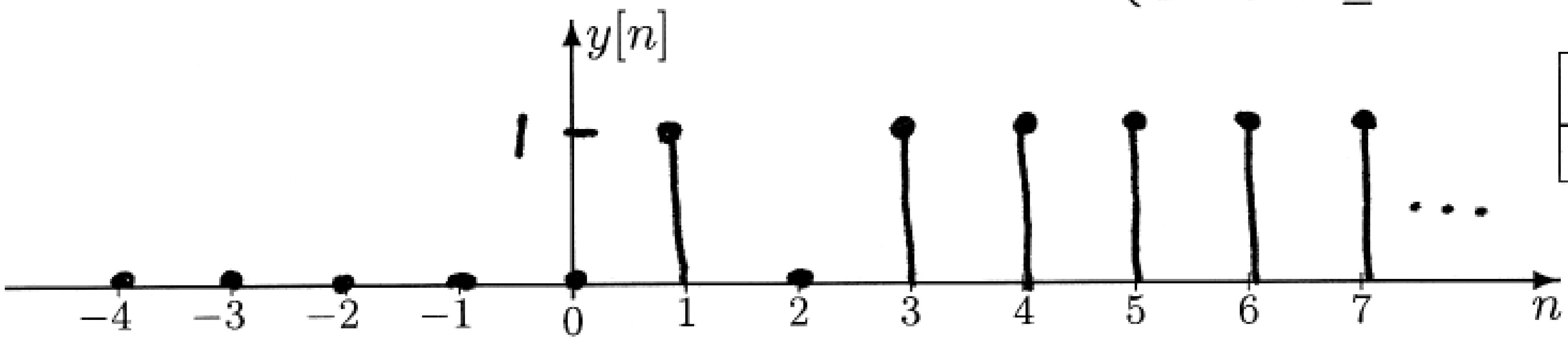
$\therefore$

$a_0 = 1$	$a_{-3} = e^{-j\pi/4}$
$a_3 = e^{j\pi/4}$	$a_{-5} = \frac{1}{2} e^{j\pi/2}$
$a_5 = \frac{1}{2} e^{-j\pi/2}$	$a_k = 0$ FOR ALL OTHER k.

Problem F-01-Q.2.3:



(a) If the filter coefficients of an FIR filter are $\{b_k\} = \{0, 1, -1, 1\}$, make a plot of the output when the input is the unit step signal: $x[n] = u[n] = \begin{cases} 0 & \text{for } n < 0 \\ 1 & \text{for } n \geq 0 \end{cases}$



Label Carefully

Plot zero values also

$x[n-1]$: 0 0 0 0 1 1 1 1 1 1 . . .

$-x[n-2]$: 0 0 0 0 −1 −1 −1 −1 −1 . . .

$x[n-3]$: 0 0 0 0 1 1 1 1 . . .

$y[n]$: 0 0 0 0 1 0 1 1 1 1 . . .

(b) Suppose that the frequency response of a different FIR filter is

$$\mathcal{H}(\hat{\omega}) = \cos(\tfrac{1}{2}\hat{\omega})e^{-j\hat{\omega}}$$

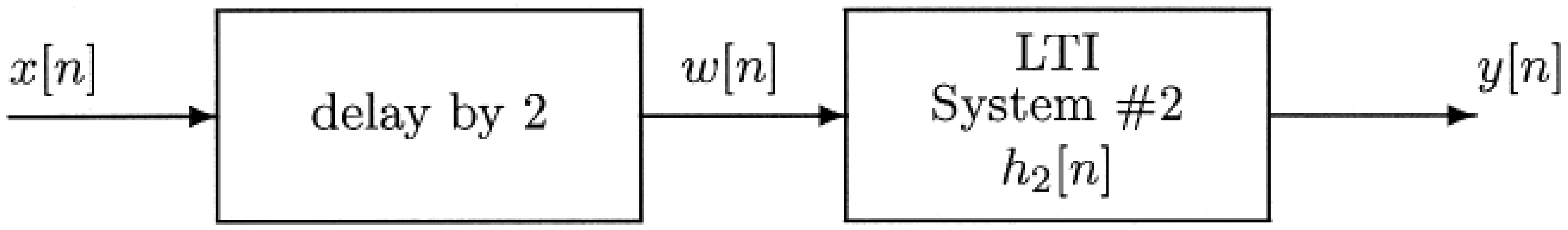
If the input signal is $x[n] = 1 + 3\cos(\pi n + \pi)$ for $-\infty < n < \infty$, determine a simple mathematical expression for the output signal $y[n]$.

$y[n] = 1$

$\mathcal{H}(0) = 1 \qquad \mathcal{H}(\pi) = 0$

Problem F-01-Q.2.4:

Consider the following cascade system:

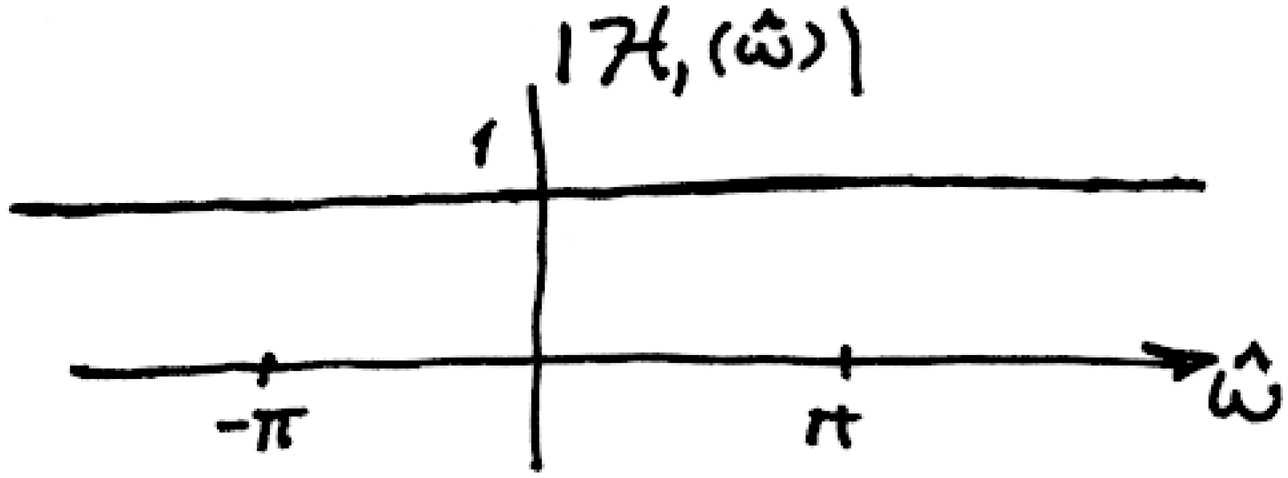


(a) Find and plot the magnitude of the frequency response of the first filter $|\mathcal{H}_1(\hat{\omega})|$.

$$h_1[n] = \delta[n-2]$$

$$\mathcal{H}_1(\hat{\omega}) = e^{-j2\hat{\omega}}$$

$$\boxed{|\mathcal{H}_1(\hat{\omega})| = 1}$$



(b) If the overall impulse response of the cascade is

$$h_{eq}[n] = \delta[n-3] + \tfrac{1}{2}\delta[n-4]$$

determine the impulse response of the second filter $h_2[n]$.

$$\delta[n-2] * h_2[n] = \delta[n-3] + \tfrac{1}{2}\delta[n-4]$$

$$\Longrightarrow \boxed{h_2[n] = \delta[n-1] + \tfrac{1}{2}\delta[n-2]}$$

Problem F-01-Q.1.5:

We have shown that an LTI system can be represented in several equivalent ways. In each part below, you are given one representation of an LTI system and you are to provide the other representations requested. (Frequency response formulas can be given in any convenient form. You do **NOT** have to simplify them.)

(a) Frequency response: $\mathcal{H}(\hat{\omega}) = 1 + 2e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$

Impulse response: $h[n] = \delta[n] + 2\delta[n-1] + \delta[n-2]$

Difference equation: $y[n] = x[n] + 2x[n-1] + x[n-2]$

(b) Frequency response: $\mathcal{H}(\hat{\omega}) = e^{\ j\hat{\omega}}(2\cos(\hat{\omega}))$ $= e^{j\hat{\omega}}(e^{j\hat{\omega}} + e^{-j\hat{\omega}}) = e^{j2\hat{\omega}} + 1$

Impulse response: $h[n] = \delta[n+2] + \delta[n]$

Difference equation: $y[n] = x[n+2] + x[n]$

(b) Frequency response: $\mathcal{H}(\hat{\omega}) = e^{-j\hat{\omega}}(2\cos(\hat{\omega}))$ $= e^{-j\hat{\omega}}(e^{j\hat{\omega}} + e^{-j\hat{\omega}}) = 1 + e^{-j2\hat{\omega}}$

Impulse response: $h[n] = \delta[n] + \delta[n-2]$

Difference equation: $y[n] = x[n] + x[n-2]$

(c) Frequency response: $\mathcal{H}(\hat{\omega}) = e^{-j\hat{\omega}} - e^{-j3\hat{\omega}}$

Impulse response: $h[n] = \delta[n-1] - \delta[n-3]$

MATLAB Implementation: `y = conv([0,1,0,-1],x)`