PROBLEM sp-04-Q.2.1:

A periodic signal $x(t)$ is represented as a Fourier series of the form

$$x(t) = \sum_{k=-\infty}^{\infty} \left(10\delta[k] + k^2 - 15\right) e^{j10\pi kt}$$

(a) Determine the fundamental period of the signal $x(t)$, i.e., the minimum period.

$\boxed{T_0 = \qquad}$ sec. (Give a numerical answer.)

(b) Determine the DC value of $x(t)$. Give your answer as a number.

$\boxed{DC = \qquad}$

(c) Define a new signal by adding a sinusoid to $x(t)$

$$y(t) = 12\cos(30\pi t - \pi/2) + x(t)$$

The new signal, $y(t)$ can be expressed in the following Fourier Series with new coefficients $\{b_k\}$:

$$y(t) = \sum_{k=-\infty}^{\infty} b_k\, e^{j10\pi kt}$$

Fill in the following table, giving *numerical values* for each $\{b_k\}$ in polar form:.
Hint: Find a simple relationship between $\{b_k\}$ and $\{a_k\}$.

b_k	Mag	Phase
b_{-3}		
b_{-2}		
b_{-1}		
b_0		
b_1		
b_2		
b_3		

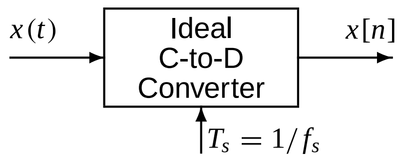
PROBLEM sp-04-Q.2.2:

For each short question, pick a correct frequency[1] (from the list on the right only) and enter the number in the answer box[2]:

Question

(a) If the C/D converter output is $x[n] = 7\cos(0.5\pi n)$, and the sampling rate is 2000 samples/sec, then determine one possible value for the input frequency of $x(t)$:

ANS =



(b) If the following MATLAB code is implemented, what is the frequency of the sound that will be produced at the output of the computer's D-to-A converter.

```
soundsc( cos(1.6*pi*(0:9999)), 2000);
```

ANS =

(c) Determine the Nyquist rate for sampling the signal $x(t)$ defined by: $x(t) = \Re e\{e^{j1200\pi t} + e^{j2000\pi t}\}$.

ANS =

Frequency

8000 Hz

4000 Hz

2000 Hz

1600 Hz

1200 Hz

1000 Hz

800 Hz

500 Hz

400 Hz

[1]Some questions might have more than one answer, but you only need to pick one correct answer.

[2]It is possible to use an answer more than once.

PROBLEM sp-04-Q.2.3:

Pick the correct output signal (from the list on the right) and enter the number in the answer box:

System Description and Input Signal

(a) $x[n] = 1 + \cos(2\pi n/3)$ for all n

and $h[n] = \delta[n] + \delta[n-1] + \delta[n-2]$

ANS =

(b) $x[n] = \delta[n-1] - \delta[n-2]$

and $y[n] = x[n] + x[n-1]$

ANS =

(c) `yy = conv([0,1,0,-1], [0,1,0,0,0])`

ANS =

(d) $x[n] = \delta[n-2]$

and $y[n] = x[n-1]$

ANS =

(e) $y[n] = \delta[n-3] * (\delta[n] - \delta[n-2])$

ANS =

Output Signal

1 $y[n] = \delta[n-3] - \delta[n-5]$

2 $y[n] = 3\sin(2\pi n/3 - 5\pi/6)$ for all n

3 $y[n] = \delta[n-2] - \delta[n-4]$

4 $y[n] = \delta[n-1] - \delta[n-3]$

5 $y[n] = 3$ for all n

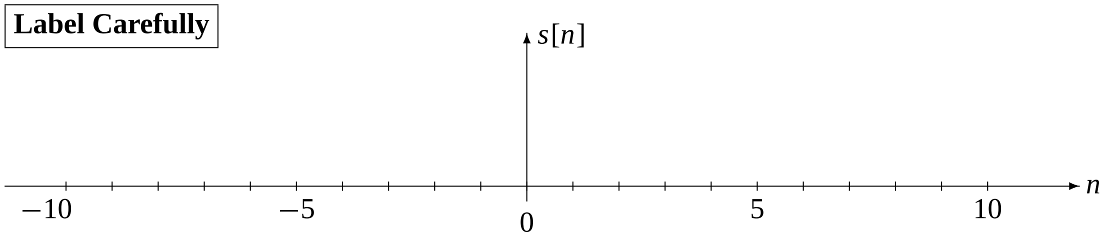
6 $y[n] = 0$ for all n

7 $y[n] = \delta[n-3]$

8 None of the above

(f) Plot the signal $s[n] = u[n+2] - \delta[n-2]$.

Label Carefully



PROBLEM sp-04-Q.2.4:

Pick the correct frequency response (from the list on the right) and enter the number in the answer box:

Time-Domain Description

(a) $y[n] = x[n] + x[n-1] + x[n-2]$

ANS =

(b) $y[n] = x[n] + x[n-1]$

ANS =

(c) $h[n] = \delta[n-1] + \delta[n-3]$

ANS =

(d) $h[n] = \delta[n-1] - \delta[n-3]$

ANS =

(e) $\{b_k\} = \{\,1,\ 0,\ -1\,\}$

ANS =

(f) Select **all** systems (from the list on the right) that **null out** DC. Enter all numbers that apply.

ANS =

Frequency Response

$\boxed{1}$ $H(e^{j\hat{\omega}}) = 1 - e^{-j2\hat{\omega}}$

$\boxed{2}$ $H(e^{j\hat{\omega}}) = 2e^{-j2\hat{\omega}}\cos(\hat{\omega})$

$\boxed{3}$ $H(e^{j\hat{\omega}}) = 2je^{-j2\hat{\omega}}\sin(\hat{\omega})$

$\boxed{4}$ $H(e^{j\hat{\omega}}) = \dfrac{\sin(\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})}\, e^{-j\hat{\omega}/2}$

$\boxed{5}$ $H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}(1 + 2\cos(\hat{\omega}))$

$\boxed{6}$ $H(e^{j\hat{\omega}}) = \dfrac{\sin(2\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})}\, e^{-j3\hat{\omega}/2}$

$\boxed{7}$ $H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}$

$\boxed{8}$ None of the above

PROBLEM sp-04-Q.2.1:

A periodic signal $x(t)$ is represented as a Fourier series of the form

$$x(t) = \sum_{k=-\infty}^{\infty} \left(10\delta[k] + k^2 - 15\right) e^{j10\pi kt}$$

(a) Determine the fundamental period of the signal $x(t)$, i.e., the minimum period.

$\boxed{T_0 = 1/5}$ sec. (Give a numerical answer.)

$\omega_0 = 10\pi$ rad/sec $\Rightarrow$ $T_0 = 2\pi/\omega_0 = 2\pi/10\pi = 1/5$

(b) Determine the DC value of $x(t)$. Give your answer as a number.

$\boxed{DC = -5}$ or $5e^{j\pi}$

At $k=0$ $\quad a_k = 10\delta[k] + k^2 - 15 = 10 + 0 - 15 = -5$

(c) Define a new signal by adding a sinusoid to $x(t)$

$$y(t) = 12\cos(30\pi t - \pi/2) + x(t)$$

The new signal, $y(t)$ can be expressed in the following Fourier Series with new coefficients $\{b_k\}$:

$$y(t) = \sum_{k=-\infty}^{\infty} b_k e^{j10\pi kt}$$

Fill in the following table, giving *numerical values* for each $\{b_k\}$ in polar form:.
Hint: Find a simple relationship between $\{b_k\}$ and $\{a_k\}$.

b_k	Mag	Phase
b_{-3}	$6\sqrt{2}$	$3\pi/4$
b_{-2}	11	$-\pi$
b_{-1}	14	$-\pi$
b_0	5	π
b_1	14	π
b_2	11	π
b_3	$6\sqrt{2}$	$-3\pi/4$

3rd harmonic

$y(t) = 6e^{j30\pi t}e^{-j\pi/2} + 6e^{-j30\pi t}e^{j\pi/2} + x(t)$

So, $a_k = b_k$ except for $k = \pm 3$

$b_3 = a_3 + 6e^{-j\pi/2}$

$= 9 - 15 - 6j = -6 - 6j = 6\sqrt{2}e^{-j3\pi/4}$

$b_{-3} = a_{-3} + 6e^{j\pi/2} = -6 + 6j = b_3^*$

$b_1 = 1^2 - 15 = -14 = 14e^{j\pi}$

$b_2 = 2^2 - 15 = -11 = 11e^{j\pi}$

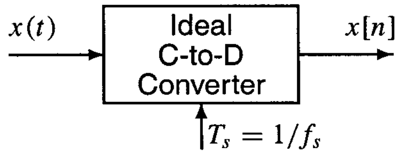
PROBLEM sp-04-Q.2.2:

For each short question, pick a correct frequency[1] (from the list on the right) and enter the number in the answer box[2]:

Question

(a) If the C/D converter output is $x[n] = 7\cos(0.5\pi n)$, and the sampling rate is 2000 samples/sec, then determine one possible value for the input frequency of $x(t)$:

ANS = 500



$\hat{\omega} = 2\pi f / f_s$

$0.5\pi = 2\pi f / 2000 \Rightarrow f = \frac{0.5\pi}{2\pi} \times 2000 = 500\text{ Hz}$

(b) If the following MATLAB code is implemented, what is the frequency of the sound that will be produced at the output of the computer's D-to-A converter.

```
soundsc( cos(1.6*pi*(0:9999)), 2000);
```

ANS = 400

$\hat{\omega} = \pm 1.6\pi$ alias to $\hat{\omega} = \pm 1.6\pi \mp 2\pi$

$= \pm 0.4\pi$

$f = \frac{\hat{\omega}}{2\pi} f_s = \frac{0.4\pi}{2\pi} \times 2000 = 400\text{ Hz}$

(c) Determine the Nyquist rate for sampling the signal $x(t)$ defined by: $x(t) = \Re e\{e^{j1200\pi t} + e^{j2000\pi t}\}$.

ANS = 2000

$\omega_{MAX} = 2000\pi$ rad/s

$f_{MAX} = 1000\text{ Hz}$

Sampling Thm $\Rightarrow f_s \geq 2 f_{MAX} = 2000\text{ Hz}$

Frequency

- 8000 Hz
- 4000 Hz
- 2000 Hz
- 1600 Hz
- 1200 Hz
- 1000 Hz
- 800 Hz
- 500 Hz
- 400 Hz

[1] Some questions might have more than one answer, but you only need to pick one correct answer.

[2] It is possible to use an answer more than once.

PROBLEM sp-04-Q.2.3:

Pick the correct output signal (from the list on the right) and enter the number in the answer box:

System Description and Input Signal

(a) $x[n] = 1 + \cos(2\pi n/3)$ for all n

and $h[n] = \delta[n] + \delta[n-1] + \delta[n-2]$

ANS = 5

Running Sum of 3

$1 \to 3$

$\cos(2\pi n/3) \to 0$

(b) $x[n] = \delta[n-1] - \delta[n-2]$

and $y[n] = x[n] + x[n-1]$

ANS = 4

```
0 1 -1
  1 1
------
0 1 -1
  0 1 -1
```

OUT = [0, 1, 0, -1]

(c) `yy = conv([0,1,0,-1], [0,1,0,0,0])`

ANS = 3

delay by one

OUT = [0, 0, 1, 0, -1]

(d) $x[n] = \delta[n-2]$

and $y[n] = x[n-1]$ delay by one

ANS = 7

(e) $y[n] = \delta[n-3] * (\delta[n] - \delta[n-2])$

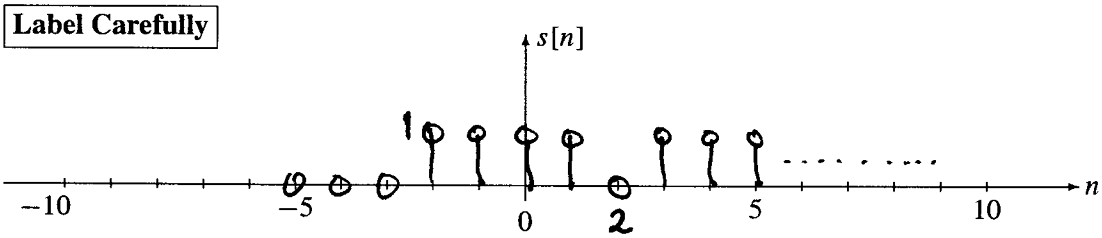
ANS = 1

delay by 3

$\delta[n-3] - \delta[n-5]$

(f) Plot the signal $s[n] = u[n+2] - \delta[n-2]$.

Label Carefully



Output Signal

1. $y[n] = \delta[n-3] - \delta[n-5]$
2. $y[n] = 3\sin(2\pi n/3 - 5\pi/6)$ for all n
3. $y[n] = \delta[n-2] - \delta[n-4]$
4. $y[n] = \delta[n-1] - \delta[n-3]$
5. $y[n] = 3$ for all n
6. $y[n] = 0$ for all n
7. $y[n] = \delta[n-3]$
8. None of the above

PROBLEM sp-04-Q.2.4:

Pick the correct frequency response (from the list on the right) and enter the number in the answer box:

Time-Domain Description

(a) $y[n] = x[n] + x[n-1] + x[n-2]$

$\boxed{\textbf{ANS} = 5}$

$1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$

$= e^{-j\hat{\omega}}(e^{j\hat{\omega}} + 1 + e^{-j\hat{\omega}})$ $1 + 2\cos\hat{\omega}$

(b) $y[n] = x[n] + x[n-1]$

$\boxed{\textbf{ANS} = 4}$ Length-2 running sum

$1 + e^{-j\hat{\omega}}$ $\quad \dfrac{\sin(L\hat{\omega}/2)}{\sin(\hat{\omega}/2)} e^{-j\hat{\omega}(L-1)/2}$

(c) $h[n] = \delta[n-1] + \delta[n-3]$

$\boxed{\textbf{ANS} = 2}$

$e^{-j\hat{\omega}} + e^{-j\hat{\omega}3} = e^{-j2\hat{\omega}}(\underbrace{e^{j\hat{\omega}} + e^{-j\hat{\omega}}}_{2\cos\hat{\omega}})$

(d) $h[n] = \delta[n-1] - \delta[n-3]$

$\boxed{\textbf{ANS} = 3}$

$e^{-j\hat{\omega}} - e^{-j3\hat{\omega}} = e^{-j2\hat{\omega}}(\underbrace{e^{j\hat{\omega}} - e^{-j\hat{\omega}}}_{2j\sin\hat{\omega}})$

(e) $\{b_k\} = \{1, 0, -1\}$

$\boxed{\textbf{ANS} = 1}$

$1 - e^{-j2\hat{\omega}}$

(f) Select **all** systems (from the list on the right) that null out DC. Enter all numbers that apply.

$\boxed{\textbf{ANS} = 1, 3}$

Look for $H(e^{j0}) = 0$

Frequency Response

$\boxed{1}$ $H(e^{j\hat{\omega}}) = 1 - e^{-j2\hat{\omega}}$

$H(e^{j0}) = 1 - 1 = 0$

$\boxed{2}$ $H(e^{j\hat{\omega}}) = 2e^{-j2\hat{\omega}}\cos(\hat{\omega})$

$H(e^{j0}) = 2\cos(0) = 2$

$\boxed{3}$ $H(e^{j\hat{\omega}}) = 2je^{-j2\hat{\omega}}\sin(\hat{\omega})$

$H(e^{j0}) = 0$ $\quad \sin(0) = 0$

$\boxed{4}$ $H(e^{j\hat{\omega}}) = \dfrac{\sin(\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})} e^{-j\hat{\omega}/2}$ Dirichlet

$H(e^{j0}) \neq 0$

$\boxed{5}$ $H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}(1 + 2\cos(\hat{\omega}))$

$H(e^{j0}) = 1 + 2 = 3$

$\boxed{6}$ $H(e^{j\hat{\omega}}) = \dfrac{\sin(2\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})} e^{-j3\hat{\omega}/2}$

$H(e^{j0}) \neq 0$

$\boxed{7}$ $H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}$

$H(e^{j0}) = 1$

$\boxed{8}$ None of the above