

GEORGIA INSTITUTE OF TECHNOLOGY
 SCHOOL of ELECTRICAL & COMPUTER ENGINEERING
QUIZ #2

DATE: 17-Mar-06

COURSE: ECE-2025

NAME: _____
 LAST, FIRST

GT #: _____
 (ex: gtz123a)

3 points

3 points

3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

- | | | |
|-----------------------|-------------------------|--------------------------|
| | L05:Tues-Noon (Juang) | L06:Thur-Noon (Verriest) |
| | L07:Tues-1:30pm (Juang) | |
| L01:M-3pm (McClellan) | L09:Tues-3pm (Chang) | L02:W-3pm (Zhou) |
| | | L10:Thur-3pm (Taylor) |
| L03:M-4:30pm (Fekri) | L11:Tues-4:30pm (Chang) | L04:W-4:30pm (Zhou) |

- Write your name on the front page **ONLY**. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning clearly to receive partial credit.
 Explanations are also required to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
 Only these answers will be graded. Circle your answers, or write them in the boxes provided.
 If space is needed for scratch work, use the backs of previous pages.

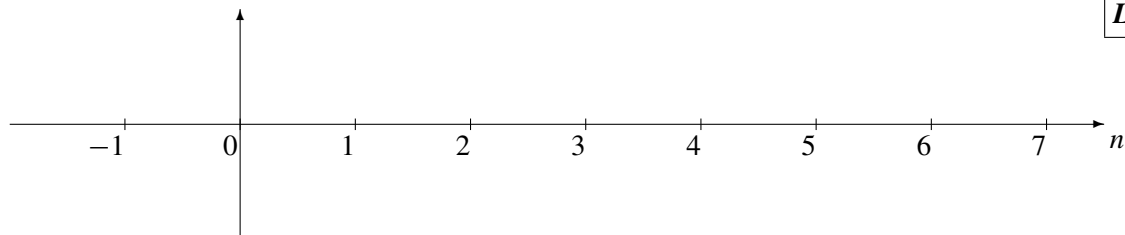
<i>Problem</i>	<i>Value</i>	<i>Score</i>
1	20	
2	20	
3	20	
4	20	
5	20	
No/Wrong Rec	-3	

PROBLEM sp-06-Q.2.1:

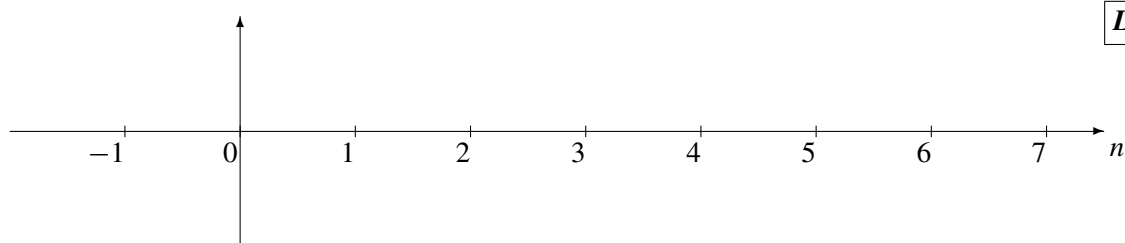
(a) Evaluate the convolution: $\mathbf{y_n = conv([100\ 0\ 100\ 0\ 100], [1\ 2\ 3])};$

Note: a MATLAB vector implicitly defines a signal to have its starting point at $n = 0$.

Give your answer as a *stem plot*.



(b) Make a *stem plot* of the signal $s[n] = \pi (u[n - 2] - u[n - 5])$.

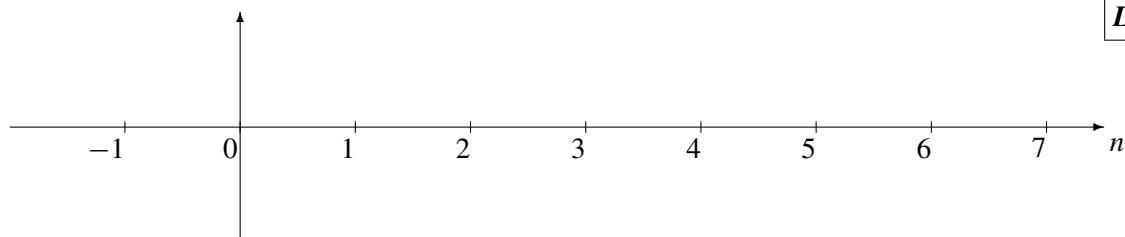


PROBLEM sp-06-Q.2.2:

(a) Determine the impulse response of the system:

$$y[n] = 10x[n - 2] + 10x[n - 5]$$

Give your answer as a *stem plot*.



(b) Determine the frequency response of the FIR system:

$$y[n] = 10x[n - 2] + 10x[n - 5]$$

Give your answer as a formula **in the following form:** $H(e^{j\hat{\omega}}) = e^{-j\beta\hat{\omega}} A \cos(\mu\hat{\omega})$
by finding numerical values for A , β and μ .

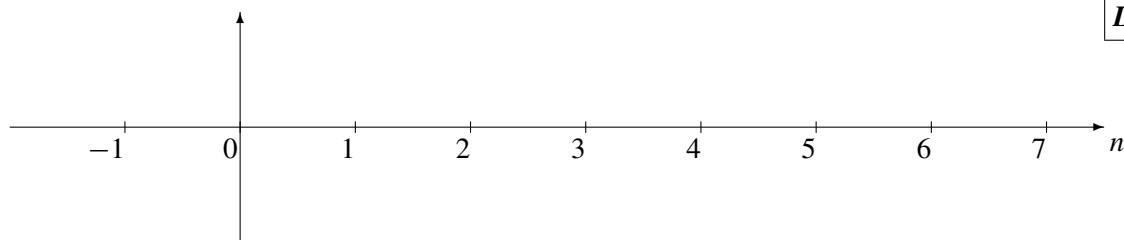
$A =$ <input style="width: 60px;" type="text"/>	$\beta =$ <input style="width: 60px;" type="text"/>	$\mu =$ <input style="width: 60px;" type="text"/>
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PROBLEM sp-06-Q.2.3:

Suppose that the system function of an FIR filter is

$$H(z) = 2z^{-2} (1 - e^{j\pi/5} z^{-1}) (1 - e^{-j\pi/5} z^{-1})$$

- (a) Determine the impulse response, $h[n]$, of the FIR filter. Give your answer as a *stem plot*.



Label all points

- (b) Evaluate the magnitude and phase of the frequency response of the FIR filter at $\hat{\omega} = 0.6\pi$.

Magnitude =

Phase =

PROBLEM sp-06-Q.2.4:

Here are some operations that are often done in MATLAB. In each case, you should analyze the code and determine the answer via mathematics.

- (a) Suppose that a student enters the following MATLAB code:

```
nn = 0:4480099;  
zz = (10 + j*pi) * exp(j*(1.8*pi*nn + 14.92));  
soundsc( real(zz), 10000 )
```

Determine the analog frequency (in Hertz) that will be heard. (There might be folding or aliasing!)

FREQ = Hz

Explain your reasoning.

- (b) Suppose that a student writes the following MATLAB code to generate a sine wave:

```
tt = 0:1/12000:10000;  
xx = cos( 2*pi*2400*tt + pi/3 );  
soundsc( xx, fsamp );
```

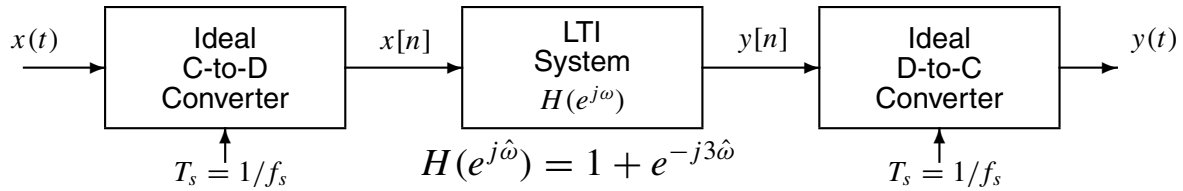
Although the sinusoid was not written to have a frequency of 3000 Hz, it is possible to play out the vector `xx` so that it sounds like a 3000 Hz tone. Determine the value of `fsamp` that should be used to play the vector `xx` as a 3000 Hz tone.

fsamp = Hz

Explain your reasoning.

PROBLEM sp-06-Q.2.5:

Consider the following system for discrete-time filtering of a continuous-time signal:

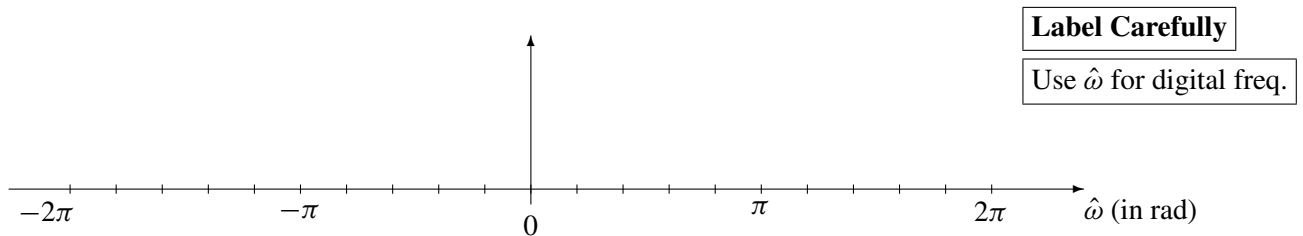


For all parts below, the input to the ideal C-to-D converter is $x(t) = 1500 \cos(16\pi t)$.

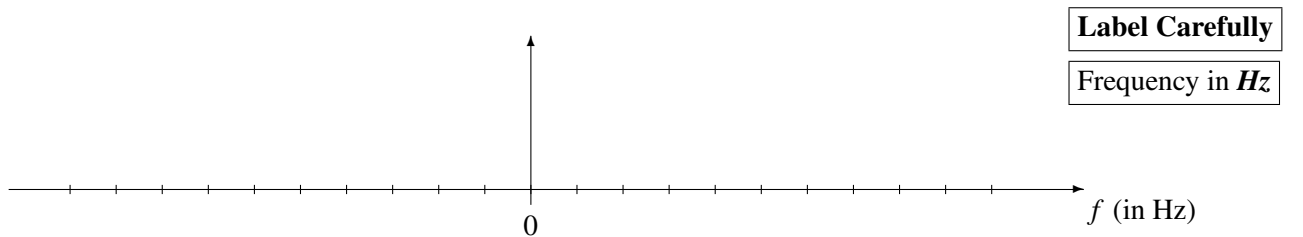
- (a) Determine the Nyquist rate (in hertz) for sampling the input signal $x(t)$. Explain.

$f_{\text{Nyquist}} =$ Hz

- (b) If the sampling rate of the C-to-D converter is $f_s = 20$ samples/sec, make a plot of the spectrum of the discrete-time signal $x[n]$ over the range of frequencies $-2\pi \leq \hat{\omega} \leq 2\pi$. Make sure to show all spectrum lines and label the frequency, amplitude and phase of each spectral component.



- (c) If the sampling rate of the the ideal D-to-C converter is $f_s = 20$ samples/sec, draw the spectrum for the continuous-time output signal, $y(t)$. Use $x(t)$ defined above.



PROBLEM sp-06-Q.2.1:

(a) Evaluate the convolution: $y_n = \text{conv}([100\ 0\ 100\ 0\ 100], [1\ 2\ 3])$;

Note: a MATLAB vector implicitly defines a signal to have its starting point at $n = 0$.

Give your answer as a stem plot.

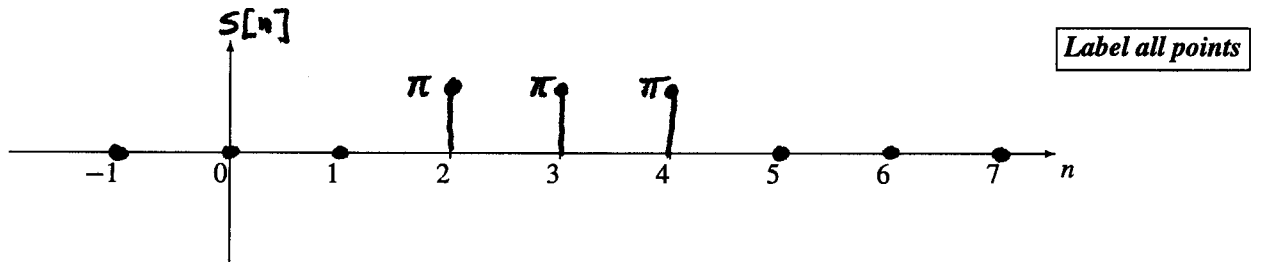


The convolution table is:

$$\begin{aligned}
 y_n &= [100\ 0\ 100\ 0\ 100\ 0\ 0\] \\
 &+ [0\ 200\ 0\ 200\ 0\ 200\ 0\] \\
 &+ [0\ 0\ 300\ 0\ 300\ 0\ 300\] \\
 &= [100\ 200\ 400\ 200\ 400\ 200\ 300\]
 \end{aligned}$$

$\nearrow n=0$
 $\nwarrow n=5$

(b) Make a stem plot of the signal $s[n] = \pi (u[n - 2] - u[n - 5])$.



The difference of two unit-step signals is a pulse. The unit-step $u[n - 2]$ goes “up” at $n = 2$; the unit-step $-u[n - 5]$ goes “down” at $n = 2$, so the final result has two cases:

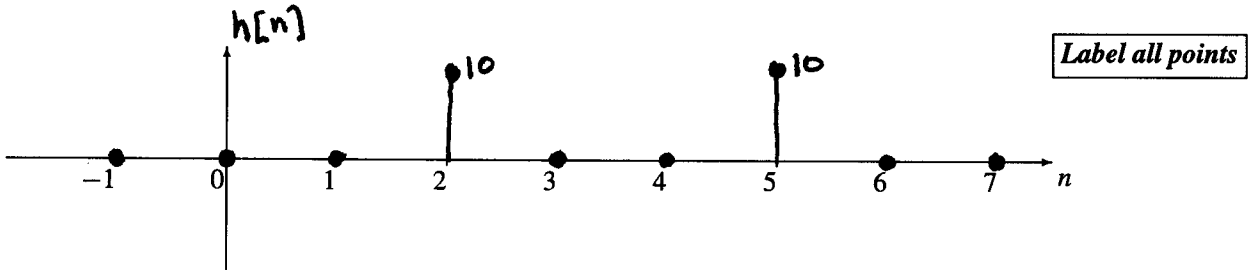
$$\pi (u[n - 2] - u[n - 5]) = \begin{cases} \pi & 2 \leq n < 5 \\ 0 & \text{elsewhere} \end{cases}$$

PROBLEM sp-06-Q.2.2:

(a) Determine the impulse response of the system:

$$y[n] = 10x[n - 2] + 10x[n - 5]$$

Give your answer as a *stem plot*.



When $x[n] = \delta[n]$, you get the impulse response, $h[n]$. Thus,

$$h[n] = 10\delta[n - 2] + 10\delta[n - 5]$$

(b) Determine the frequency response of the FIR system:

$$y[n] = 10x[n - 2] + 10x[n - 5]$$

Give your answer as a formula *in the following form*: $H(e^{j\hat{\omega}}) = e^{-j\beta\hat{\omega}} A \cos(\mu\hat{\omega})$ by finding numerical values for A , β and μ .

$A = 20$	$\beta = 3.5$	$\mu = 1.5$
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The frequency response is $H(e^{j\hat{\omega}}) = \sum_{n=0}^M h[n]e^{-jk\hat{\omega}}$. Thus,

$$H(e^{j\hat{\omega}}) = 10e^{-j2\hat{\omega}} + 10e^{-j5\hat{\omega}} = 10e^{-j3.5\hat{\omega}} (e^{j1.5\hat{\omega}} + e^{-j1.5\hat{\omega}})$$

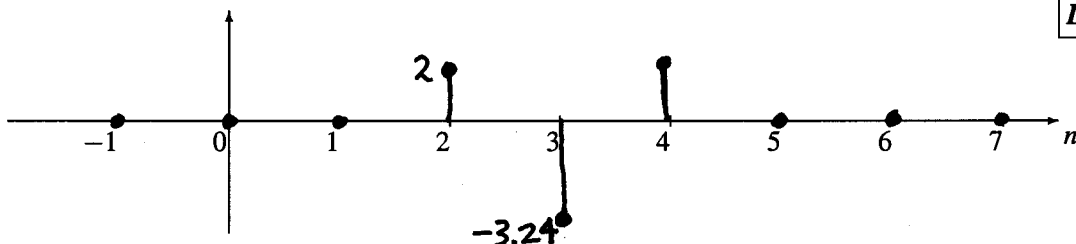
$$H(e^{j\hat{\omega}}) = 20e^{-j3.5\hat{\omega}} \cos(1.5\hat{\omega})$$

PROBLEM sp-06-Q.2.3:

Suppose that the system function of an FIR filter is

$$H(z) = 2z^{-2} (1 - e^{j\pi/5} z^{-1}) (1 - e^{-j\pi/5} z^{-1})$$

- (a) Determine the impulse response, $h[n]$, of the FIR filter. Give your answer as a *stem plot*.



Multiply out the factors and then use the definition of the z -Transform, $H(z) = \sum_{n=0}^M h[n]z^{-n}$, to get the impulse response from the coefficients of the z -Transform polynomial:

$$H(z) = 2z^{-2} (1 - 2 \cos(\pi/5)z^{-1} + z^{-2}) = 2z^{-2} - 3.24z^{-3} + 2z^{-4}$$

$$\Rightarrow h[n] = 2\delta[n - 2] - 3.24\delta[n - 3] + 2\delta[n - 4]$$

- (b) Evaluate the magnitude and phase of the frequency response of the FIR filter at $\hat{\omega} = 0.6\pi$.

$$\text{Magnitude} = 4.472$$

$$\text{Phase} = -0.8\pi = -2.513 \text{ radians}$$

Evaluating $H(e^{j\hat{\omega}})$ at $\hat{\omega} = 0.6\pi$ is the same as evaluating $H(z)$ at $z = e^{j0.6\pi}$, so we obtain

$$H(e^{j0.6\pi}) = 2e^{-j1.2\pi} (1 - e^{j0.2\pi} e^{-j0.6\pi}) (1 - e^{-j0.2\pi} e^{-j0.6\pi})$$

which reduces to

$$\begin{aligned} H(e^{j0.6\pi}) &= 2 (e^{-j1.2\pi} - e^{-j1.6\pi} - e^{-j2\pi} + e^{-j2.4\pi}) \\ &= 4.472e^{-j0.8\pi} = 4.472e^{-j2.513} \end{aligned}$$

PROBLEM sp-06-Q.2.4:

Here are some operations that are often done in MATLAB. In each case, you should analyze the code and determine the answer via mathematics.

- (a) Suppose that a student enters the following MATLAB code:

```
nn = 0:4480099;  
zz = (10 + j*pi) * exp(j*(1.8*pi*nn + 14.92) );  
soundsc( real(zz), 10000 )
```

Determine the analog frequency (in Hertz) that will be heard. (There might be folding or aliasing!)

FREQ = 1000 Hz

The discrete-time frequency of $\hat{\omega} = \pm 1.8\pi$ will alias to $\hat{\omega} = \mp 0.2\pi = \mp 2\pi(0.1)$, so the analog frequency of the output is $f_{\text{out}} = \frac{\hat{\omega}}{2\pi} f_s = (0.1)(10000) = 1000$ Hz.

- (b) Suppose that a student writes the following MATLAB code to generate a sine wave:

```
tt = 0:1/12000:10000;  
xx = cos( 2*pi*2400*tt + pi/3 );  
soundsc( xx, fsamp );
```

Although the sinusoid was not written to have a frequency of 3000 Hz, it is possible to play out the vector **xx** so that it sounds like a 3000 Hz tone. Determine the value of **fsamp** that should be used to play the vector **xx** as a 3000 Hz tone.

fsamp = 15000 Hz

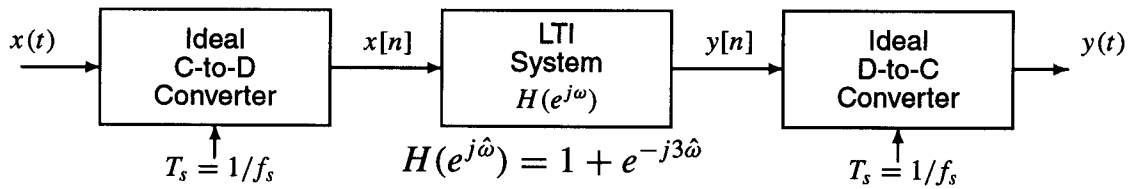
The **xx** signal is actually a discrete-time signal that came from a sinusoid of frequency 2400 Hz, sampled at 12000 Hz. Thus the discrete-time frequency is $\hat{\omega} = 2\pi(2400/12000) = 2\pi(0.2)$ which does **not** alias. In order to get a specified output frequency of 3000 Hz, we must solve for f_s in

$$f_{\text{out}} = \frac{\hat{\omega}}{2\pi} f_s \quad \Rightarrow \quad 3000 = \frac{2\pi(1/5)}{2\pi} f_s$$

Thus, the sampling frequency should be $f_s = 5(3000) = 15000$ Hz.

PROBLEM sp-06-Q.2.5:

Consider the following system for discrete-time filtering of a continuous-time signal:



For all parts below, the input to the ideal C/D converter is $x(t) = 1500 \cos(16\pi t)$.

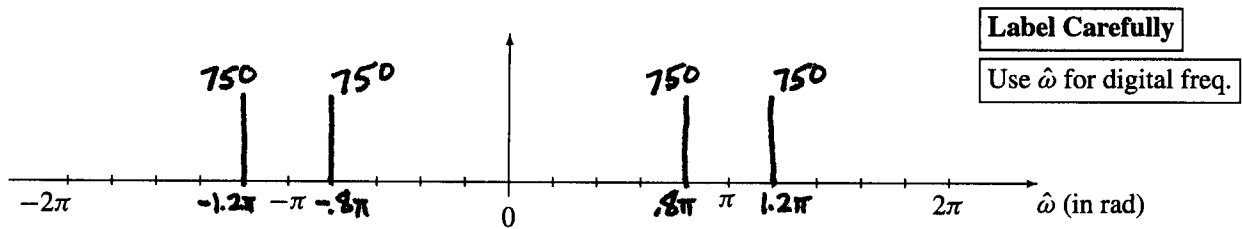
- (a) Determine the Nyquist rate (in hertz) for sampling the input signal $x(t)$. Explain.

$$f_{\text{Nyquist}} = \boxed{16} \text{ Hz}$$

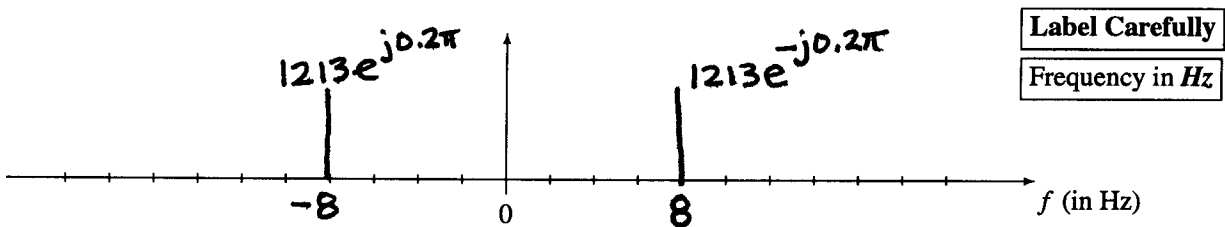
The Nyquist Rate is two times the highest frequency in $x(t)$, whose only frequency is 8 Hz. The Sampling Theorem says that you must sample at greater than the Nyquist Rate to be able to recover the signal.

- (b) If the sampling rate of the C-to-D converter is $f_s = 20$ samples/sec, make a plot of the spectrum of the discrete-time signal $x[n]$ over the range of frequencies $-2\pi \leq \hat{\omega} \leq 2\pi$. Make sure to show all spectrum lines and label the frequency, amplitude and phase of each spectral component.

The signal $x[n]$ has a discrete-time frequency of $\hat{\omega} = 16\pi/20 = 0.8\pi$, so there will be spectral lines at $\hat{\omega} = \pm 0.8\pi \pm 2\pi$. In the figure the spectrum lines will be at $\hat{\omega} = \{0.8\pi, -1.2\pi, -0.8\pi, 1.2\pi\}$.



- (c) If the sampling rate of the the ideal D-to-C converter is $f_s = 20$ samples/sec, draw the spectrum for the continuous-time output signal, $y(t)$. Use $x[n]$ determined in the previous part.



The frequency response at $\hat{\omega} = 0.8\pi$ is

$$H(e^{j\hat{\omega}}) = 1 + e^{-j2.4\pi} = 1.618e^{-j0.2\pi}$$

Therefore, the output of the filter is $y[n] = 2427 \cos(0.8\pi n - 0.2\pi)$, and converting from $\hat{\omega}$ to the output frequency ($\omega_{\text{out}} = \hat{\omega} f_s$) gives

$$y(t) = 2427 \cos(16\pi t - 0.2\pi)$$

Thus the complex amplitudes are $1213.5e^{\mp j0.2\pi}$ at $f = \pm 8$ Hz.