



**Problem Q3.1:**

Use the properties of the continuous-time delta function to simplify the following expressions:

(a)  $t^4\delta(t - 2) =$

(b)  $\frac{d}{dt}\{t^4u(t - 2)\} =$

(c)  $\int_{-\infty}^{\infty} t^4\delta(t - 2)dt =$

(be sure to explain your reasoning)

**Problem Q3.2:**

Suppose an LTI filter has a system function given by

$$H(z) = \frac{1 + z^{-1} + z^{-2}}{1 + 0.75z^{-1}}$$

(a) Plot the poles and zeros of  $H(z)$ .

(b) What frequencies  $\hat{\omega}$  between  $-\pi$  and  $\pi$  of any input signal will be completely nulled out by the filter?

- (c) Find the difference equation describing the input/output relationship of the system in terms of an input  $x[n]$  and an output  $y[n]$ .

- (d) Suppose that a signal  $x[n]$  with  $z$ -transform given by

$$X(z) = \frac{z^{-5}}{1 + z^{-1} + z^{-2}}$$

is input to the system. Find the output signal  $y[n]$ .

**Problem Q3.3:**

Suppose  $h(t) = t^3[u(t) - u(t - 5)]$  and  $x(t) = \tan(t)[u(t - 1) - u(t - 3)]$ . Fill in the boxes in the following first step of computing the convolution of  $h(t)$  with  $x(t)$ , where we are choosing to let  $h(t)$  be the function that we are flipping and shifting:

$$(h * x)(t) = \begin{cases} 0 & \text{for } t \leq \boxed{\phantom{00}} \\ \int_{\boxed{\phantom{00}}}^{\boxed{\phantom{00}}} (t - \tau)^3 \tan(\tau) d\tau & \text{for } \boxed{\phantom{00}} < t \leq \boxed{\phantom{00}} \\ \int_{\boxed{\phantom{00}}}^{\boxed{\phantom{00}}} (t - \tau)^3 \tan(\tau) d\tau & \text{for } \boxed{\phantom{00}} < t \leq \boxed{\phantom{00}} \\ \int_{\boxed{\phantom{00}}}^{\boxed{\phantom{00}}} (t - \tau)^3 \tan(\tau) d\tau & \text{for } \boxed{\phantom{00}} < t \leq \boxed{\phantom{00}} \\ 0 & \text{for } t > \boxed{\phantom{00}} \end{cases}$$

Just fill in the boxes. Don't try to work any more of the convolution, i.e. don't try actually working out the integrals! (Notice we picked  $t^3$  and  $\tan(t)$  rather arbitrarily. The point of the problem is to see if you can figure out the limits of the integrals and what the different regions are.)

**Problem Q3.4:**

Suppose a continuous-time LTI system has an impulse response given by  $h(t) = 400\pi \exp(-10\pi t)u(t)$ .

- (a) Find an expression for the output  $y(t)$  if the input is  $x(t) = \delta(t - 3)$ . Draw a labeled sketch of  $y(t)$ . (Your sketch need not be a work of art; just put in enough details so the grader can be sure you know understand what  $y(t)$  looks like.)

- (b) Find the output  $y(t)$  if the input is  $x(t) = \sqrt{2} \cos(10\pi t)$ . (Express your answer as simply as possible; for instance, we don't want to see a convolution operator in the final answer.)

**Problem Q3.5:**

Consider a system whose impulse response is given by

$$h(t) = \frac{\sin(500\pi t)}{\pi t}$$

Suppose the system is given an input  $x(t)$  specified by the Fourier series

$$\sum_{k=-\infty}^{\infty} \frac{1}{|k|^2 + 3} e^{jk300\pi t}$$

- (a) Find  $H(j\omega)$ , the Fourier transform of  $h(t)$ . Give your answer as either a formula or a carefully labeled sketch, whichever you find most convenient.
- (b) Find the output  $y(t)$ . (Be sure to write your answer entirely using “real” quantities, i.e., don’t leave any complex exponentials in your answer.)