**GEORGIA INSTITUTE OF TECHNOLOGY**

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

# QUIZ #3

DATE: 20-Nov-00 COURSE: ECE 2025

NAME: ______________________________ STUDENT #: ________________

LAST, FIRST

---

Recitation Section: **Circle the day & time** when your Recitation Section meets:

| | | |
|---|---|---|
| L01:Tues-9:30am (Casinovi) | L02:Thur-9:30am (Bordelon) | L03:Tues-12:00pm (Casinovi) |
| L04:Thur-12:00pm (Bordelon) | L05:Tues-1:30pm (Bordelon) | L06:Thur-1:30pm (Cassinovi) |
| L07:Tues-3:00pm (Bordelon) | L08:Thur-3:00pm (Hayes) | L09:Tues-4:30pm (Fekri) |
| L10:Thur-4:30pm (Li) | L11:Tues-6:00pm (Fekri) | L12:Thur-6:00pm (Li) |
| L13:Mon -3:00pm (Williams) | L14:Weds-3:00pm (Bordelon) | L15:Mon -4:30pm (Verriest) |
| L16:Weds-4:30pm (Dansereau) | L17:Mon -6:00pm (Verriest) | L18:Weds-6:00pm (Dansereau) |
| L19:Weds-1:30pm (Bordelon) | L20:Mon -1:30pm (Williams) | |

---

- Write your name on your exam, and **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted. However, one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes permitted. OK to write on both sides.
- Justify your reasoning CLEARLY to receive any partial credit.
  Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

| *Problem* | *Value* | *Score* |
|---|---|---|
| 1 | 20 | |
| 2 | 20 | |
| 3 | 20 | |
| 4 | 20 | |
| 5 | 20 | |

**Problem Fall-00-Q.3.1:**

For each of the following problems, simplify the expression using the properties of the continuous-time unit impulse signal. Provide some **explanation** or intermediate steps for each answer.

(a) $\displaystyle\int_{-\infty}^{t} \frac{\tau}{3+\tau}\delta(\tau-5)\,d\tau =$

(b) $\displaystyle\frac{d}{dt}\Big\{\cos(4t)u(t-4)\Big\} =$

**Problem Fall-00-Q.3.2:**

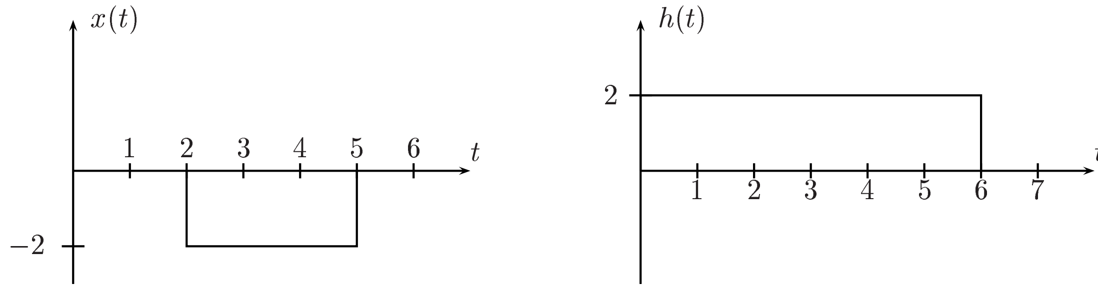
In each of the following problems, find the Fourier transform, or inverse Fourier transform. Give your answer as a simple formula or a plot. **Explain** each answer by stating which property and transform pair you used.

(a) Find $V(j\omega)$ when $v(t) = \begin{cases} 1 & 4 \le t < 12 \\ 0 & \text{otherwise} \end{cases}$.

(b) Find $X(j\omega)$ when $x(t) = 1 + \delta(t - 10)$.

(c) Find $h(t)$ when $H(j\omega) = \dfrac{e^{-j2\omega}}{3 + 2j\omega}$.

**Problem Fall-00-Q.3.3:**
The following figure shows the signal $x(t) = -2u(t-2)+2u(t-5)$, which is the input to a continuous-time LTI system whose impulse response (shown on the right) is $h(t) = 2u(t) - 2u(t-6)$.

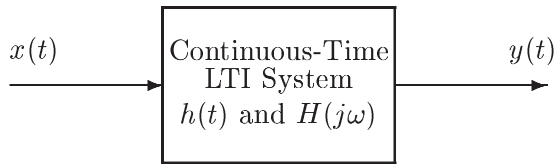



(a) Sketch $h(3-\tau)$ as a function of $\tau$ in the space below.

(b) Determine the value of the output of the LTI system, $y(t)$, at $t = 3$; that is, determine $y(3)$. It is not necessary to evaluate $y(t)$ for all $t$, only for $t = 3$. Note: This problem may be answered without performing any integration.

(c) What is the minimum value of $y(t)$? For what value(s) of $t$ does $y(t)$ reach this minimum value?

**Problem Fall-00-Q.3.4:**
The input to the LTI system shown below is a periodic signal $x(t)$ that has a period $\boxed{T_0 = 1/3}$ seconds.



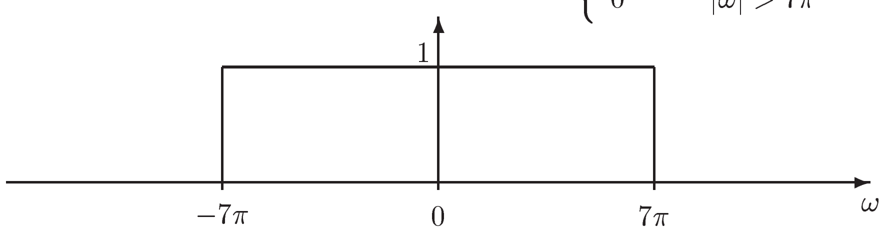

The Fourier series representation for the input $x(t)$ is

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \qquad \text{where} \qquad a_k = \begin{cases} 3 & k = 0 \\ \dfrac{\sin(\pi k/2)}{4\pi k} & k \neq 0. \end{cases}$$

(a) What is the fundamental frequency $\omega_0$ of the input signal $x(t)$? $\boxed{\omega_0 = \underline{\qquad\qquad} \text{ rad/sec}}$

(b) Suppose that the frequency response of the system is an ideal lowpass filter as illustrated below.

$$H(j\omega) = \begin{cases} 1 & |\omega| < 7\pi \\ 0 & |\omega| > 7\pi \end{cases}$$




Give an equation for the output of the system, $y(t)$, that is valid for $-\infty < t < \infty$. (Your answer should be expressed in terms of only real quantities). Justify your answer by sketching the Fourier transform of $y(t)$.

**Problem Fall-00-Q.3.5:**

(a) Evaluate the following integral: $\int_{-\infty}^{\infty} \delta(\omega + 0.3\pi)e^{j\omega t}d\omega$.

Provide some **explanation** or intermediate steps to justify your answer.

(b) Plot $x(t) = \frac{\sin(4\pi t)}{3t}$ versus $t$ for all $t$ between $-1$ and 1. Label all important features such as peaks and zero crossings.

(c) Use the **Fourier transform** to find the DC value of the sinc function in part (b). In other words, evaluate the integral $\int_{-\infty}^{\infty} \frac{\sin(4\pi t)}{3t}dt$. Again, provide some **explanation** or intermediate steps to justify your answer.

# GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #3

DATE: 20-Nov-00 COURSE: ECE 2025

NAME: Solutions ______ STUDENT #: ______

LAST, FIRST

**Recitation Section**: **Circle the day & time** when your Recitation Section meets:

| | | |
|---|---|---|
| L01:Tues-9:30am (Casinovi) | L02:Thur-9:30am (Bordelon) | L03:Tues-12:00pm (Casinovi) |
| L04:Thur-12:00pm (Bordelon) | L05:Tues-1:30pm (Bordelon) | L06:Thur-1:30pm (Cassinovi) |
| L07:Tues-3:00pm (Bordelon) | L08:Thur-3:00pm (Hayes) | L09:Tues-4:30pm (Fekri) |
| L10:Thur-4:30pm (Li) | L11:Tues-6:00pm (Fekri) | L12:Thur-6:00pm (Li) |
| L13:Mon -3:00pm (Williams) | L14:Weds-3:00pm (Bordelon) | L15:Mon -4:30pm (Verriest) |
| L16:Weds-4:30pm (Dansereau) | L17:Mon -6:00pm (Verriest) | L18:Weds-6:00pm (Dansereau) |
| L19:Weds-1:30pm (Bordelon) | L20:Mon -1:30pm (Williams) | |

- Write your name on your exam, and **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted. However, one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes permitted. OK to write on both sides.
- Justify your reasoning CLEARLY to receive any partial credit. Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers, or write them in the boxes provided. If space is needed for scratch work, use the backs of previous pages.

| *Problem* | *Value* | *Score* |
|---|---|---|
| 1 | 20 | |
| 2 | 20 | |
| 3 | 20 | |
| 4 | 20 | |
| 5 | 20 | |

**Problem Fall-00-Q.3.1:**

For each of the following problems, simplify the expression using the properties of the continuous-time unit impulse signal. Provide some **explanation** or intermediate steps for each answer.

(a) $\int_{-\infty}^{t} \frac{\tau}{3+\tau}\delta(\tau-5)d\tau =$

$$\frac{\tau}{3+\tau}\delta(\tau-5) = \frac{5}{3+5}\delta(\tau-5) = \frac{5}{8}\delta(\tau-5)$$

Therefore,

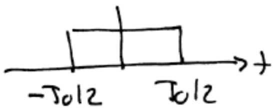
$$\int_{-\infty}^{t} \frac{\tau}{3+\tau}\delta(\tau-5)d\tau = \frac{5}{8}\int_{-\infty}^{t}\delta(\tau-5)d\tau = \frac{5}{8}u(t-5)$$

(b) $\frac{d}{dt}\left\{\cos(4t)u(t-4)\right\} = -4\sin(4t)u(t-4) + \cos(4t)\delta(t-4)$

$$= -4\sin(4t)u(t-4) + \cos(16)\delta(t-4)$$

**Problem Fall-00-Q.3.2:**
In each of the following problems, find the Fourier transform, or inverse Fourier transform. Give your answer as a simple formula or a plot. **Explain** each answer by stating which property and transform pair you used.

(a) Find $V(j\omega)$ when $v(t) = \begin{cases} 1 & 4 \le t < 12 \\ 0 & \text{otherwise} \end{cases}$.




$$\Longleftrightarrow \quad \frac{\sin(\omega T_0/2)}{\omega/2}$$

Since $v(t)$ is a pulse of width $T_0 = 8$ that has been delayed by $t_0 = 8$, then

$$V(j\omega) = e^{-j8\omega} \frac{\sin(\omega 4)}{\omega/2}$$

(b) Find $X(j\omega)$ when $x(t) = 1 + \delta(t - 10)$.

$$1 \Longleftrightarrow 2\pi\,\delta(\omega)$$

$$\delta(t-10) \Longleftrightarrow e^{-j10\omega}$$

Using linearity, we have

$$1 + \delta(t-10) \Longleftrightarrow 2\pi\,\delta(\omega) + e^{-j10\omega}$$

(c) Find $h(t)$ when $H(j\omega) = \dfrac{e^{-j2\omega}}{3 + 2j\omega}$.

$$\frac{e^{-j2\omega}}{3 + 2j\omega} = e^{-j2\omega} \frac{1/2}{3/2 + j\omega}$$

Since

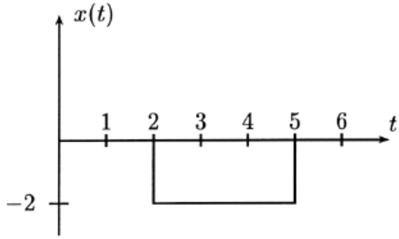
$$e^{-at}u(t) \Longleftrightarrow \frac{1}{a + j\omega}$$

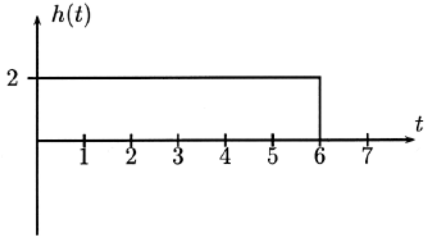
Using the linearity and delay properties we have

$$e^{-j2\omega} \frac{1/2}{3/2 + j\omega} \Longleftrightarrow \frac{1}{2} e^{-\frac{3}{2}(t-2)} u(t-2)$$

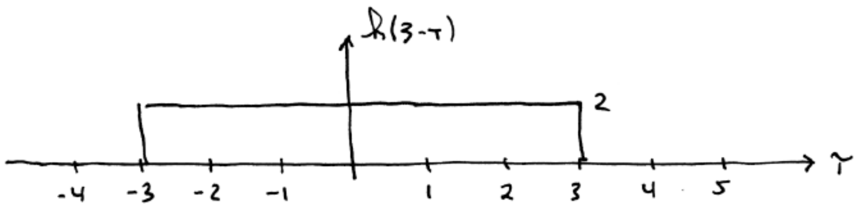
**Problem Fall-00-Q.3.3:**

The following figure shows the signal $x(t) = -2u(t-2)+2u(t-5)$, which is the input to a continuous-time LTI system whose impulse response (shown on the right) is $h(t) = 2u(t) - 2u(t-6)$.




(a) Sketch $h(3-\tau)$ as a function of $\tau$ in the space below.




(b) Determine the value of the output of the LTI system, $y(t)$, at $t = 3$; that is, determine $y(3)$. It is not necessary to evaluate $y(t)$ for all $t$, only for $t = 3$. Note: This problem may be answered without performing any integration.

$$y(3) = \int_{-\infty}^{\infty} x(\tau) h(3-\tau)\, d\tau$$

Note that the integrand, $x(\tau)h(3-\tau)$, is equal to $(2)(-2) = 4$ for $2 \le \tau \le 3$, and is zero otherwise. Therefore, the integral is equal to $\boxed{-4}$.

(c) What is the minimum value of $y(t)$? For what value(s) of $t$ does $y(t)$ reach this minimum value?

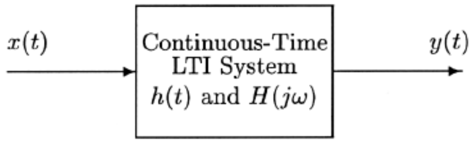
The minimum value occurs when $h(t-\tau)$ completely overlaps $x(\tau)$. This occurs when $5 \le t \le 8$.

For these values,

$$y(t) = \int_{2}^{5} (2)(-2)\, d\tau = -12$$

**Problem Fall-00-Q.3.4:**

The input to the LTI system shown below is a periodic signal $x(t)$ that has a period $\boxed{T_0 = 1/3}$ seconds.



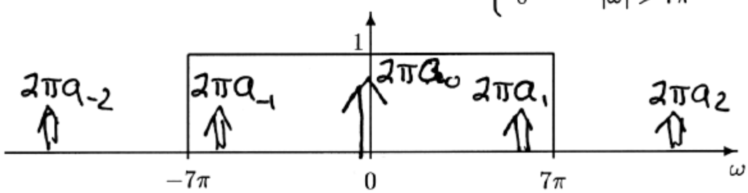

The Fourier series representation for the input $x(t)$ is

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \qquad \text{where} \qquad a_k = \begin{cases} 3 & k = 0 \\ \dfrac{\sin(\pi k/2)}{4\pi k} & k \neq 0. \end{cases}$$

(a) What is the fundamental frequency $\omega_0$ of the input signal $x(t)$? $\boxed{\omega_0 = 2\pi/T_0 = 6\pi \text{ rad/sec}}$

(b) Suppose that the frequency response of the system is an ideal lowpass filter as illustrated below.

$$H(j\omega) = \begin{cases} 1 & |\omega| < 7\pi \\ 0 & |\omega| > 7\pi \end{cases}$$




Give an equation for the output of the system, $y(t)$, that is valid for $-\infty < t < \infty$. (Your answer should be expressed in terms of only real quantities). Justify your answer by sketching the Fourier transform of $y(t)$.

The spectrum of $x(t)$ is plotted above. Note that the filter only passes three terms in the FS expansion,

$$y(t) = a_0 + a_1 e^{j\omega_0 t} + a_{-1} e^{-j\omega_0 t}$$

$$= a_0 + 2a_1 \cos(\omega_0 t)$$

$$= 3 + \frac{1}{2\pi}\cos(6\pi t)$$

**Problem Fall-00-Q.3.5:**

(a) Evaluate the following integral: $\int_{-\infty}^{\infty} \delta(\omega + 0.3\pi) e^{j\omega t} d\omega$.

Provide some **explanation** or intermediate steps to justify your answer.

Use the property $\int_{-\infty}^{\infty} x(t)\delta(t-t_0)dt = x(t_0)$.

Here, we have

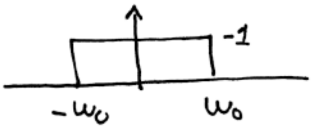
$$\int_{-\infty}^{\infty} \delta(\omega+0.3\pi)e^{j\omega t}d\omega = e^{-j0.3\pi t}$$

(b) Plot $x(t) = \dfrac{\sin(4\pi t)}{3t}$ versus $t$ for all $t$ between $-1$ and $1$. Label all important features such as peaks and zero crossings.


(c) Use the **Fourier transform** to find the DC value of the sinc function in part (b). In other words, evaluate the integral $\int_{-\infty}^{\infty} \frac{\sin(4\pi t)}{3t} dt$. Again, provide some **explanation** or intermediate steps to justify your answer.

With $X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$, note that the DC value at $\omega = 0$ is the integral of $x(t)$. Since we know that

$$\frac{\sin(\omega_0 t)}{\pi t} \Leftrightarrow$$




with

$$\frac{\sin(4\pi t)}{3t} = \frac{\pi}{3}\frac{\sin(4\pi t)}{\pi t}$$

it follows that $X(j\omega)$ at $\omega = 0$ is $\boxed{\pi/3}$.