GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #3

DATE: 9-Apr-04 COURSE: ECE-2025

NAME: ______________________________ GT #: ____________________

LAST, FIRST

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

	L03:Tues-Noon (Ji)		L04:Thur-Noon (Bordelon)
	L05:Tues-1:30pm (Ji)		L06:Thurs-1:30pm (Bordelon)
L11:M-3pm (McClellan)	L07:Tues-3pm (Fan)	L12:W-3pm (Bordelon)	
	L09:Tues-4:30pm (Fan)	L14:W-4:30pm (Bordelon)	GTREP: (Barnes)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning clearly to receive partial credit.
  Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	25	
2	25	
3	25	
4	25	

PROBLEM sp-04-Q.3.1:

For each of the following expressions, reduce the expression to the simplest possible form:
(The operator $*$ denotes convolution.)

(a) $\displaystyle\int_{-\infty}^{0} 4\delta(t+4)dt$

(b) $\displaystyle\frac{\sin(4\omega)}{\omega/2}\bigg|_{\omega=0}$

(c) $\left\{e^{-4(t-1)}u(t-1)\right\} * \delta(t+4)$

(d) $\left\{e^{-4(t-1)}u(t-1)\right\} \delta(t+4)$

(e) $\delta(t-1) * \delta(t+4)$

(f) $\displaystyle\frac{d}{dt}\left\{e^{-4t}u(t+4)\right\}$

PROBLEM sp-04-Q.3.2:

For each of the following time-domain signals, select the correct match from the list of Fourier transforms below. ***Write each answer in the box provided.*** (The operator $*$ denotes convolution.)

(a) $x(t) = -3e^{-3t/4}u(t) + 4\delta(t)$

(b) $x(t) = 4e^{(-3+j4)t}u(t)$

(c) $x(t) = \delta(t-4)\ \sin(\pi t)$

(d) $x(t) = u(t-3) - u(t-5)$

(e) $x(t) = \Im m\{\delta(t-4) * e^{j\pi t}\}$

Each of the time signals above has a Fourier transform that can be found in the list below.

[0] $X(j\omega) = \dfrac{j16\omega}{3 + j4\omega}$

[1] $X(j\omega) = 2e^{-j4\omega}\ \dfrac{\sin(4\omega)}{\omega}$

[2] $X(j\omega) = \dfrac{-12}{3 + j4\omega}$

[3] $X(j\omega) = j2e^{-j4\omega}\ \sin(4\omega)$

[4] $X(j\omega) = 0$

[5] $X(j\omega) = 2e^{-j4\omega}\ \dfrac{\sin(\omega)}{\omega}$

[6] $X(j\omega) = \dfrac{\sin(\omega)}{\omega/2}$

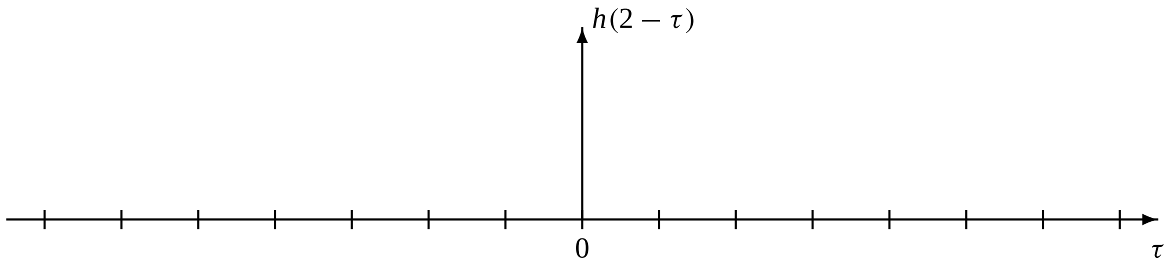
[7] $X(j\omega) = e^{-j4\omega}\,[j\pi\delta(\omega+\pi) - j\pi\delta(\omega-\pi)]$

[8] $X(j\omega) = \frac{1}{2\pi}e^{-j4\omega} * [j\pi u(\omega+\pi) - j\pi u(\omega-\pi)]$

[9] $X(j\omega) = \dfrac{4}{3 + j(\omega - 4)}$

PROBLEM sp-04-Q.3.3:

(a) Assume that $h(t) = u(t+3) - u(t-1)$. Plot $h(2-\tau)$ as a function of τ.



(b) When two finite-duration signals are convolved, the result is a finite-duration signal, $y(t) = x(t) * h(t)$. Suppose that $h(t)$ is the signal defined in part (a), and that the input signal is:

$$x(t) = e^{t-2}\{u(t-2) - u(t-9)\}$$

Determine the duration (in secs.) of the output signal $y(t) = x(t) * h(t)$.

Duration =

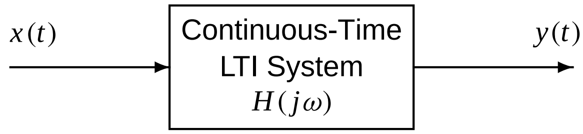
(c) If the input is changed to $x(t) = 7u(t-2)$, and $h(t)$ is still defined as in part (a), then it will be true that the output $y(t) = h(t) * x(t)$ from the convolution can be written as

$$y(t) = B(t - T_{12})\{u(t - T_{12}) - u(t - T_{23})\} + Cu(t - T_{23})$$

where B and C and the times T_{12} and T_{23} are constants. Determine the values of these four parameters.

$B =$
$C =$
$T_{12} =$
$T_{23} =$

PROBLEM sp-04-Q.3.4:

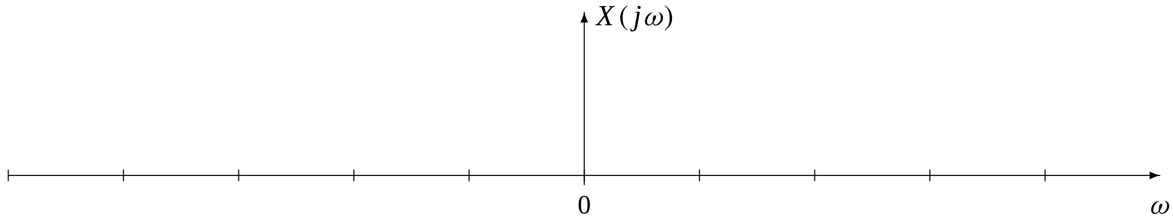


The periodic input to the above system is defined by the equation:

$$x(t) = \sum_{k=-2}^{2} a_k e^{j10kt}, \quad \text{where} \quad a_k = \begin{cases} \dfrac{1/\pi}{1+k^2} & k \neq 0 \\ 0.1 & k = 0 \end{cases}$$

(a) Determine the Fourier transform of the periodic signal $x(t)$. Give a formula and then plot it on the graph below. Label your plot with numerical values to receive full credit.

$X(j\omega) =$



(b) The frequency response of the LTI system is given by the following equation:

$$H(j\omega) = \frac{j10\omega}{20 + j\omega}$$

Evaluate the frequency response at $\omega = 10$, giving your answer in polar form (with numerical values):

at $\omega = 10$, $|H(j\omega)| =$

at $\omega = 10$, $\angle H(j\omega) =$

(c) For $x(t)$ given above, the output signal can be written as $y(t) = \sum_{k=-2}^{2} b_k e^{jk\omega_0 t}$

Determine the numerical values of the parameters ω_0, b_0 and b_1.

$\omega_0 =$

$b_0 =$

$b_1 =$

GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #3

DATE: 9-Apr-04 COURSE: ECE-2025

NAME: Answer Key (LAST, FIRST) GT #: Version-1

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

	L03:Tues-Noon (Ji)		L04:Thur-Noon (Bordelon)
	L05:Tues-1:30pm (Ji)		L06:Thurs-1:30pm (Bordelon)
L11:M-3pm (McClellan)	L07:Tues-3pm (Fan)	L12:W-3pm (Bordelon)	
	L09:Tues-4:30pm (Fan)	L14:W-4:30pm (Bordelon)	GTREP: (Barnes)

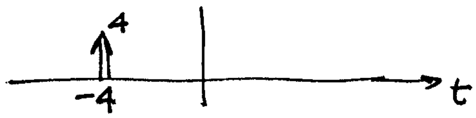
- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning clearly to receive partial credit.
  Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	25	
2	25	
3	25	
4	25	

PROBLEM sp-04-Q.3.1:

For each of the following expressions, reduce the expression to the simplest possible form:
(The operator $*$ denotes convolution.)

(a) $\int_{-\infty}^{0} 4\delta(t+4)dt$ **Answer** = 4



Integral range includes the impulse

(b) $\left.\frac{\sin(4\omega)}{\omega/2}\right|_{\omega=0}$ **Answer** = 8

$$\frac{\sin(4\omega)}{\omega/2} \longrightarrow \frac{4\omega}{\omega/2} = 8$$

(c) $\left\{e^{-4(t-1)}u(t-1)\right\} * \delta(t+4)$ **Answer** = $e^{-4(t+3)}u(t+3)$

shift by -4

$\longrightarrow e^{-4(t+4-1)}u(t+4-1)$

(d) $\left\{e^{-4(t-1)}u(t-1)\right\} \delta(t+4)$ **Answer** = 0

starts at $t=1$

impulse at $t=-4$

(e) $\delta(t-1) * \delta(t+4)$ **Answer** = $\delta(t+3)$

$= \delta(t+4-1)$

(f) $\frac{d}{dt}\left\{e^{-4t}u(t+4)\right\}$ **Answer** = $-4e^{-4t}u(t+4) + e^{16}\delta(t+4)$

$$= -4e^{-4t}u(t+4) + \underbrace{e^{-4t}\delta(t+4)}_{e^{-4(-4)}\delta(t+4)}$$

$\longleftarrow$ eval at $t=-4$

PROBLEM sp-04-Q.3.2:

For each of the following time-domain signals, select the correct match from the list of Fourier transforms below. ***Write each answer in the box provided.*** (The operator $*$ denotes convolution.)

(a) $x(t) = -3e^{-3t/4}u(t) + 4\delta(t)$ $\xrightarrow{F.T.}$ $\frac{-3}{\frac{3}{4}+j\omega} + 4 = \frac{-12}{3+j4\omega} + 4 = \frac{j16\omega}{3+j4\omega}$

[0] $H(j\omega) = \frac{j16\omega}{3+j4\omega}$

(b) $x(t) = 4e^{(-3+j4)t}u(t)$ $\xrightarrow{F.T.}$ $\frac{4}{(3-j4)+j\omega} = \frac{4}{3+j(\omega-4)}$

[9] $H(j\omega) = \frac{4}{3+j(\omega-4)}$

(c) $x(t) = \delta(t-4)\,\sin(\pi t)$ $= \sin(4\pi)\delta(t-4) = 0$

[4] $H(j\omega) = 0$

(d) $x(t) = u(t-3) - u(t-5)$ ← length-2 pulse centered at $t=4$

[5] $H(j\omega) = 2e^{-j4\omega}\frac{\sin(\omega)}{\omega}$

(e) $x(t) = \Im m\{\delta(t-4) * e^{j\pi t}\}$ $= \sin(\pi(t-4))$

[7] $H(j\omega) = e^{-j4\omega}[j\pi\delta(\omega+\pi) - j\pi\delta(\omega-\pi)]$

Each of the time signals above has a Fourier transform that can be found in the list below.

[0] $X(j\omega) = \frac{j16\omega}{3+j4\omega}$

[1] $X(j\omega) = 2e^{-j4\omega}\frac{\sin(4\omega)}{\omega}$

[2] $X(j\omega) = \frac{-12}{3+j4\omega}$

[3] $X(j\omega) = j2e^{-j4\omega}\sin(4\omega)$

[4] $X(j\omega) = 0$

[5] $X(j\omega) = 2e^{-j4\omega}\frac{\sin(\omega)}{\omega}$

[6] $X(j\omega) = \frac{\sin(\omega)}{\omega/2}$

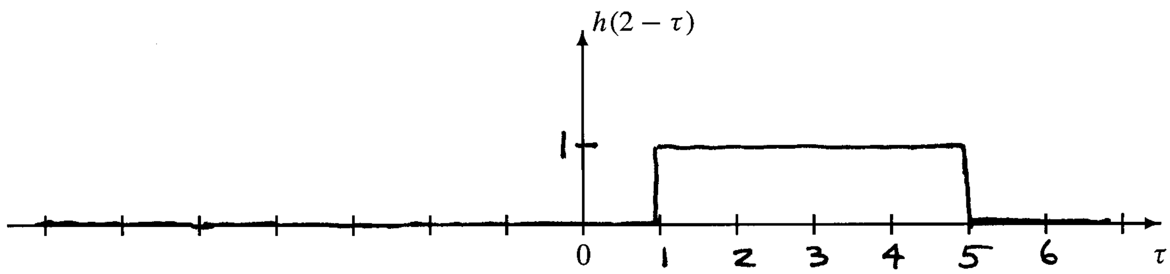
[7] $X(j\omega) = e^{-j4\omega}[j\pi\delta(\omega+\pi) - j\pi\delta(\omega-\pi)]$

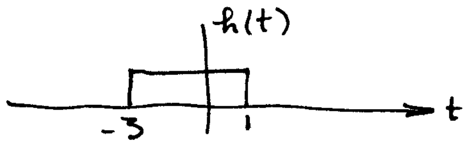
[8] $X(j\omega) = \frac{1}{2\pi}e^{-j4\omega} * [j\pi u(\omega+\pi) - j\pi u(\omega-\pi)]$

[9] $X(j\omega) = \frac{4}{3+j(\omega-4)}$

PROBLEM sp-04-Q.3.3:

(a) Assume that $h(t) = u(t+3) - u(t-1)$. Plot $h(2-\tau)$ as a function of τ.





(b) When two finite-duration signals are convolved, the result is a finite-duration signal, $y(t) = x(t) * h(t)$. Suppose that the input signal is:

$$x(t) = e^{t-2}\{u(t-2) - u(t-9)\}$$

Determine the duration (in secs.) of the output signal $y(t) = x(t) * h(t)$.

Duration = 11 secs.

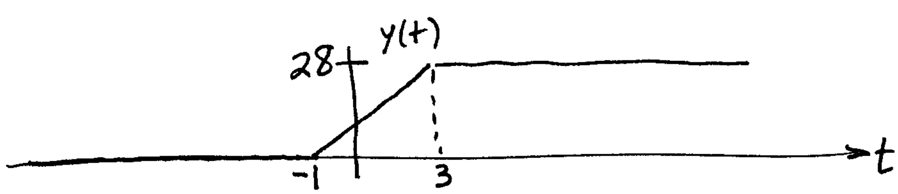
Length of x(t) = 7 secs
Length of h(t) = 4 secs.
11 secs.

(c) If the input is changed to $x(t) = 7u(t-2)$, then it will be true that the output $y(t) = h(t) * x(t)$ from the convolution can be written as

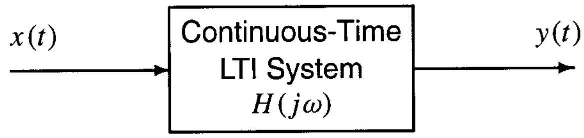
$$y(t) = B(t - T_{12})\{u(t - T_{12}) - u(t - T_{23})\} + Cu(t - T_{23})$$

where B and C and the times T_{12} and T_{23} are constants. Determine the values of these four parameters.

$B = 7$	$C = 28$	$T_{12} = -1$	$T_{23} = 3$



PROBLEM sp-04-Q.3.4:

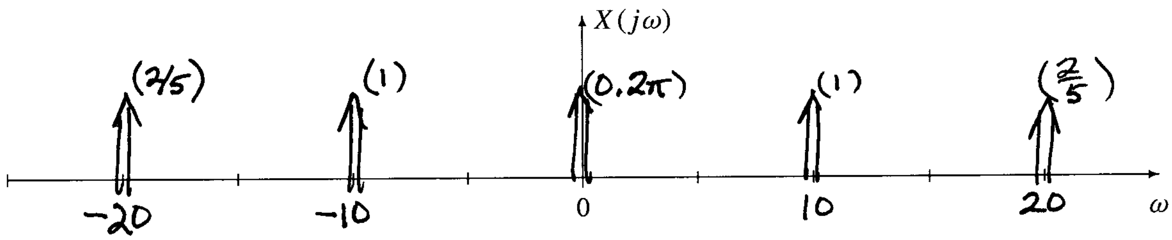


The periodic input to the above system is defined by the equation:

$$x(t) = \sum_{k=-2}^{2} a_k e^{j10kt}, \quad \text{where} \quad a_k = \begin{cases} \dfrac{1/\pi}{1+k^2} & k \neq 0 \\ 0.1 & k = 0 \end{cases}$$

(a) Determine the Fourier transform of the periodic signal $x(t)$. Give a formula and then plot it on the graph below. Label your plot with numerical values to receive full credit.

$$X(j\omega) = \sum_{k=-2}^{2} (2\pi a_k)\delta(\omega - 10k) \qquad \{a_k\} = \left\{ \frac{1}{5\pi}, \frac{1}{2\pi}, 0.1, \frac{1}{2\pi}, \frac{1}{5\pi} \right\}$$



(b) The frequency response of the LTI system is given by the following equation:

$$H(j\omega) = \frac{j10\omega}{20 + j\omega}\Bigg|_{\omega=10} = \frac{j\,100}{20 + j\,10}$$

Evaluate the frequency response at $\omega = 10$, giving your answer in polar form (with numerical values):

$|H(j10)| = 4.472$

$\angle H(j10) = 1.107 \text{ rads} = 0.352\pi = 63.43°$

(c) For $x(t)$ given above, the output signal can be written as $y(t) = \sum_{k=-2}^{2} b_k e^{jk\omega_0 t}$

Determine the numerical values of the parameters ω_0, b_0 and b_1.

$\omega_0 = 10$ rad/sec

$b_0 = 0$

$b_1 = 0.7118 e^{j1.107}$

$b_k = a_k H(j\omega_0 k) = a_k H(j10k)$

at $k=0$, $H(j0) = 0 \Rightarrow b_0 = 0$

$k=1$: $b_k = \frac{1}{2\pi} \times 4.472 e^{j1.107}$