GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #3

DATE: 21-Apr-06 COURSE: ECE-2025

NAME: ______________________________ GT #: ____________________

LAST, FIRST (ex: gtz123q)

3 points 3 points 3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

	L05:Tues-Noon (Juang)		L06:Thur-Noon (Verriest)
	L07:Tues-1:30pm (Juang)		
L01:M-3pm (McClellan)	L09:Tues-3pm (Chang)	L02:W-3pm (Zhou)	L10:Thur-3pm (Taylor)
L03:M-4:30pm (Fekri)	L11:Tues-4:30pm (Chang)	L04:W-4:30pm (Zhou)	

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning clearly to receive partial credit.
  Explanations are also required to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	25	
2	25	
3	25	
4	25	
No/Wrong Rec	$-\lfloor \pi \rfloor$	

PROBLEM sp-06-Q.3.1:

In each of the following cases, use properties of the unit-impulse function to simplify the expression ***as much as possible.*** Provide some $\boxed{\textbf{explanation}}$ or intermediate steps for each answer. *Note:* Star $*$ is the convolution operator.

(a) Simplify $x(t) = \dfrac{d}{dt}\left\{t^2\, u(t-\tfrac{1}{2})\right\}$

(b) Simplify $H(j\omega) = \delta(\omega - 3\pi) * \displaystyle\sum_{\ell=0}^{3} \sin(\omega/4)\,\delta(\omega - 2\pi\ell)$

(c) Simplify $q(t) = \displaystyle\int_{-\infty}^{t+1} \delta(\tau - 2)\, e^{-2(\tau - t)} d\tau$

PROBLEM sp-06-Q.3.2:

In each of the following cases, determine the (inverse or forward) Fourier transform. Give your answer as a plot, or a $\boxed{\text{simple}}$ formula (two of the answers will be ***real-valued.***)
Explain each answer (briefly) by stating which property and/or transform pair you used.

(a) Find $X(j\omega)$ when $x(t) = 7\cos(4\pi t - \pi/3)$.

(b) Find $s(t)$ when $S(j\omega) = \dfrac{\sin(5\omega)}{\omega}\, e^{-j2\omega}$.

(c) Find $h(t)$ when $H(j\omega) = \dfrac{12j\omega}{8 + j2\omega}$.

PROBLEM sp-06-Q.3.3:

Two questions about convolution:

(a) Find $y(t) = \pi u(t-4) * e^{-(t-2)}u(t-2)$. Give the answer as a **simple** formula.

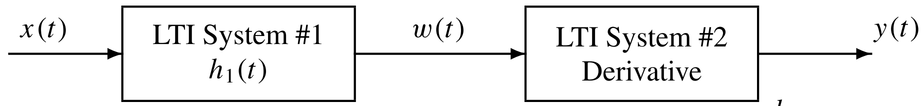
(b) If the signal $r(t)$ is a rectangular pulse, then $r(t) * r(t)$ is a triangle. Suppose that

$$r(t) * r(t) = y(t) = \begin{cases} 9t & \text{for } 0 \le t \le 2 \\ 18 - 9(t-2) & \text{for } 2 \le t \le 4 \\ 0 & \text{elsewhere} \end{cases}$$

Determine the rectangular signal $r(t)$.

PROBLEM sp-06-Q.3.4:

A cascade of linear time-invariant systems is depicted by the following block diagram:



$$h_1(t) = \sqrt{7}u(t-4) - \sqrt{7}u(t-9) \qquad\qquad y(t) = \frac{d}{dt}w(t)$$

(a) Determine the overall impulse response for this cascade of two systems. Give your answer in the ***simplest possible form.***

(b) The overall frequency response of this system, $H(j\omega)$, is zero for infinitely many values of ω. Derive a general formula that gives **all** the zeros of $H(j\omega)$. ***Explain.***

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QUIZ #3

DATE: 21-Apr-06 COURSE: ECE-2025

NAME: **ANSWER KEY** GT #: **VERSION #1**

LAST, FIRST (ex: gtz123q)

3 points 3 points 3 points

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PROBLEM sp-06-Q.3.1:

In each of the following cases, use properties of the unit-impulse function to simplify the expression ***as much as possible.*** Provide some $\boxed{\textbf{explanation}}$ or intermediate steps for each answer. *Note:* Star $*$ is the convolution operator.

(a) Simplify $x(t) = \dfrac{d}{dt}\left\{t^2\, u(t - \frac{1}{2})\right\}$

Take the derivative and evaluate at the location of the impulse ($t = \frac{1}{2}$):

$$\begin{aligned}\frac{d}{dt}\left\{t^2\, u(t - \tfrac{1}{2})\right\} &= 2t\, u(t - \tfrac{1}{2}) + t^2\, \delta(t - \tfrac{1}{2}) \\ &= 2t\, u(t - \tfrac{1}{2}) + \frac{1}{4}\delta(t - \tfrac{1}{2})\end{aligned}$$

(b) Simplify $H(j\omega) = \delta(\omega - 3\pi) * \sum\limits_{\ell=0}^{3} \sin(\omega/4)\, \delta(\omega - 2\pi\ell)$

$$\begin{aligned}H(j\omega) &= \delta(\omega - 3\pi) * \sum_{\ell=0}^{3} \sin(\omega/4)\, \delta(\omega - 2\pi\ell) \\ &= \delta(\omega - 3\pi) * \sum_{\ell=0}^{3} \sin(2\pi\ell/4)\, \delta(\omega - 2\pi\ell) \\ &= \delta(\omega - 3\pi) * \big(\delta(\omega - 2\pi) - \delta(\omega - 6\pi)\big) \\ &= \delta(\omega - 5\pi) - \delta(\omega - 9\pi)\end{aligned}$$

(c) Simplify $q(t) = \displaystyle\int_{-\infty}^{t+1} \delta(\tau - 2)\, e^{-2(\tau - t)} d\tau$

$$\begin{aligned}q(t) &= \int_{-\infty}^{t+1} \delta(\tau - 2)\, e^{-2(\tau - t)} d\tau \\ &= \int_{-\infty}^{t+1} \delta(\tau - 2)\, e^{-2(2 - t)} d\tau \\ &= e^{-2(2-t)} \int_{-\infty}^{t+1} \delta(\tau - 2)\, d\tau \\ &= e^{-2(2-t)} u(t + 1 - 2) = e^{-4} e^{2t} u(t - 1)\end{aligned}$$

PROBLEM sp-06-Q.3.2:

In each of the following cases, determine the (inverse or forward) Fourier transform. Give your answer as a plot, or a $\boxed{\text{simple}}$ formula (two of the answers will be ***real-valued.***)
Explain each answer (briefly) by stating which property and/or transform pair you used.

(a) Find $X(j\omega)$ when $x(t) = 7\cos(4\pi t - \pi/3)$.

Use the transform pair: $e^{j\omega_c t}$ transforms to $2\pi\delta(\omega - \omega_c)$. First, we expand the cosine via Euler's formula:

$$x(t) = 7\cos(4\pi t - \pi/3) = \frac{7}{2} e^{j(4\pi t - \pi/3)} + \frac{7}{2} e^{-j(4\pi t - \pi/3)}$$

Then we transform the two complex exponentials to get:

$$X(j\omega) = 7\pi\, e^{-j\pi/3}\, \delta(\omega - 4\pi) + 7\pi\, e^{j\pi/3}\, \delta(\omega + 4\pi)$$

(b) Find $s(t)$ when $S(j\omega) = \dfrac{\sin(5\omega)}{\omega} e^{-j2\omega}$.

Multiplication by a complex exponential in the frequency domain corresponds to a time shift. Also, the inverse transform of a "sinc" function is a rectangle. First, we write $S(j\omega)$ in standard form

$$S(j\omega) = \tfrac{1}{2} \frac{\sin(10\omega/2)}{\omega/2} e^{-j2\omega}$$

Thus, the inverse transform is:

$$s(t) = \tfrac{1}{2}\big(u(t+5-2) - u(t-5-2)\big) = \tfrac{1}{2}\big(u(t+3) - u(t-7)\big)$$

(c) Find $h(t)$ when $H(j\omega) = \dfrac{12j\omega}{8 + j2\omega}$.

Use the transform pair: $e^{-at}u(t)$ transforms to $1/(a + j\omega)$, and the property that multiplying the transform by $j\omega$ corresponds to differentiation in the time domain. So, we write $H(j\omega)$ as:

$$H(j\omega) = \frac{12j\omega}{8 + j2\omega} = (j\omega)\frac{6}{4 + j\omega}$$

Then we get the inverse transform:

$$\begin{aligned} h(t) &= \frac{d}{dt}\left\{6\, e^{-4t} u(t)\right\} \\ &= -24\, e^{-4t} u(t) + 6\, e^{-4t}\delta(t) = -24\, e^{-4t} u(t) + 6\,\delta(t) \end{aligned}$$

PROBLEM sp-06-Q.3.3:

Two questions about convolution:

(a) Find $y(t) = \pi u(t-4) * e^{-(t-2)}u(t-2)$. Give the answer as a **simple** formula.

There is a general formula for convolving one-sided exponentials:

$$e^{-at}u(t) * e^{-bt}u(t) = \left(\frac{1}{b-a}\right)\left(e^{-at}u(t) - e^{-bt}u(t)\right)$$

In addition, there are time shifts of both signals. In this case, $a = 0$ and $b = 1$ and the total delay will be be $4 + 2 = 6$. Thus, the answer becomes

$$\begin{aligned} y(t) &= \pi\left(\frac{1}{1-0}\right)\left(u(t-6) - e^{-(t-6)}u(t-6)\right) \\ &= \pi u(t-6) - \pi e^{-(t-6)}u(t-6) \end{aligned}$$

(b) If the signal $r(t)$ is a rectangular pulse, then $r(t) * r(t)$ is a triangle. Suppose that

$$r(t) * r(t) = y(t) = \begin{cases} 9t & \text{for } 0 \le t \le 2 \\ 18 - 9(t-2) & \text{for } 2 \le t \le 4 \\ 0 & \text{elsewhere} \end{cases}$$

Determine the rectangular signal $r(t)$.

First of all, we can exploit a fact about convolution, $y(t) = x(t) * h(t)$, that the duration of $y(t)$ is the sum of the durations of $x(t)$ and $h(t)$. Since the duration of $y(t)$ is 4 secs, and $x(t)$ and $h(t)$ are both equal to $r(t)$, the duration of $r(t)$ will be have to be 2 secs.

Next, we can recognize that if the starting time of $r(t)$ is $t = 0$, then the starting time of $r(t) * r(t)$ is also $t = 0$.

Finally, we need to figure out the amplitude of the rectangular pulse. To find the amplitude of $r(t)$, visualize convolution as "flip and slide." The maximum value of the output is obtained when there is complete overlap of $r(\tau)$ and $r(t-\tau)$. The convolution integral at this time will be:

$$y(2) = \int_0^2 r(\tau)r(2-\tau)\,d\tau = \int_0^2 A^2 d\tau = 2A^2$$

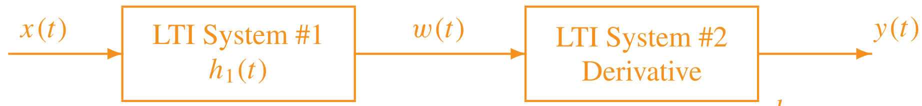
where A is the amplitude of $r(t)$. From the maximum value, $y(2) = 18$, we get

$$2A^2 = 18 \qquad \Rightarrow \qquad A = \sqrt{18/2} = 3$$

Thus, the final answer for the rectangular signal is: $r(t) = 3u(t) - 3u(t-2)$

PROBLEM sp-06-Q.3.4:

A cascade of linear time-invariant systems is depicted by the following block diagram:



$h_1(t) = \sqrt{7}u(t-4) - \sqrt{7}u(t-9)$ $\qquad$ $y(t) = \frac{d}{dt}w(t)$

(a) Determine the overall impulse response for this cascade of two systems. Give your answer in the ***simplest possible form.***

Since the second system takes the derivative and we are given the impulse response of the first system, we merely take the derivative of $h_1(t)$.

$$h(t) = \frac{d}{dt}h_1(t) = \sqrt{7}\delta(t-4) - \sqrt{7}\delta(t-9)$$

Note: the derivative of the unit-step, $u(t)$, is the unit-impulse signal, $\delta(t)$.

(b) The overall frequency response of this system, $H(j\omega)$, is zero for infinitely many values of ω. Derive a general formula that gives **all** the zeros of $H(j\omega)$. ***Explain.***

In a cascade, the overall frequency response is the product of the individual frequency responses

$$H(j\omega) = H_1(j\omega)H_2(j\omega)$$

so the zeros of $H(j\omega)$ are the zeros of $H_1(j\omega)$ plus the zeros of $H_2(j\omega)$. From the Fourier transform tables, we can find $H_1(j\omega)$ and $H_2(j\omega)$ easily.

$$H_1(j\omega) = \sqrt{7}\,\frac{\sin(5\omega/2)}{\omega/2}\,e^{-j6.5\omega} \qquad \text{and} \qquad H_2(j\omega) = j\omega$$

The zeros of $H_1(j\omega)$ are the zeros of a "sinc" function, so we get zeros of $H_1(j\omega)$ at:

$$\omega = \frac{2\pi k}{5}, \quad \text{except for } k = 0.$$

However, $H_2(j\omega)$ has one zero at $\omega = 0$, so the final answer is that the zeros of $H(j\omega)$ are at

$$\omega = \frac{2\pi k}{5}, \quad \text{for } k = 0, \pm 1, \pm 2, \pm 3, \ldots$$