

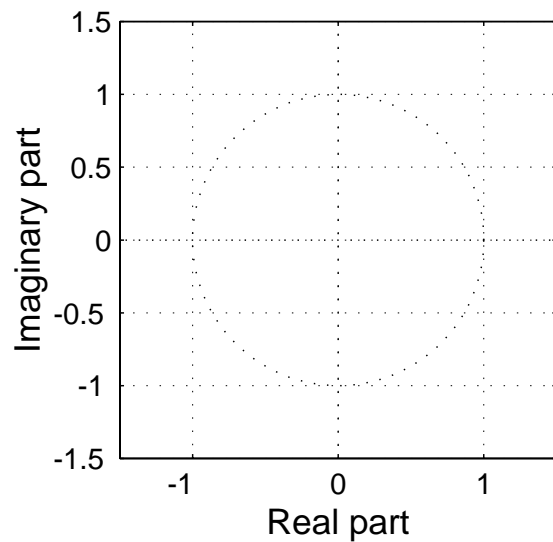
Problem FALL-01-Q.3.1:

A discrete-time system (FIR filter) is defined by the following z -transform system function:

$$H(z) = (1 + 0.75z^{-1})(1 - e^{j\pi/2}z^{-1})(1 - e^{-j\pi/2}z^{-1})$$

- (a) Write down the difference equation that is satisfied by the input $x[n]$ and output $y[n]$ of the system. Give the numerical values of all filter coefficients.

- (b) Determine *all* the zeros of $H(z)$ and plot them in the z -plane.



- (c) If the input is of the form $x[n] = A \sin(\omega_0 n + \phi)$, for what value of frequency ω_0 (in the range $0 < \omega_0 < \pi$) will the filter completely remove the sinusoidal component? **EXPLAIN your answer.**

Problem FALL-01-Q.3.2:

In each of the following cases, simplify the expression using the properties of the continuous-time unit impulse signal. Provide some explanation or intermediate steps for each answer.

$$(a) \int_{-\infty}^{t-2} \delta(\tau - 1) e^{-\tau} d\tau =$$

$$(b) \frac{d}{dt} \{ \cos(3t) u(t - 2) \} =$$

Problem FALL-01-Q.3.3:

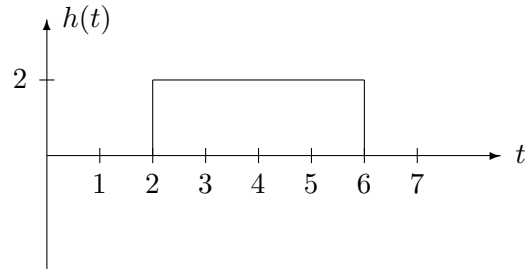
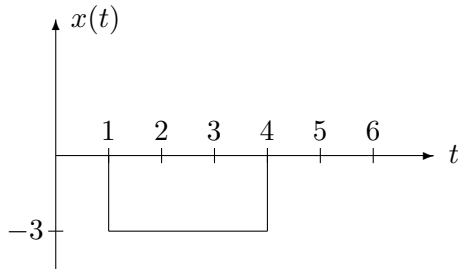
In each of the following cases, determine the Fourier transform, or inverse Fourier transform. Give your answer as a **simple** formula or a plot. **Explain** each answer by stating which property and transform pair you used.

(a) Find $X(j\omega)$ when $x(t) = \sin(t - 3)$.

(b) Find $s(t)$ when $S(j\omega) = \frac{\sin(25\omega/2)}{20\omega} e^{-j25\omega}$.

Problem FALL-01-Q.3.4:

The following figure shows the signal $x(t) = -3u(t-1)+3u(t-4)$, which is the input to a continuous-time LTI system whose impulse response (shown on the right) is $h(t) = 2u(t-2)-2u(t-6)$.



(a) Sketch $h(4 - \tau)$ as a function of τ in the space below.

(b) Determine the value of the output of the LTI system, $y(t)$, at $t = 4$; that is, determine $y(4)$. It is not necessary to evaluate $y(t)$ for all t , only for $t = 4$. Note: This problem may be answered without performing any integration.

(c) $y(t)$ reaches its minimum value for $T_1 \leq t \leq T_2$. Find the minimum value, y_{min} and also the values for T_1 and T_2 .

| |
|-------------------|
| $y_{min} =$ _____ |
|-------------------|

| |
|-------------------|
| $T_1 =$ _____ sec |
|-------------------|

| |
|-------------------|
| $T_2 =$ _____ sec |
|-------------------|

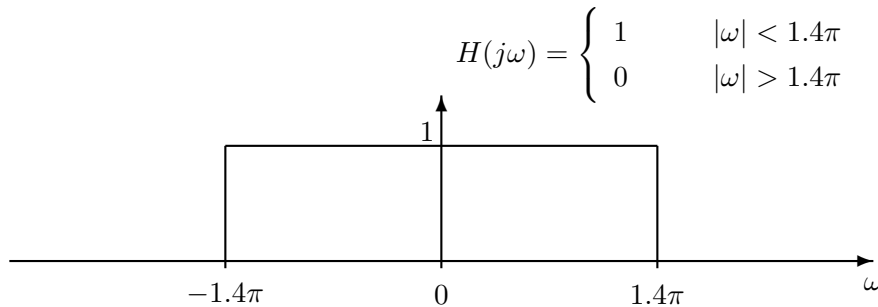
Problem FALL-01-Q.3.5:

The input to the LTI system shown below is a periodic signal $x(t)$ that has a period $T_0 = 2$ seconds. The Fourier series representation for the input $x(t)$ is

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \quad \text{where} \quad a_k = \begin{cases} 2 & k = 0 \\ \frac{3 \sin(\pi k/2)}{2\pi k} & k \neq 0. \end{cases}$$

(a) What is the fundamental frequency ω_0 of the input signal $x(t)$? $\omega_0 = \underline{\hspace{2cm}}$ rad/sec

(b) Suppose that the frequency response of the system is an ideal lowpass filter as illustrated below.



Give an equation for the output of the system, $y(t)$, that is valid for $-\infty < t < \infty$. (Your answer should be expressed in terms of only real quantities). **Justify your answer** by sketching the spectrum of $y(t)$ (or its Fourier transform).

(c) Draw the spectrum of the output signal superimposed on the plot of $H(j\omega)$.