



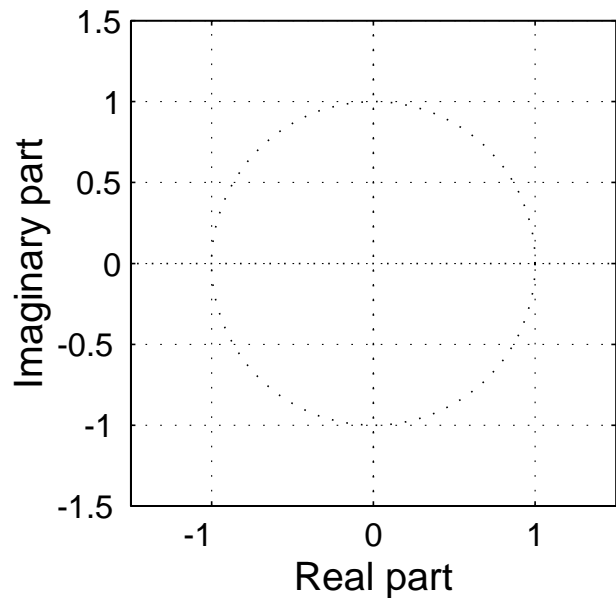
**PROBLEM FALL-04-Q.3.1:**

A discrete-time system (FIR filter) is defined by the following  $z$ -transform system function:

$$H(z) = (1 - 0.7z^{-1})(1 - e^{j\pi/2}z^{-1})(1 - e^{-j\pi/2}z^{-1})$$

- (a) Write down the difference equation that is satisfied by the input  $x[n]$  and output  $y[n]$  of the system. Give the numerical values of all filter coefficients.

- (b) Determine *all* the zeros of  $H(z)$  and plot them in the  $z$ -plane.



- (c) If the input is of the form  $x[n] = s[n] + A \cos(\omega_0 n + \phi)$ , where  $s[n]$  is a speech signal, for what value of frequency  $\omega_0$  (in the range  $0 < \omega_0 < \pi$ ) will the filter completely remove the sinusoidal component? **EXPLAIN your answer.**

**PROBLEM FALL-04-Q.3.2:**

For each of the following expressions, select the correct match from the second list below.  
(The operator  $*$  denotes convolution.)

(a)   $u(t-1) * u(t-3)$

(b)   $e^{-t}u(t) * \delta(t-4)$

(c)   $\int_{-\infty}^0 \delta(t-4) dt$

(d)   $u(4)$

(e)   $\frac{d}{dt} \{e^{-t}u(t-4)\}$

(f)   $e^{-t}u(t)\delta(t-4)$

(g)   $\delta(t-1) * \delta(t-3)$

(h)   $e^{-t}u(t) * u(t-4)$

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Each of the expressions above is equivalent to one (and only one) of the expressions below:

[1]  $u(t-4)$

[2]  $-e^{-t}u(t-4) + e^{-4}\delta(t-4)$

[3]  $(t-4)u(t-4)$

[4]  $(1 - e^{-t+4})u(t-4)$

[5]  $e^{-(t-4)}u(t-4)$

[6]  $e^{-4}\delta(t-4)$

[7]  $0$

[8]  $\delta(t-4)$

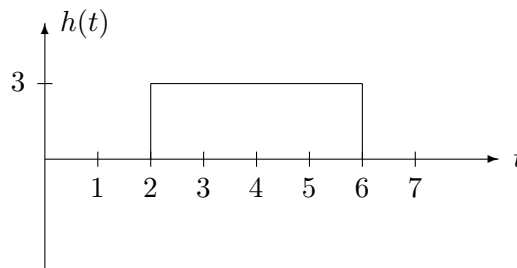
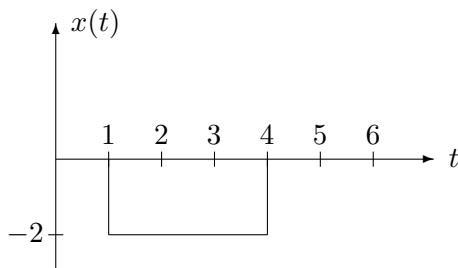
[9]  $1$

[10]  $e^{-4}$

[11]  $-e^{-t}u(t-4)$

**PROBLEM FALL-04-Q.3.3:**

The following figure shows the signal  $x(t) = -2u(t-1) + 2u(t-4)$ , which is the input to a continuous-time LTI system whose impulse response (shown on the right) is  $h(t) = 3u(t-2) - 3u(t-6)$ .



(a) Sketch  $h(7 - \tau)$  as a function of  $\tau$  in the space below.

(b) Determine the value of the output of the LTI system,  $y(t)$ , at  $t = 7$ ; that is, determine  $y(7)$ . It is not necessary to evaluate  $y(t)$  for all  $t$ , only for  $t = 7$ . Note: This problem may be answered without performing any integration.

(c)  $y(t)$  reaches its minimum value for  $T_1 \leq t \leq T_2$ . Find the minimum value,  $y_{min}$  and also the values for  $T_1$  and  $T_2$ .

$$y_{min} = \underline{\hspace{2cm}}$$

$$T_1 = \underline{\hspace{2cm}} \text{ sec}$$

$$T_2 = \underline{\hspace{2cm}} \text{ sec}$$

**PROBLEM FALL-04-Q.3.4:**

For each of the following time-domain signals, select the correct match from the list of Fourier transforms below. **Write each answer in the box provided.** (The operator  $*$  denotes convolution.)

(a)  $x(t) = u(t - 2) - u(t - 4)$

(b)  $x(t) = \Im\{\delta(t - 4) * e^{j\pi t}\}$

(c)  $x(t) = -\frac{4}{3}e^{-2t/3}u(t) + 2\delta(t)$

(d)  $x(t) = \frac{4}{3}e^{(-2+j3)t}u(t)$

(e)  $x(t) = \delta(t - 3) \sin(\pi t)$

Each of the time signals above has a Fourier transform that can be found in the list below.

[0]  $X(j\omega) = \frac{1}{2\pi}e^{-j3\omega} * [j\pi u(\omega + \pi) - j\pi u(\omega - \pi)]$

[1]  $X(j\omega) = 2e^{-j3\omega} \frac{\sin(3\omega)}{\omega}$

[2]  $X(j\omega) = j2e^{-j3\omega} \sin(3\omega)$

[3]  $X(j\omega) = \frac{j6\omega}{2 + j3\omega}$

[4]  $X(j\omega) = \frac{\sin(\omega)}{\omega/2}$

[5]  $X(j\omega) = \frac{4/3}{2 + j(\omega - 3)}$

[6]  $X(j\omega) = 0$

[7]  $X(j\omega) = 2e^{-j3\omega} \frac{\sin(\omega)}{\omega}$

[8]  $X(j\omega) = \frac{-9}{2 + j3\omega}$

[9]  $X(j\omega) = j\pi\delta(\omega + \pi) - j\pi\delta(\omega - \pi)$