

PROBLEM SPR-04-Q.3.1:

For each of the following expressions, reduce the expression to the simplest possible form:
(The operator $*$ denotes convolution.)

$$(a) \frac{d}{dt} \{e^{-3t} u(t+3)\}$$

$$(b) \{e^{-4(t-1)} u(t-1)\} \delta(t+3)$$

$$(c) \delta(t-1) * \delta(t+3)$$

$$(d) \int_{-\infty}^0 3\delta(t+3) dt$$

$$(e) \{e^{-4(t-1)} u(t-1)\} * \delta(t+3)$$

$$(f) \left. \frac{\sin(3\omega)}{\omega/2} \right|_{\omega=0}$$

PROBLEM SPR-04-Q.3.2:

For each of the following time-domain signals, select the correct match from the list of Fourier transforms below. *Write each answer in the box provided.* (The operator $*$ denotes convolution.)

(a) $x(t) = u(t - 3) - u(t - 5)$

(b) $x(t) = \Im\{\delta(t - 4) * e^{j\pi t}\}$

(c) $x(t) = -3e^{-3t/4}u(t) + 4\delta(t)$

(d) $x(t) = 4e^{(-3+j4)t}u(t)$

(e) $x(t) = \delta(t - 4) \sin(\pi t)$

Each of the time signals above has a Fourier transform that can be found in the list below.

[0] $X(j\omega) = \frac{1}{2\pi}e^{-j4\omega} * [j\pi u(\omega + \pi) - j\pi u(\omega - \pi)]$

[1] $X(j\omega) = 2e^{-j4\omega} \frac{\sin(4\omega)}{\omega}$

[2] $X(j\omega) = j2e^{-j4\omega} \sin(4\omega)$

[3] $X(j\omega) = \frac{j16\omega}{3 + j4\omega}$

[4] $X(j\omega) = \frac{\sin(\omega)}{\omega/2}$

[5] $X(j\omega) = \frac{4}{3 + j(\omega - 4)}$

[6] $X(j\omega) = 0$

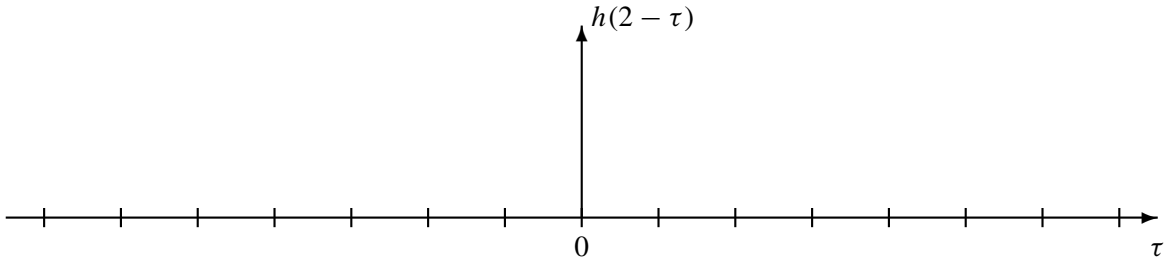
[7] $X(j\omega) = 2e^{-j4\omega} \frac{\sin(\omega)}{\omega}$

[8] $X(j\omega) = \frac{-12}{3 + j4\omega}$

[9] $X(j\omega) = e^{-j4\omega} [j\pi\delta(\omega + \pi) - j\pi\delta(\omega - \pi)]$

PROBLEM SPR-04-Q.3.3:

- (a) Assume that $h(t) = u(t + 2) - u(t - 3)$. Plot $h(2 - \tau)$ as a function of τ .



- (b) When two finite-duration signals are convolved, the result is a finite-duration signal, $y(t) = x(t) * h(t)$. Suppose that $h(t)$ is the signal defined in part (a), and that the input signal is:

$$x(t) = e^{t-2} \{u(t - 2) - u(t - 9)\}$$

Determine the duration (in secs.) of the output signal $y(t) = x(t) * h(t)$.

Duration =

- (c) If the input is changed to $x(t) = 4u(t - 5)$, and $h(t)$ is still defined as in part (a), then it will be true that the output $y(t) = h(t) * x(t)$ from the convolution can be written as

$$y(t) = B(t - T_{12}) \{u(t - T_{12}) - u(t - T_{23})\} + Cu(t - T_{23})$$

where B and C and the times T_{12} and T_{23} are constants. Determine the values of these four parameters.

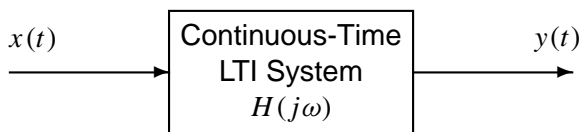
$B =$

$C =$

$T_{12} =$

$T_{23} =$

PROBLEM SPR-04-Q.3.4:

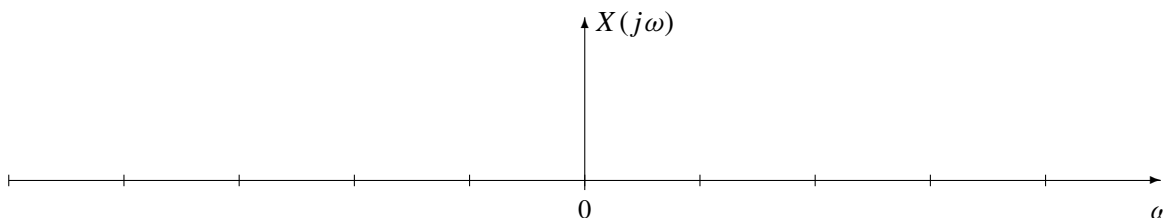


The periodic input to the above system is defined by the equation:

$$x(t) = \sum_{k=-2}^2 a_k e^{j12kt}, \quad \text{where } a_k = \begin{cases} \frac{1}{j\pi k} & k \neq 0 \\ 0.4 & k = 0 \end{cases}$$

- (a) Determine the Fourier transform of the periodic signal $x(t)$. Give a formula and then plot it on the graph below. Label your plot with numerical values to receive full credit.

$X(j\omega) =$



- (b) The frequency response of the LTI system is given by the following equation:

$$H(j\omega) = \frac{26}{5 + j\omega}$$

Evaluate the frequency response at $\omega = 12$, giving your answer in polar form (with numerical values):

at $\omega = 12$, $|H(j\omega)| =$

at $\omega = 12$, $\angle H(j\omega) =$

- (c) For $x(t)$ given above, the output signal can be written as $y(t) = \sum_{k=-2}^2 b_k e^{jk\omega_0 t}$

Determine the numerical values of the parameters ω_0 , b_0 and b_1 .

$\omega_0 =$

$b_0 =$

$b_1 =$