



**PROBLEM s-04-Q.3.1:**

For each of the following expressions, reduce the expression to the simplest possible form:  
(The operator  $*$  denotes convolution.)

(a)  $\{e^{-4(t-1)}u(t-1)\} * \delta(t+5)$

(b)  $\int_{-\infty}^0 5\delta(t+5)dt$

(c)  $\left. \frac{\sin(5\omega)}{\omega/2} \right|_{\omega=0}$

(d)  $\frac{d}{dt} \{e^{-5t}u(t+5)\}$

(e)  $\{e^{-4(t-1)}u(t-1)\} \delta(t+5)$

(f)  $\delta(t-1) * \delta(t+5)$

**PROBLEM s-04-Q.3.2:**

For each of the following time-domain signals, select the correct match from the list of Fourier transforms below. **Write each answer in the box provided.** (The operator  $*$  denotes convolution.)

(a)  $x(t) = 4e^{(-3+j4)t}u(t)$

(b)  $x(t) = u(t - 3) - u(t - 5)$

(c)  $x(t) = -3e^{-3t/4}u(t) + 4\delta(t)$

(d)  $x(t) = \delta(t - 4) \sin(\pi t)$

(e)  $x(t) = \Im\{\delta(t - 4) * e^{j\pi t}\}$

Each of the time signals above has a Fourier transform that can be found in the list below.

[0]  $X(j\omega) = \frac{-12}{3 + j4\omega}$

[1]  $X(j\omega) = e^{-j4\omega} [j\pi\delta(\omega + \pi) - j\pi\delta(\omega - \pi)]$

[2]  $X(j\omega) = 2e^{-j4\omega} \frac{\sin(4\omega)}{\omega}$

[3]  $X(j\omega) = 2e^{-j4\omega} \frac{\sin(\omega)}{\omega}$

[4]  $X(j\omega) = \frac{1}{2\pi} e^{-j4\omega} * [j\pi u(\omega + \pi) - j\pi u(\omega - \pi)]$

[5]  $X(j\omega) = \frac{j16\omega}{3 + j4\omega}$

[6]  $X(j\omega) = 0$

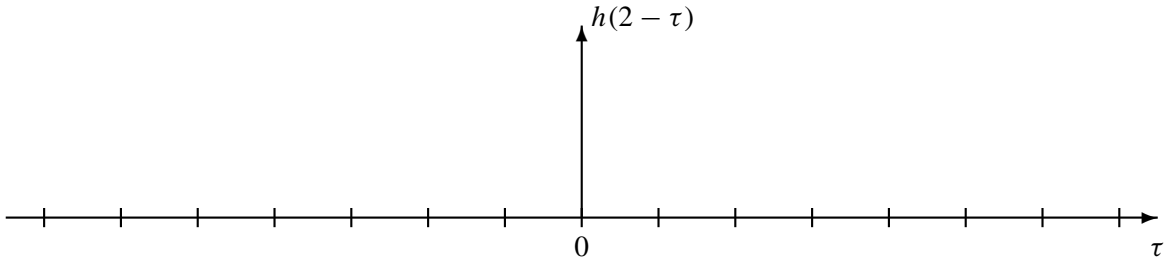
[7]  $X(j\omega) = \frac{4}{3 + j(\omega - 4)}$

[8]  $X(j\omega) = \frac{\sin(\omega)}{\omega/2}$

[9]  $X(j\omega) = j2e^{-j4\omega} \sin(4\omega)$

**PROBLEM s-04-Q.3.3:**

- (a) Assume that  $h(t) = u(t + 2) - u(t - 1)$ . Plot  $h(2 - \tau)$  as a function of  $\tau$ .



- (b) When two finite-duration signals are convolved, the result is a finite-duration signal,  $y(t) = x(t) * h(t)$ . Suppose that  $h(t)$  is the signal defined in part (a), and that the input signal is:

$$x(t) = e^{t-2} \{u(t - 2) - u(t - 9)\}$$

Determine the duration (in secs.) of the output signal  $y(t) = x(t) * h(t)$ .

Duration =

- (c) If the input is changed to  $x(t) = 2u(t - 1)$ , and  $h(t)$  is still defined as in part (a), then it will be true that the output  $y(t) = h(t) * x(t)$  from the convolution can be written as

$$y(t) = B(t - T_{12}) \{u(t - T_{12}) - u(t - T_{23})\} + Cu(t - T_{23})$$

where  $B$  and  $C$  and the times  $T_{12}$  and  $T_{23}$  are constants. Determine the values of these four parameters.

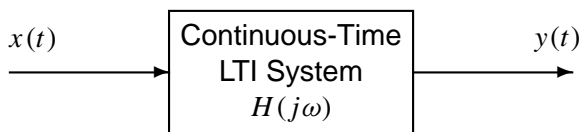
$B =$

$C =$

$T_{12} =$

$T_{23} =$

**PROBLEM s-04-Q.3.4:**

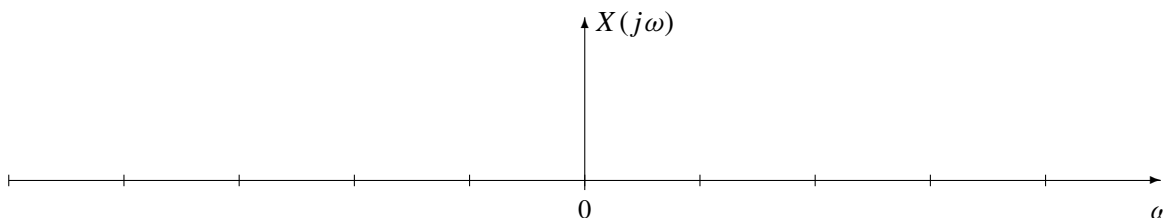


The periodic input to the above system is defined by the equation:

$$x(t) = \sum_{k=-2}^2 a_k e^{j10kt}, \quad \text{where } a_k = \begin{cases} \frac{1/\pi}{1-2k^2} & k \neq 0 \\ 0.2 & k = 0 \end{cases}$$

- (a) Determine the Fourier transform of the periodic signal  $x(t)$ . Give a formula and then plot it on the graph below. Label your plot with numerical values to receive full credit.

$X(j\omega) =$



- (b) The frequency response of the LTI system is given by the following equation:

$$H(j\omega) = \frac{32}{17.321 + j\omega}$$

Evaluate the frequency response at  $\omega = 10$ , giving your answer in polar form (with numerical values):

at  $\omega = 10$ ,  $|H(j\omega)| =$

at  $\omega = 10$ ,  $\angle H(j\omega) =$

- (c) For  $x(t)$  given above, the output signal can be written as  $y(t) = \sum_{k=-2}^2 b_k e^{jk\omega_0 t}$

Determine the numerical values of the parameters  $\omega_0$ ,  $b_0$  and  $b_1$ .

$\omega_0 =$

$b_0 =$

$b_1 =$