Signal Processing First

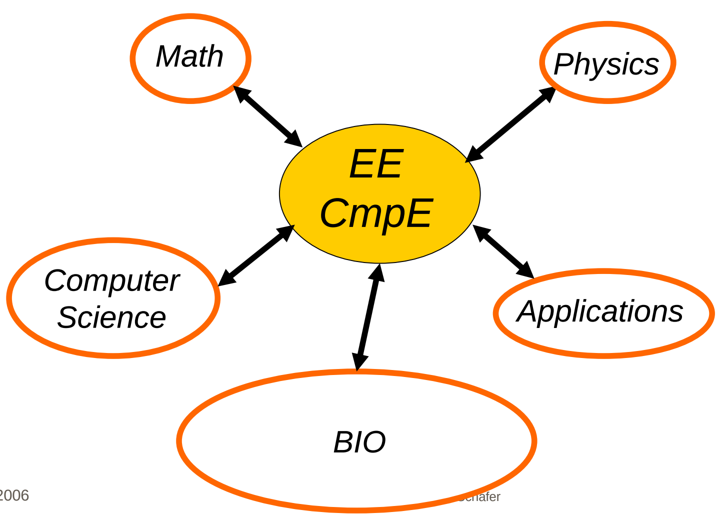
LECTURE #1

Sinusoids

READING ASSIGNMENTS

- This Lecture:
  - Chapter 2, pp. 9-17
- Appendix A: Complex Numbers
- Appendix B: MATLAB
- Chapter 1: Introduction

CONVERGING FIELDS



COURSE OBJECTIVE

- Students will be able to:
- Understand **mathematical** descriptions of signal processing **algorithms** and express those algorithms as computer **implementations** (MATLAB)
- What are your objectives?

WHY USE DSP ?

- Mathematical **abstractions** lead to generalization and discovery of new processing techniques
- Computer implementations are **flexible**
- Applications provide a **physical** context

Fourier Everywhere

- Telecommunications
- **Sound & Music**
  - CDROM, Digital Video
- Fourier Optics
- X-ray Crystallography
  - Protein Structure & DNA
- Computerized Tomography
- Nuclear Magnetic Resonance: MRI
- Radioastronomy
- Ref: Prestini, "The Evolution of Applied Harmonic Analysis"

LECTURE OBJECTIVES

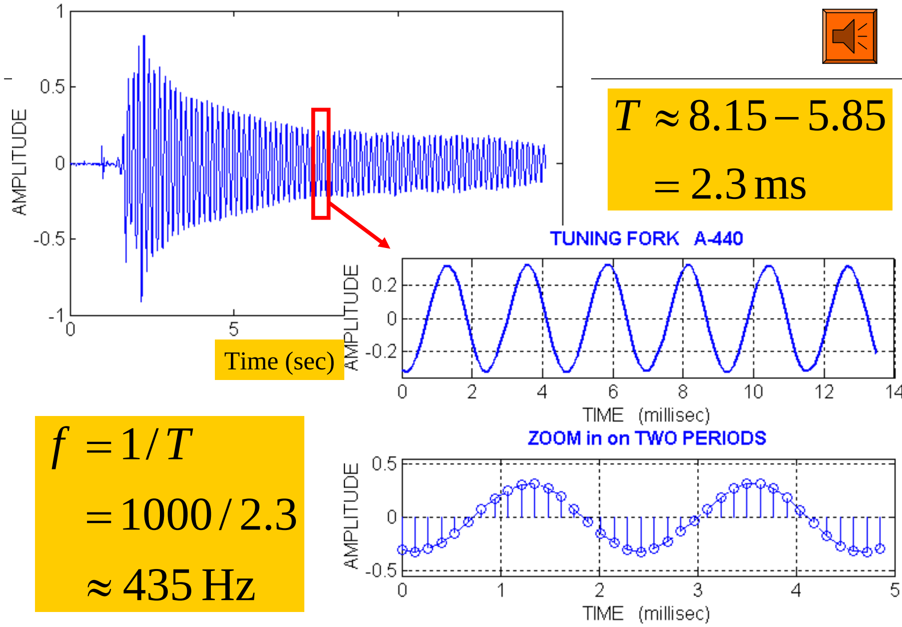
- Write general formula for a "sinusoidal" waveform, or signal
- From the formula, plot the sinusoid versus time
- What's a **signal**?
  - It's a **function** of time, x(t)
  - in the mathematical sense

TUNING FORK EXAMPLE

- CD-ROM demo
- "A" is at 440 Hertz (Hz)
- Waveform is a SINUSOIDAL SIGNAL
- Computer plot looks like a sine wave
- This should be the mathematical formula:

$$A\cos(2\pi(440)t + \varphi)$$

TUNING FORK A-440 Waveform



$$T \approx 8.15 - 5.85 = 2.3\,\text{ms}$$

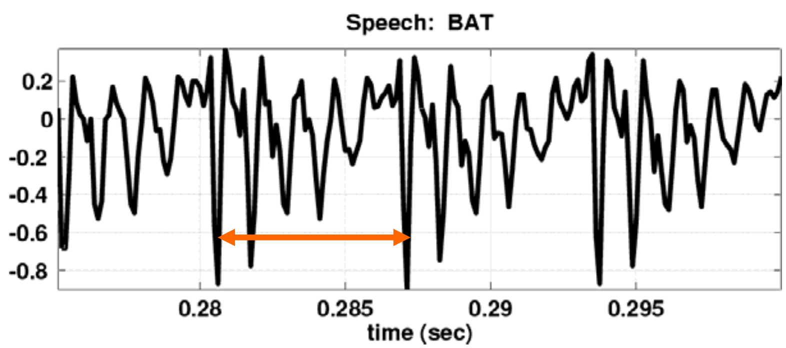
$$f = 1/T = 1000/2.3 \approx 435\,\text{Hz}$$

SPEECH EXAMPLE

- More complicated signal (BAT.WAV)
- Waveform ***x*(*t*)** is NOT a Sinusoid
- Theory will tell us
  - ***x*(*t*)** is approximately a sum of sinusoids
  - FOURIER ANALYSIS
    - Break ***x*(*t*)** into its sinusoidal components
  - Called the FREQUENCY SPECTRUM

Speech Signal: BAT

- Nearly **Periodic** in Vowel Region
  - Period is (Approximately) T = 0.0065 sec



DIGITIZE the WAVEFORM

- ***x*[*n*]** is a SAMPLED SINUSOID
  - A list of numbers stored in memory
- Sample at 11,025 samples per second
  - Called the SAMPLING RATE of the A/D
  - Time between samples is
    - 1/11025 = 90.7 microsec
- Output via D/A hardware (at F_{samp})

STORING DIGITAL SOUND

- ***x*[*n*]** is a SAMPLED SINUSOID
  - A list of numbers stored in memory
- CD rate is 44,100 samples per second
- 16-bit samples
- Stereo uses 2 channels
- Number of bytes for 1 minute is
  - 2 X (16/8) X 60 X 44100 = 10.584 Mbytes

SINES and COSINES

- Always use the COSINE FORM

$$A\cos(2\pi(440)t+\varphi)$$

- Sine is a special case:

$$\sin(\omega t)=\cos(\omega t-\tfrac{\pi}{2})$$

SINUSOIDAL SIGNAL

$$A\cos(\omega t+\varphi)$$

- **FREQUENCY** ω
  - Radians/sec
  - Hertz (cycles/sec)

$$\omega=(2\pi)f$$

- **PERIOD** (in sec)

$$T=\frac{1}{f}=\frac{2\pi}{\omega}$$

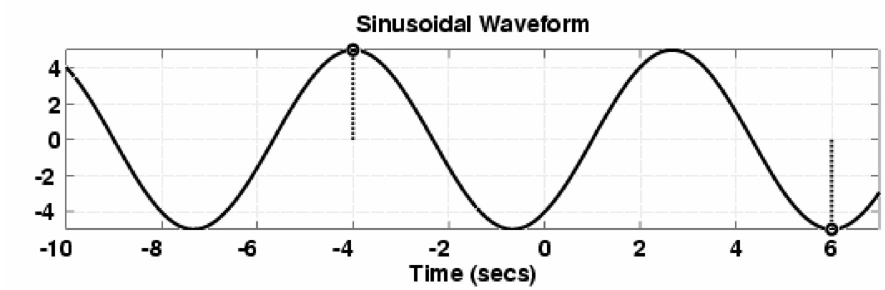
- **AMPLITUDE** A
  - Magnitude
- **PHASE** φ

EXAMPLE of SINUSOID

- Given the Formula

$$5\cos(0.3\pi t+1.2\pi)$$

- Make a plot



PLOT COSINE SIGNAL

$$5\cos(0.3\pi t + 1.2\pi)$$

- Formula defines A, ω, and ϕ

$$A = 5$$
$$\omega = 0.3\pi$$
$$\varphi = 1.2\pi$$

PLOTTING COSINE SIGNAL from the FORMULA

$$5\cos(0.3\pi t + 1.2\pi)$$

- Determine **period**:

$$T = 2\pi / \omega = 2\pi / 0.3\pi = 20/3$$

- Determine a **peak** location by solving

$$(\omega t + \varphi) = 0 \quad \Rightarrow (0.3\pi t + 1.2\pi) = 0$$

- **Zero** crossing is T/4 before or after
- Positive & Negative peaks spaced by T/2

PLOT the SINUSOID

$$5\cos(0.3\pi t + 1.2\pi)$$

- Use T=20/3 and the peak location at t=-4