ECE-2025 Fall-09

LECTURE #2
Phase & Time-Shift
Complex Exponentials
21-Aug-09

INFORMATION

- MATLAB issues: ask your TA for help
- LABS start NEXT week **(Monday)**
  - Attend correct section (in Klaus-2440)
  - Login for ECE Lab computers:
    - Georgia Tech password
    - Select your "Windows Domain" to be **AD**
- RECITATIONS
  - Bring Calculator to Recitation next week
    - Practice Complex arithmetic
  - Attend your assigned time
  - **ACTIVE** participation
  - **5 POINTS**

T-SQUARE Info

- Check Chat Room & Announcements for msgs
  - **MAKE YOUR OWN POSTINGS**
- PDF Files on T-square
  - Lectures are being posted (4 per page)
  - Lab #1 has been posted
    - Get PDF file of Lab #1 from t-square
    - Hard copy of Instructor Verification Sheet
  - HW #1 was posted as PDF
    - HW #1 due next week
    - HW #2 should be posted also

Homework Info

- HWs will be posted on Friday/Sat
  - Covered in Rec during the following Week
  - Due the week after that (9+ days later)
- Format info on T-square
  - Cover page for Homework
- Prob Set #1 due **in RECITATION next week**
  - **At the beginning of class**
  - Solutions will be posted to T-square
    - after the last Recitation on Thursday afternoon

Homework Formatting

- Include a Cover page with
  - Name
  - Lab section, ie, L05, L20, etc.
  - Recitation Prof's name
  - **Download example from t-square**
- Write on **ONE** side only
  - Use Engineer's paper or plain white paper
  - **STAPLE**
- **TWO parts to every solution**
  - **Description of your approach to the solution**
  - **Actual solution**

Becoming an Expert

- 10,000 hours
- Factor of 3 (class+outside)
  - **3** x 4 x 15

READING ASSIGNMENTS

- This Lecture:
  - Chapter 2, Sects. 2-3 to 2-5
- Appendix A: Complex Numbers
- Appendix B: MATLAB
- Next Lecture: finish Chap. 2,
  - Section 2-6 to end

LECTURE OBJECTIVES

- Define Sinusoid Formula from a plot
- Relate TIME-SHIFT to PHASE

*Introduce an **ABSTRACTION**:*
*Complex Numbers **represent** Sinusoids*
Complex Exponential Signal

$$z(t) = Xe^{j\omega t}$$

SINUSOIDAL SIGNAL

$$A\cos(\omega t + \varphi)$$

- **FREQUENCY** ω
  - **Radians/sec**
  - or, Hertz (cycles/sec)

$$\omega = (2\pi) f$$

- **PERIOD** (in sec)

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

- **AMPLITUDE** A
  - Magnitude
- **PHASE** φ

PLOTTING COSINE SIGNAL from the FORMULA

$$5\cos(0.3\pi t + 1.2\pi)$$

- Determine **period**:

$$T = 2\pi / \omega = 2\pi / 0.3\pi = 20/3$$

- Determine a **peak** location by solving

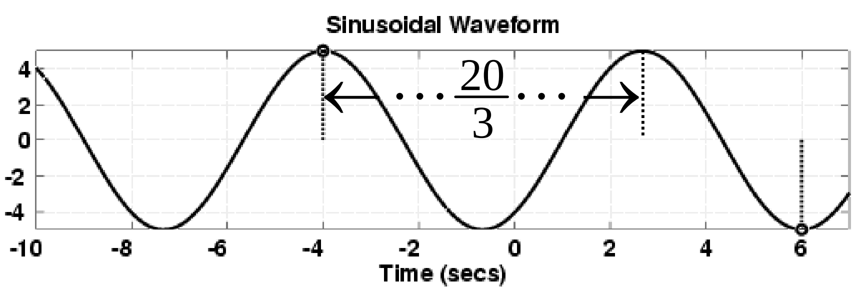
$$(\omega t + \varphi) = 0$$

- **Peak at t=-4**

ANSWER for the PLOT

$$5\cos(0.3\pi t + 1.2\pi)$$

- Use T=20/3 and the peak location at t=-4



TIME-SHIFT

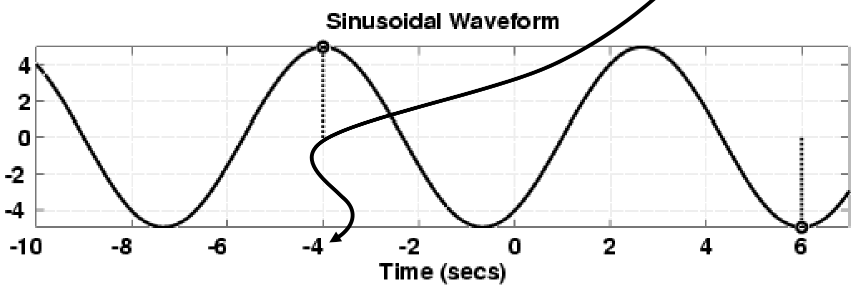
- In a formula we can replace t with $t-t_m$

$$x(t - t_m) = A\cos(\omega(t - t_m))$$

- Then the t=0 point moves to $t=t_m$
- Peak value of $\cos(\omega(t-t_m))$ is now at $t=t_m$

TIME-SHIFTED SINUSOID

$$x(t+4) = 5\cos(0.3\pi(t+4)) = 5\cos(0.3\pi(t-(-4)))$$



PHASE ←→ TIME-SHIFT

- Equate the formulas:

$$A\cos(\omega(t-t_m)) = A\cos(\omega t+\varphi)$$

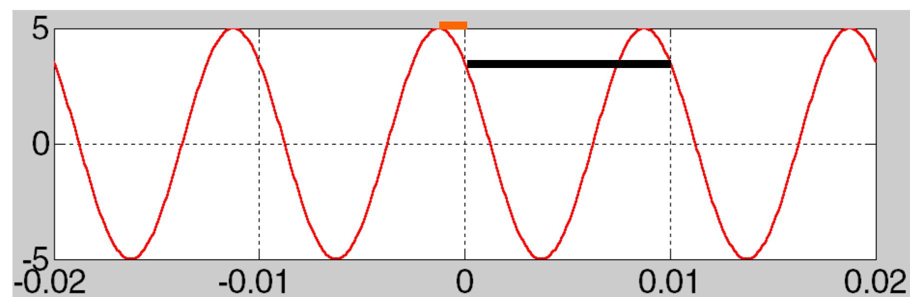
- and we obtain: $-\omega t_m = \varphi$
- or, $t_m = -\dfrac{\varphi}{\omega}$

SINUSOID from a PLOT

- Measure the period, T
  - Between peaks or zero crossings
  - Compute frequency: $\omega = 2\pi/T$
- Measure time of a peak: t_m
  - Compute phase: $\phi = -\omega\, t_m$
- Measure height of positive peak: A

3 steps

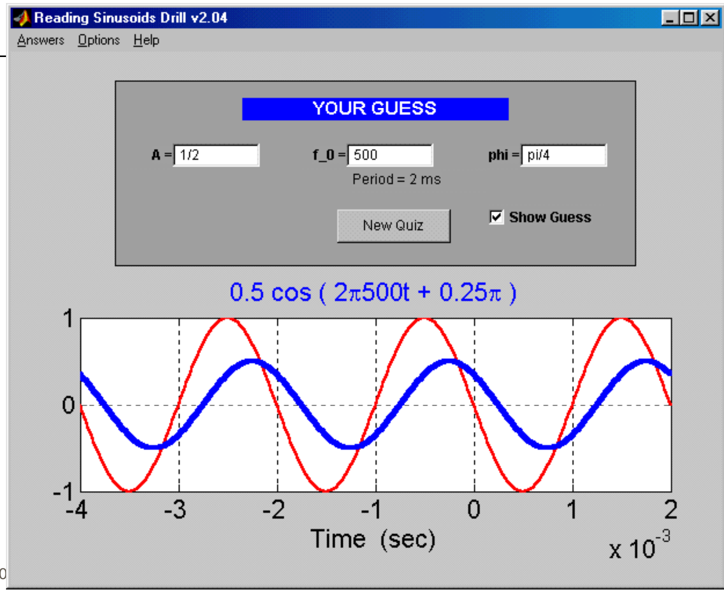
(A, ω, ϕ) from a PLOT



$T = \frac{0.01\,\text{sec}}{1\,\text{period}} = \frac{1}{100}$ → $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.01} = 200\pi$

$t_m = -0.00125\,\text{sec}$ → $\varphi = -\omega t_m = -(200\pi)(t_m) = 0.25\pi$

SINE DRILL (MATLAB GUI)



PHASE is AMBIGUOUS

- The cosine signal is periodic
  - Period is 2π

$$A\cos(\omega t + \varphi + 2\pi) = A\cos(\omega t + \varphi)$$

  - **Thus adding any multiple of 2π to the phase leaves x(t) unchanged**
  - Equivalent to time-shifting by one period:

$$A\cos(\omega t + \varphi + 2\pi) =$$
$$A\cos(\omega(t + 2\pi/\omega) + \varphi) = A\cos(\omega(t + T) + \varphi)$$

Complex Numbers

- WHY? need to ADD sinusoids

$$s(t) = \sum_{k=1}^{20} \sqrt{k}\cos(120\pi(t - 0.002k))$$

- Use an ABSTRACTION
  - Complex Amplitude, ***X***, has mag & phase
  - Complex Exponential
  - **Euler's Formula**

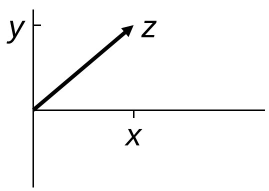
$$z(t) = Xe^{j\omega t}$$

WHY? What do we gain?

- Sinusoids will be critically important to us,
  - but trig identities will be too cumbersome
- Abstraction of complex numbers
  - lets us be efficient by replacing trig with **algebra**
- **Avoid all Trigonometric manipulations**

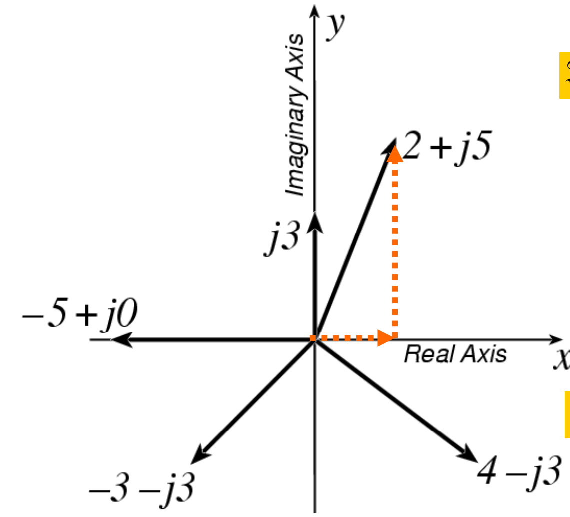
COMPLEX NUMBERS

- To solve: $z^2 = -1$
  - $z = j$
  - Math and Physics use $z = i$
- Complex number: $z = x + j\,y$



Cartesian coordinate system

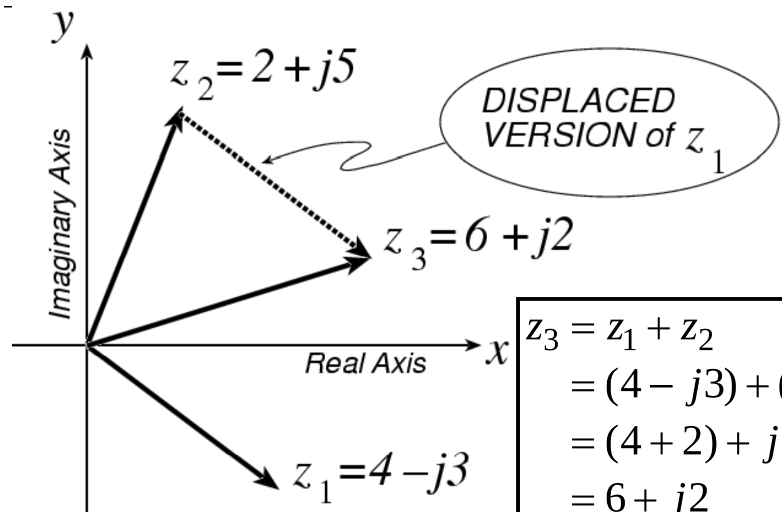
PLOT COMPLEX NUMBERS



$\Im\{2+j5\}=5$

$\Re\{4-j3\}=4$

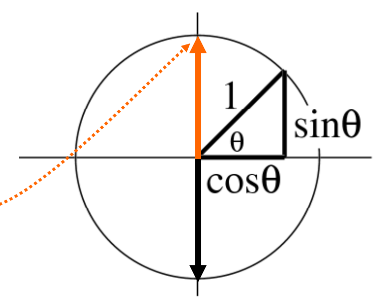
COMPLEX ADDITION = VECTOR Addition



$$\begin{aligned} z_3 &= z_1 + z_2 \\ &= (4-j3)+(2+j5) \\ &= (4+2)+j(-3+5) \\ &= 6+j2 \end{aligned}$$

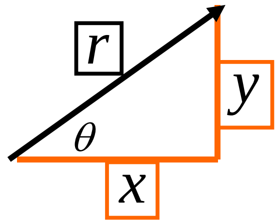
*** POLAR FORM ***

- **Vector** Form
  - **Length** =1
  - **Angle** = θ
- Common Values
  - **j** has angle of 0.5π
  - -1 has angle of π
  - **– j** has angle of 1.5π
  - also, angle of **–j could** be $-0.5\pi = 1.5\pi - 2\pi$
  - because the PHASE is **AMBIGUOUS**



POLAR <--> RECTANGULAR

- Relate (x,y) to (r,θ)



$$r^2 = x^2 + y^2$$

$$\theta = \text{Tan}^{-1}\left(\frac{y}{x}\right)$$

Most calculators do Polar-Rectangular

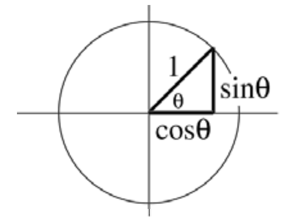
$$x = r\cos\theta$$

$$y = r\sin\theta$$

Need a notation for POLAR FORM

Euler's FORMULA

- **Complex Exponential**
  - **Real part is cosine**
  - Imaginary part is sine
  - Magnitude is one

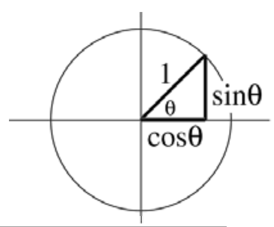


$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$re^{j\theta} = r\cos(\theta) + jr\sin(\theta)$$

Cosine = Real Part

- Complex Exponential
  - **Real part is cosine**
  - Imaginary part is sine



$$re^{j\theta} = r\cos(\theta) + jr\sin(\theta)$$

$$\Re\{re^{j\theta}\} = r\cos(\theta)$$

AVOID Trigonometry

- Algebra, even complex, is EASIER !!!
- Can you recall $\cos(\theta_1+\theta_2)$?
- Use: real part of $e^{j(\theta_1+\theta_2)} = \cos(\theta_1+\theta_2)$

$$\cos(\theta_1+\theta_2) = \Re\{e^{j(\theta_1+\theta_2)}\} = \Re\{e^{j\theta_1}e^{j\theta_2}\}$$

$$= \Re\{(\cos\theta_1 + j\sin\theta_1)(\cos\theta_2 + j\sin\theta_2)\}$$

$$= (\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2) + j(\ldots)$$