ECE-2025 Fall-2009

LECTURE #3
Phasor Addition Theorem
24-Aug-09

Info

- **Bring Calculator to Recitation**
  - **Practice Complex arithmetic**
    - **Add two sinusoids in 14 seconds**
- Help Sessions (during the semester)
  - MATLAB++ help: 6pm M,T,W in K-2440
- Labs: ECE Login
  - Use the Windows domain called “AD”

T-square Info

- Check **Chat Room** & Announcements daily
- SP First URL quick link on T-square
  `Username = gt, password = student`
- Lectures are being posted
  - PDF format (4 per page)
- Get PDF files of Labs & HWs from T-square

Lab Info

- **Be prepared for Lab**
  - Work the Pre-Lab section
  - Even better, do some or all of the Exercises for Verification
- Lab Verifications: Turn in at end of Lab
  - Counts as part of the Lab Report score
  - ITS also (ITS = Intelligent Tutoring System)
- Lab Report due one week later
  - Learn **your** Lab TA’s format requirements
- Lab FAQs are posted, but relate to old labs
- **Bring your laptop to lab if you want to use your own environment.**

Teams for Lab Projects

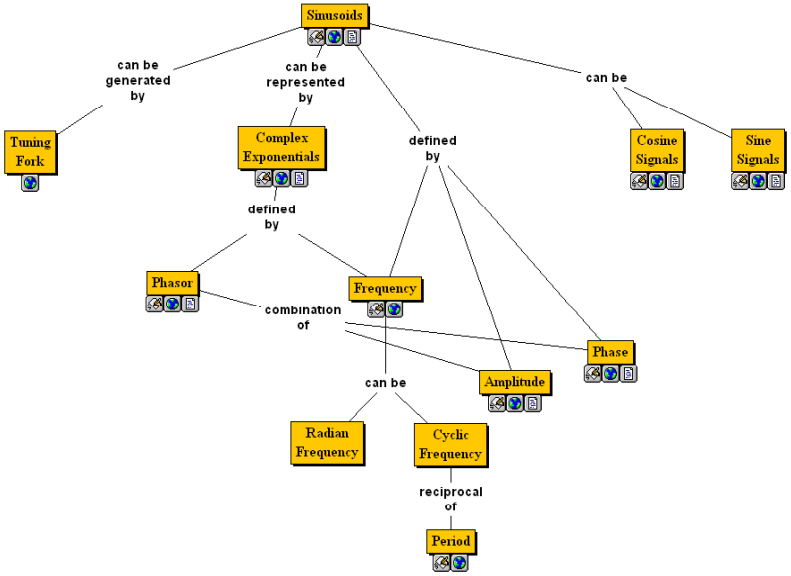
- Two-student teams will be assigned
- One report per team
- New teams at mid-semester
- Peer Evaluation (on-Line)
  - Diary of your work/meetings
  - Sharing the workload with your partner

Homework Formatting

- Include a Cover page with
  - Name
  - Lab section, ie, L05, L20, etc.
  - Recitation Prof's name
  - **Download example from t-square**
- Write on **ONE** side only
  - Use Engineer's paper or plain paper
- STAPLE
- **TWO parts to every solution**
  - **Description of your approach to the solution**
  - **Actual solution**

JUMP

Concept Map for Chapter 2



JUMP

READING ASSIGNMENTS

- This Lecture:
  - Chapter 2, Section 2-6
- Other Reading:
  - Appendix A: Complex Numbers
  - Appendix B: MATLAB
  - Next Lecture: start Chapter 3

LECTURE OBJECTIVES

- Phasors = Complex Amplitude
  - Complex Numbers **represent** Sinusoids

$$A\cos(\omega t+\varphi)=\Re\{(Ae^{j\varphi})e^{j\omega t}\}$$

- *Develop the ABSTRACTION:*
  - *Adding Sinusoids = Complex Addition*
  - ***PHASOR ADDITION THEOREM***

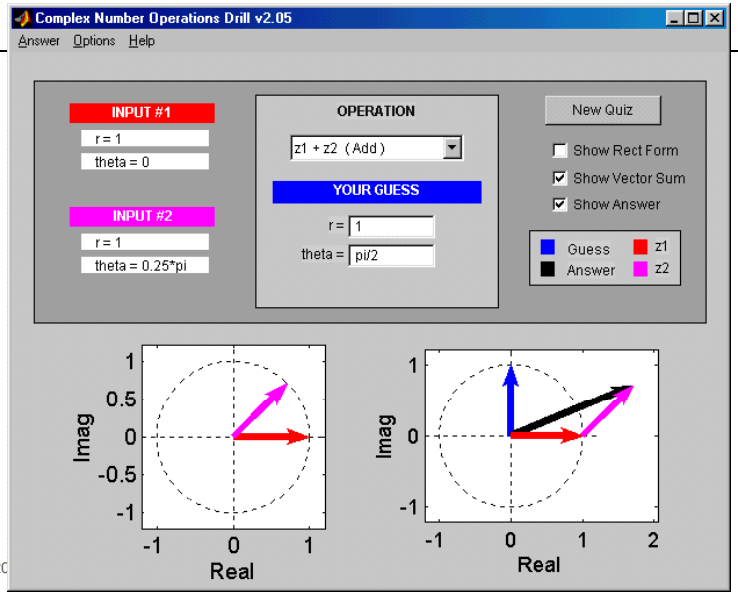
Adding Complex Numbers

- Polar Form
  - Could convert to Cartesian and back out
  - **Use Calculator that does complex ops !**
  - Use MATLAB
  - Visualize the vectors

$$1.7e^{j70\pi/180}+1.9e^{j200\pi/180}=Ae^{j\varphi}\ ?$$

$$1.532e^{j141.79\pi/180}$$

Z DRILL (Complex Arith)



Complex number relations for SCALARS

Cartesian and polar forms

$$r^2=x^2+y^2 \qquad \theta=\mathrm{Tan}^{-1}\left(\frac{y}{x}\right)$$

$$x=r\cos\theta \qquad y=r\sin\theta$$

Euler's formula

$$re^{j\theta}=r\cos(\theta)+jr\sin(\theta)$$

Real part of Euler's

$$\cos(\theta)=\Re\{e^{j\theta}\}$$

Cos = REAL PART

What about sinusoidal signals over time?
Real part of Euler's

$$\cos(\omega t) = \Re\{e^{j\omega t}\}$$

General Sinusoid

$$A\cos(\omega t + \varphi) = \Re\{Ae^{j(\omega t+\varphi)}\}$$
$$= \Re\{Ae^{j\varphi}e^{j\omega t}\}$$

Constant *Varies with time*

COMPLEX AMPLITUDE

General Sinusoid

$$x(t) = A\cos(\omega t + \varphi) = \Re\{Ae^{j\varphi}e^{j\omega t}\}$$

Sinusoid = REAL PART of complex exp: $z(t)=(Ae^{j\phi})e^{j\omega t}$

$$x(t) = \Re\{Xe^{j\omega t}\} = \Re\{z(t)\}$$

X is a (complex) constant -> amplitude and phase
Called COMPLEX AMPLITUDE or PHASOR

$$X = Ae^{j\varphi} \qquad \text{when } z(t) = Xe^{j\omega t}$$

POP QUIZ: Complex Amp

- Find the COMPLEX AMPLITUDE for:

$$x(t) = \sqrt{3}\cos(77\pi t + 0.5\pi)$$

- Use EULER's FORMULA:

$$x(t) = \Re\{\sqrt{3}e^{j(77\pi t+0.5\pi)}\}$$
$$= \Re\{\sqrt{3}e^{j0.5\pi}e^{j77\pi t}\}$$

$$X = \sqrt{3}e^{j0.5\pi}$$

POP QUIZ-2: Complex Amp

- Determine the 60-Hz sinusoid whose COMPLEX AMPLITUDE is:

$$X = \sqrt{3} + j3$$

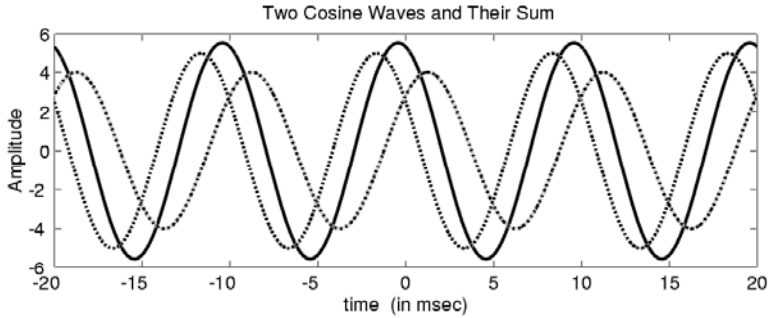
- Convert ***X*** to **POLAR**:

$$x(t) = \Re\{(\sqrt{3} + j3)e^{j(120\pi t)}\}$$
$$= \Re\{\sqrt{12}e^{j\pi/3}e^{j120\pi t}\}$$

$$\Rightarrow x(t) = \sqrt{12}\cos(120\pi t + \pi/3)$$

WANT to ADD SINUSOIDS

- **Main point to remember**: Adding sinusoids of common frequency results in sinusoid with **SAME** frequency



ADD SINUSOIDS: Amp/Phase

- ALL SINUSOIDS have SAME FREQUENCY
- HOW to GET {Amp,Phase} of RESULT ?

$$x_1(t) = 1.7\cos(2\pi(10)t + 70\pi/180)$$

$$x_2(t) = 1.9\cos(2\pi(10)t + 200\pi/180)$$

$$x_3(t) = x_1(t) + x_2(t)$$

$$= 1.532\cos(2\pi(10)t + 141.79\pi/180)$$

PHASOR ADDITION RULE

$$x(t) = \sum_{k=1}^{N} A_k \cos(\omega_0 t + \phi_k)$$

$$= A\cos(\omega_0 t + \phi)$$

Get the new complex amplitude by complex addition

$$\sum_{k=1}^{N} A_k e^{j\phi_k} = Ae^{j\phi}$$

Phasor Addition Proof

$$\sum_{k=1}^{N} A_k \cos(\omega_0 t + \phi_k) = \sum_{k=1}^{N} \Re e\left\{A_k e^{j(\omega_0 t + \phi_k)}\right\}$$

$$= \Re e\left\{\sum_{k=1}^{N} A_k e^{j\phi_k} e^{j\omega_0 t}\right\}$$

$$= \Re e\left\{\left(\sum_{k=1}^{N} A_k e^{j\phi_k}\right) e^{j\omega_0 t}\right\}$$

$$= \Re e\left\{\left(Ae^{j\phi}\right) e^{j\omega_0 t}\right\} = A\cos(\omega_0 t + \phi)$$

POP QUIZ: Add Sinusoids

- ADD THESE 2 SINUSOIDS:

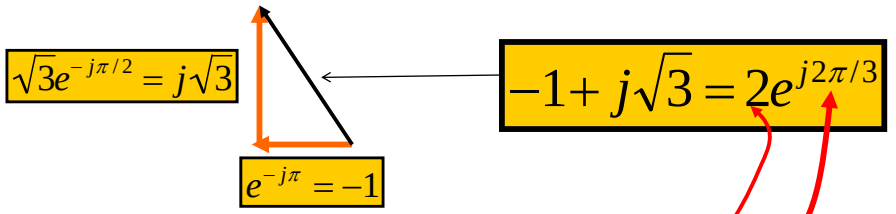
$$x_1(t) = \cos(77\pi t - \pi)$$
$$x_2(t) = \sqrt{3}\cos(77\pi t + 0.5\pi)$$

- COMPLEX (PHASOR) ADDITION:

$$1e^{-j\pi} + \sqrt{3}e^{j0.5\pi}$$

POP QUIZ (answer)

- COMPLEX ADDITION: $1e^{-j\pi} + \sqrt{3}e^{j0.5\pi}$



- CONVERT back to cosine form:

$$x_3(t) = 2\cos(77\pi t + \tfrac{2\pi}{3})$$

ADD SINUSOIDS EXAMPLE

- ALL SINUSOIDS have SAME FREQUENCY
- HOW to GET {Amp,Phase} of RESULT ?

$$x_1(t) = 1.7\cos(2\pi(10)t + 70\pi/180)$$
$$x_2(t) = 1.9\cos(2\pi(10)t + 200\pi/180)$$

$$x_3(t) = x_1(t) + x_2(t) = A\cos(\omega t + \varphi)$$
$$= \Re\{Ae^{j\varphi}e^{j20\pi t}\}$$

Convert Sinusoids to Phasors

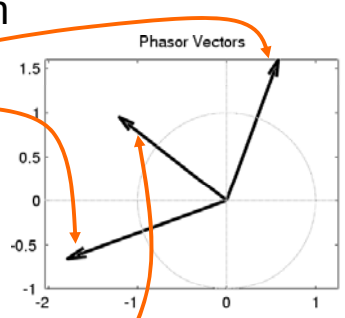
- Each sinusoid → Complex Amp

$$1.7\cos(20\pi t + 70\pi/180) \rightarrow 1.7e^{j70\pi/180}$$
$$1.9\cos(20\pi t + 200\pi/180) \rightarrow 1.9e^{j200\pi/180}$$

$$1.7e^{j70\pi/180} + 1.9e^{j200\pi/180} = ?$$
$$1.532e^{j141.79\pi 180}$$
$$\rightarrow 1.532\cos(20\pi t + 141.79\pi/180)$$

Phasor Add: Numerical

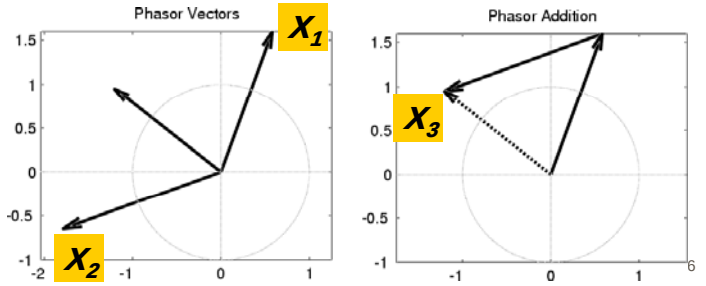
- Convert Polar to Cartesian
  - X_1 = 0.5814 + j1.597
  - X_2 = -1.785 - j0.6498
  - sum =
  - X_3 = -1.204 + j0.9476
- Convert back to Polar
  - X_3 = 1.532 at angle 141.79π/180
  - This is the sum



ADDING SINUSOIDS IS COMPLEX ADDITION

$$x_1(t) = 1.7\cos(2\pi(10)t + 70\pi/180)$$
$$x_2(t) = 1.9\cos(2\pi(10)t + 200\pi/180)$$
$$x_3(t) = x_1(t) + x_2(t)$$
$$= 1.532\cos(2\pi(10)t + 141.79\pi/180)$$

VECTOR (PHASOR) ADD



Add 20 Sinusoids (MATLAB)

$$s(t) = \sum_{k=1}^{20} \sqrt{k}\cos(120\pi(t - 0.002k))$$

- Each sinusoid → Complex Amp

$$S = Ae^{j\varphi} = \sum_{k=1}^{20} \sqrt{k}\, e^{j120\pi(-0.002k)}$$

- *MATLAB*

```
kk=1:20;
SS = sum( sqrt(kk) .* exp(120i*pi*(-0.002)*kk) );
zprint( SS )
```

$$A = 6.949, \quad \varphi = -1.545$$