ECE-2025 Fall-2009

Lecture 6

Periodic Signals, Harmonics & Time-Varying Sinusoids

4-Sept-09

General Information

- ***Office Hours on t-square WIKI***
- ***t-square: OFFICIAL ANNOUNCEMENTS***
  - Old Quizzes & Problems are part of the huge database on the SP-First CDROM website (linked on t-square)
- **Quiz #1 on 14-Sept (Monday)**
  - Allowed one page of notes (Handwritten)
  - Calculators permitted
  - Coverage: HWs 1, 2, & 3; and Labs 1 & 2
- HW #3 due NEXT WEEK in Recitation
  - **Monday Recitations turn in HW #3 during Lab on Wed, 9-Sept**

Lab Info

- Lab #2 Report
  - Notation Change in Section 4. "tau" for src-rcvr time
  - **Start work early—there's a lot to comprehend**
  - Turn in at beginning your lab time
  - Discuss lab report standards with your TA
- Lab #3
  - Bring headphones
- Miscellaneous
  - ERRORS ? ALWAYS Check Announcements
  - Complete INSTRUCTOR VERIFICATION in Lab

Vowel Challenge: Extra Credit

- Synthesize a vowel from a sum of sinusoids
  - **Make it sound good/realistic**
  - **Few Sinusoids: <10, try to do it with 5**
  - **Write up your approach**
- There only a few true vowels:
  - bat, bet, bit, bot, but
  - bait is not; it's a diphthong
  - Use spectrogram to find the "significant" sinusoids (more later)
- Max of Five Winners: +1 pt Final grade for top three
  - ½ point for others
  - earliest entries get priority; best sounding win

Lecture

READING ASSIGNMENTS

- This Lecture:
  - Chapter 3, Sections 3-2 and 3-3
  - Chapter 3, Sections 3-7 and 3-8
- Next Lecture:
  - **Fourier Series ANALYSIS**
  - Sections 3-4, 3-5 and 3-6

LECTURE OBJECTIVES

- Signals with HARMONIC Frequencies
  - Add Sinusoids with $f_k = kf_0$

$$x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi k f_0 t + \varphi_k)$$

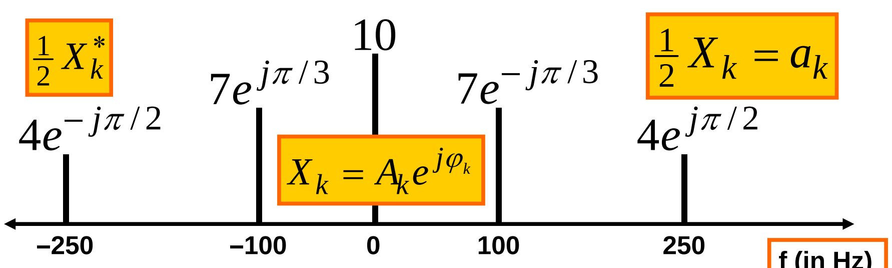
FREQUENCY can change vs. TIME

Chirps: $x(t) = \cos(\alpha t^2)$

Introduce Spectrogram Visualization **(specgram.m)** **(plotspec.m)**

SPECTRUM DIAGRAM

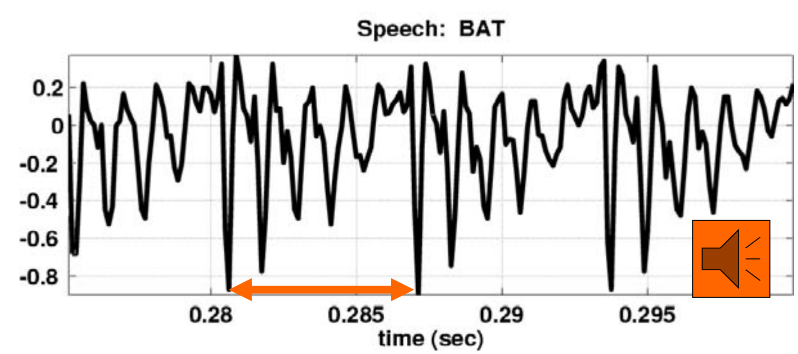
- Recall Complex Amplitude vs. Freq



$$x(t) = 10 + 14\cos(2\pi(100)t - \pi/3) + 8\cos(2\pi(250)t + \pi/2)$$

SPECTRUM for PERIODIC ?

- Nearly Periodic in the Vowel Region
  - Period is (Approximately) T = 0.0065 sec



Harmonic Signal Spectrum

Periodic signal can only have : $f_k = k\, f_0$

$$x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi k f_0 t + \varphi_k)$$

$$f_0 = \frac{1}{T}$$

$$X_k = A_k e^{j\varphi_k}$$

$$x(t) = X_0 + \sum_{k=1}^{N} \left\{ \tfrac{1}{2} X_k e^{j2\pi k f_0 t} + \tfrac{1}{2} X_k^* e^{-j2\pi k f_0 t} \right\}$$

Define FUNDAMENTAL FREQ

$$x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi k f_0 t + \varphi_k)$$

$$f_k = k\, f_0 \qquad (\omega_0 = 2\pi f_0)$$

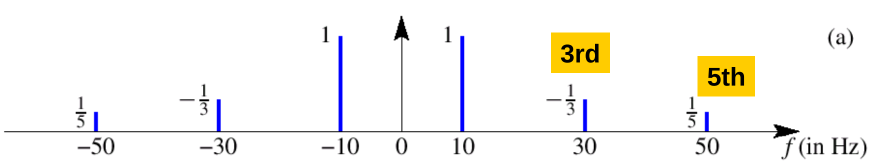
$$f_0 = \frac{1}{T_0}$$

f_0 = fundamental Frequency

f_k / f_0 = integer

T_0 = fundamental Period

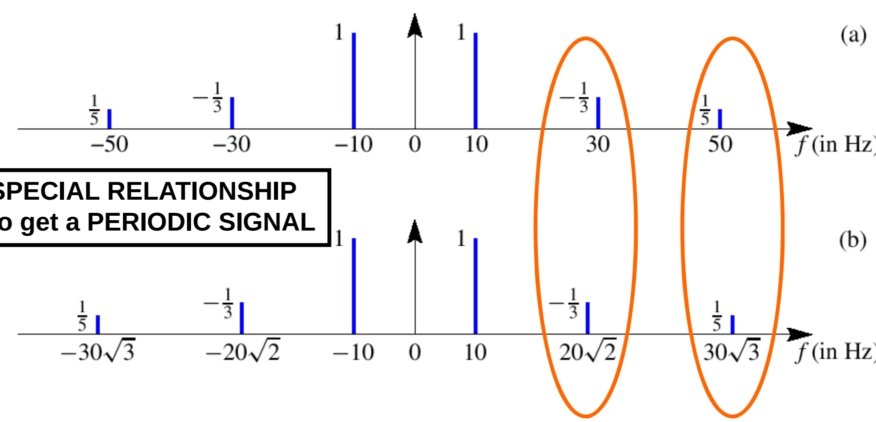
Harmonic Signal (3 Freqs)



What is the fundamental frequency?

10 Hz

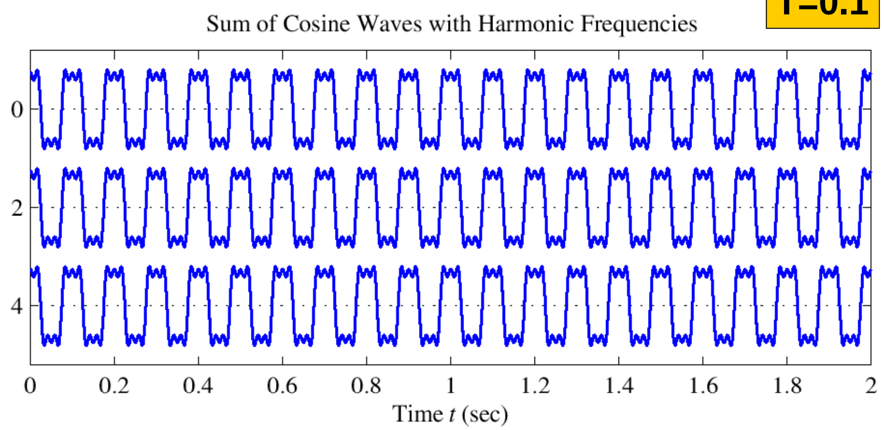
IRRATIONAL SPECTRUM



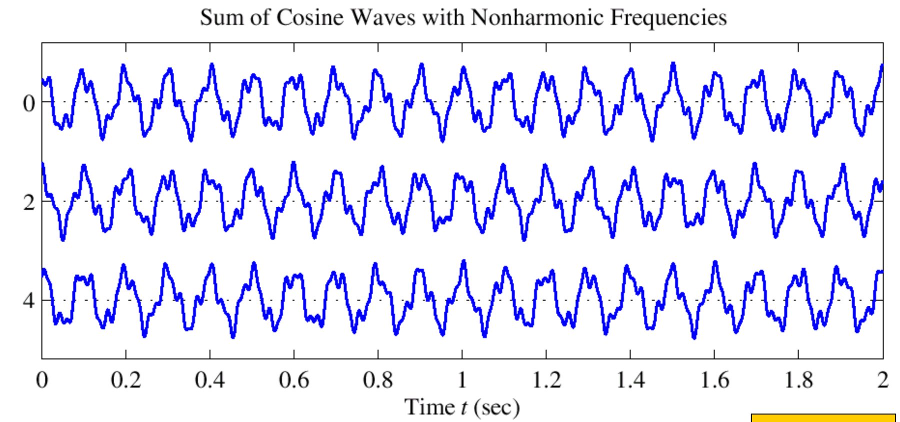
SPECIAL RELATIONSHIP to get a PERIODIC SIGNAL

Harmonic Signal (3 Freqs)

T=0.1



NON-Harmonic Signal



NOT PERIODIC

FREQUENCY ANALYSIS

- **<u>Now, a much HARDER problem</u>**
- Given a recording of a song, have the computer write the music
- Can a machine extract frequencies?
  - Yes, if we COMPUTE the spectrum for x(t)
    - During short intervals

Time-Varying FREQUENCIES Diagram

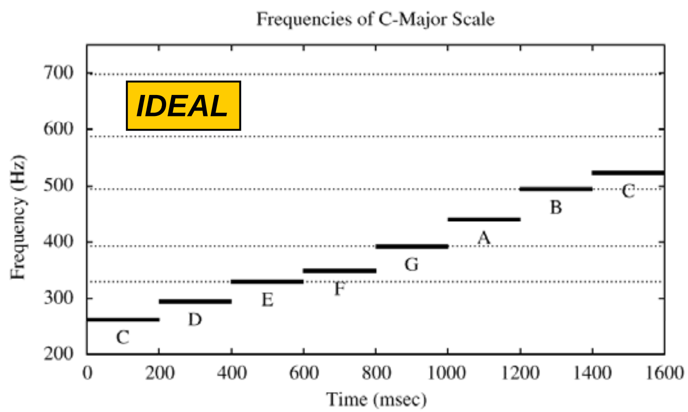
Frequency is the vertical axis

Figure 3.18 Sheet-music notation is a time–frequency diagram.

<u>Time is the horizontal axis</u>

SIMPLE TEST SIGNAL

- C-major SCALE: stepped frequencies
  - Frequency is constant for each note

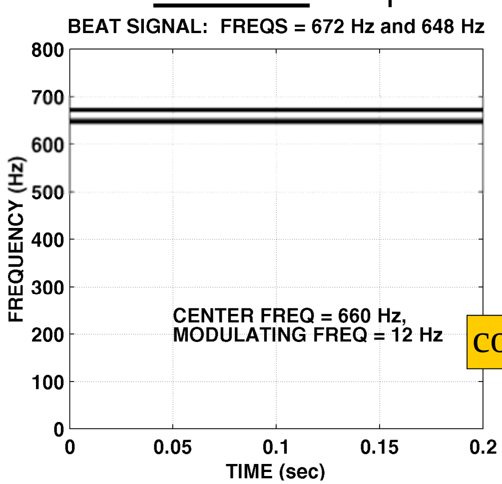


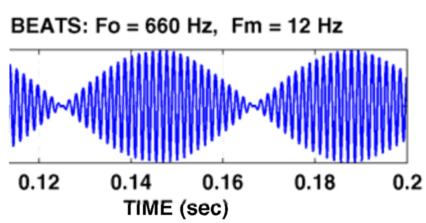
R-rated: ADULTS ONLY

- SPECTROGRAM Tool
  - MATLAB function is `specgram.m`
  - SP-First has `plotspec.m` & `spectgr.m`
- ANALYSIS program
  - Takes x(t) as input &
  - Produces spectrum values X_k
  - Breaks x(t) into SHORT TIME SEGMENTS
    - Then uses the FFT (Fast Fourier Transform)

SPECTROGRAM EXAMPLE

- Two **Constant** Frequencies: Beats





$$\cos(2\pi(660)t)\sin(2\pi(12)t)$$

AM Radio Signal

- Same as BEAT Notes

$$\cos(2\pi(660)t)\sin(2\pi(12)t)$$

$$\frac{1}{2}\left(e^{j2\pi(660)t}+e^{-j2\pi(660)t}\right)\frac{1}{2j}\left(e^{j2\pi(12)t}-e^{-j2\pi(12)t}\right)$$

$$\frac{1}{4j}\left(e^{j2\pi(672)t}-e^{-j2\pi(672)t}-e^{j2\pi(648)t}+e^{-j2\pi(648)t}\right)$$

$$\tfrac{1}{2}\cos(2\pi(672)t-\tfrac{\pi}{2})+\tfrac{1}{2}\cos(2\pi(648)t+\tfrac{\pi}{2})$$

SPECTRUM of AM (Beat)

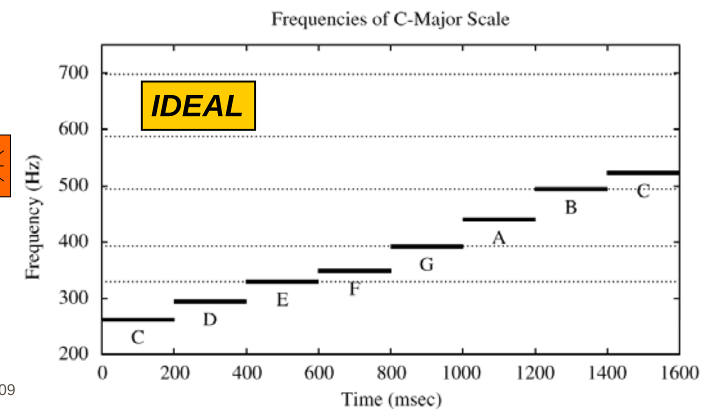
- 4 complex exponentials in AM:

What is the fundamental frequency?

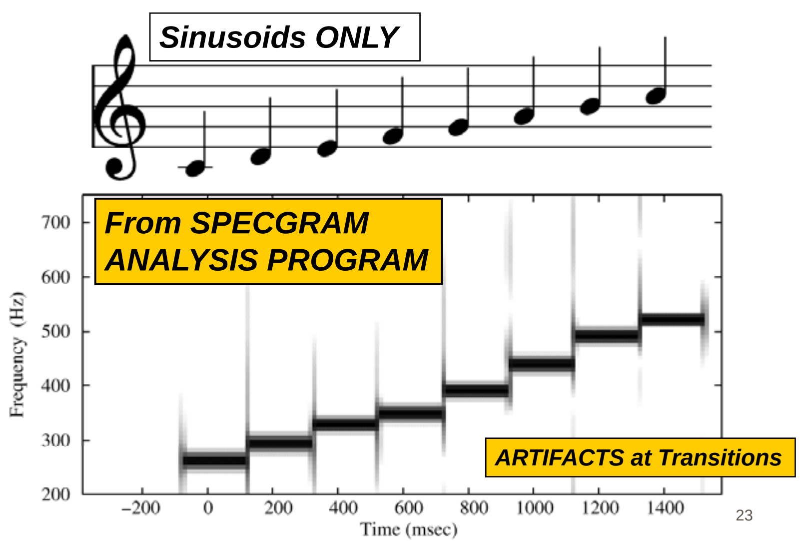
648 Hz ? **24 Hz ?**

STEPPED FREQUENCIES

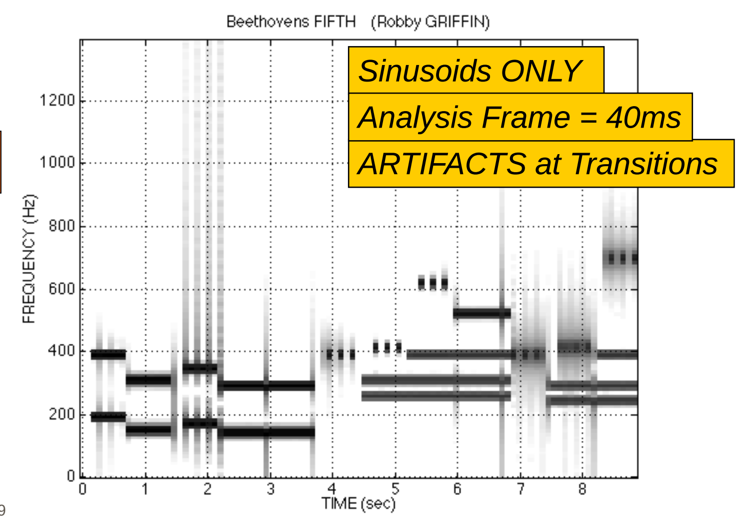
- C-major SCALE: successive sinusoids
  - Frequency is constant for each note



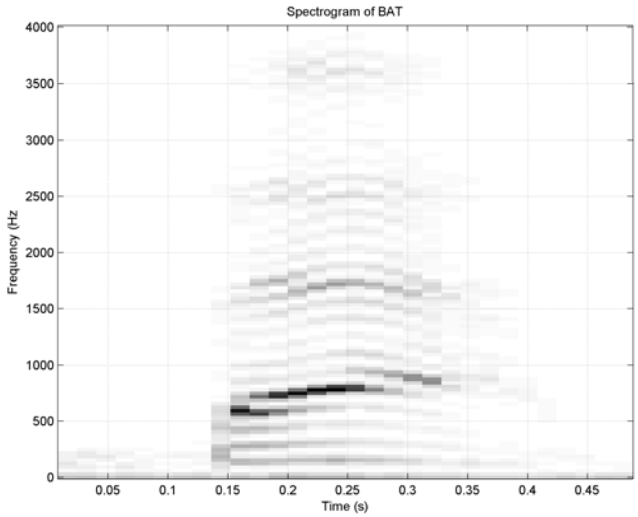
SPECTROGRAM of C-Scale



Spectrogram of LAB SONG



Spectrogram of BAT (plotspec)



Time-Varying Frequency

- Frequency can change vs. time
  - Continuously, not stepped
- **FREQUENCY MODULATION (FM)**

$$x(t) = \cos(2\pi f_c t + v(t))$$

VOICE

- CHIRP SIGNALS
  - Linear Frequency Modulation (LFM)

New Signal: Linear FM

- Called Chirp Signals (LFM)
  - Quadratic phase

$$x(t) = A\cos(\alpha t^2 + 2\pi f_0 t + \varphi)$$

QUADRATIC

- Freq will change LINEARLY vs. time
  - Example of Frequency Modulation (FM)
  - Define "instantaneous frequency"

INSTANTANEOUS FREQ

- Definition

$$x(t) = A\cos(\psi(t))$$
$$\Rightarrow \omega_i(t) = \frac{d}{dt}\psi(t)$$

Derivative of the "Angle"

- For Sinusoid:

$$x(t) = A\cos(2\pi f_0 t + \varphi)$$
$$\psi(t) = 2\pi f_0 t + \varphi$$
$$\Rightarrow \omega_i(t) = \frac{d}{dt}\psi(t) = 2\pi f_0$$

Makes sense

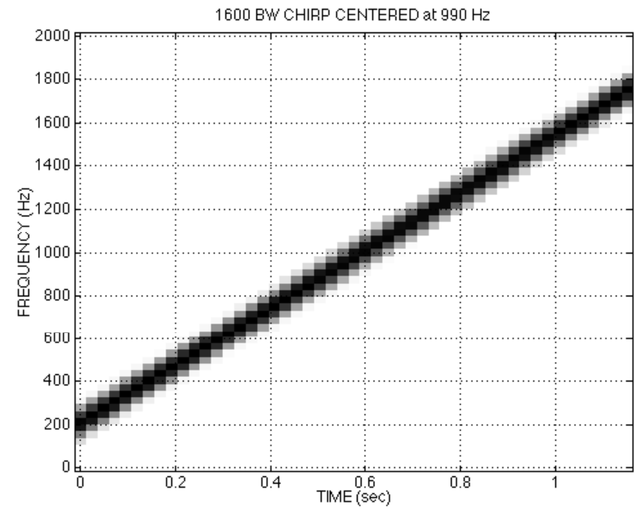
INSTANTANEOUS FREQ of the Chirp

- Chirp Signals have Quadratic phase
- Freq will change LINEARLY vs. time

$$x(t) = A\cos(\alpha t^2 + \beta t + \varphi)$$
$$\Rightarrow \psi(t) = \alpha t^2 + \beta t + \varphi$$

$$\Rightarrow \omega_i(t) = \frac{d}{dt}\psi(t) = 2\alpha t + \beta$$

CHIRP SPECTROGRAM



OTHER CHIRPS

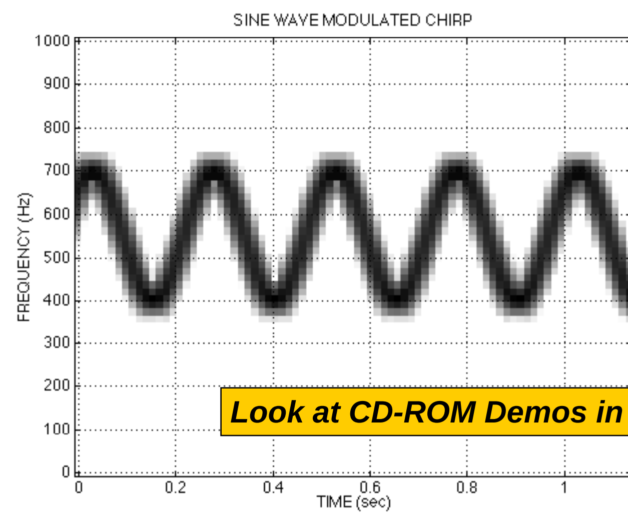
- $\psi(t)$ can be anything:

$$x(t) = A\cos(\alpha\cos(\beta t) + \varphi)$$

$$\Rightarrow \omega_i(t) = \frac{d}{dt}\psi(t) = -\alpha\beta\sin(\beta t)$$

- $\psi(t)$ could be speech or music:
  - FM radio broadcast

SINE-WAVE FREQUENCY MODULATION (FM)



Look at CD-ROM Demos in Ch 3