

## Lecture 14

Digital Filtering of Analog Signals  
12-Oct-09

## Lab &amp; HW Info

- Lab #5
  - **Formal** Lab Report (150 pts)
  - Due starting on 13-Oct (Tues)
  - **Listening Test** when report is turned in
- Lab #7 starts on 13-Oct
  - With new partner
- HW #7 due week of 12-Oct
- **Quiz #2 on 23-Oct (Friday)**
  - Coverage: HW #4, #5, #6, #7 and #8
    - Chapters 3(FourierSeries), 4(Sampling), 5(FIR Filters) and 6(Frequency Response)

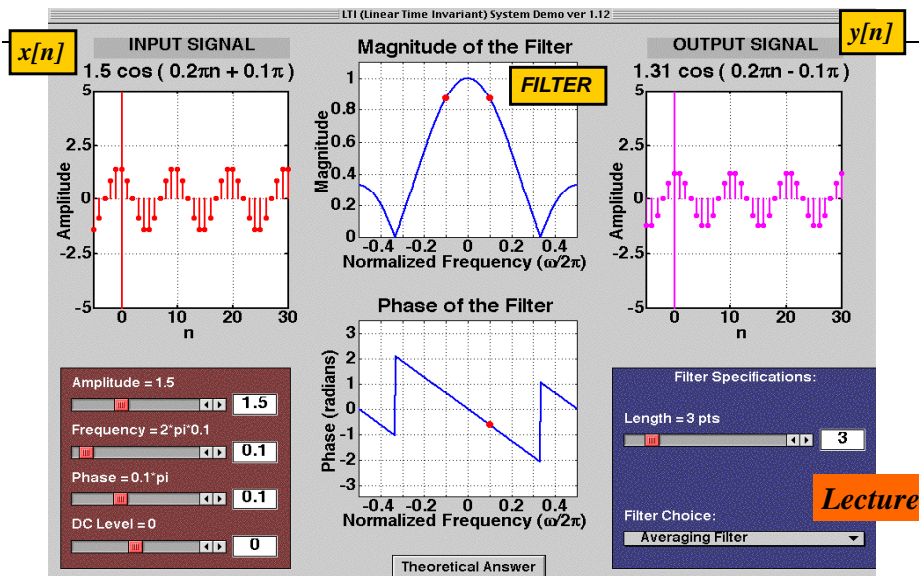
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## LTI Demo with Sinusoids



Lecture

## READING ASSIGNMENTS

- This Lecture:
  - Chapter 6, Sections 6-6, 6-7 & 6-8
- Other Reading:
  - Recitation: Chapter 6
    - FREQUENCY RESPONSE EXAMPLES
  - Next Lecture: Chapter 7

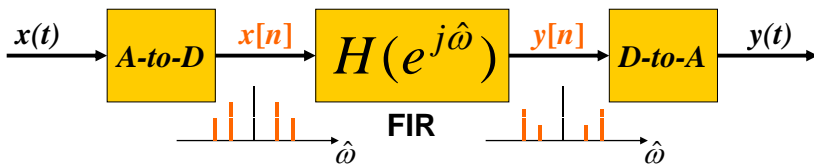
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# LECTURE OBJECTIVES

- Two Domains: Time & Frequency
- Track the spectrum of  $x[n]$  thru an FIR Filter: **Sinusoid-IN gives Sinusoid-OUT**
- UNIFICATION:** How does Frequency Response affect  $x(t)$  to produce  $y(t)$  ?



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# TIME & FREQUENCY

$$y[n] = \sum_{k=0}^M b_k x[n-k] = \sum_{k=0}^M h[k] x[n-k]$$

FIR DIFFERENCE EQUATION is the TIME-DOMAIN

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^M h[k] e^{-j\hat{\omega}k}$$

$$H(e^{j\hat{\omega}}) = h[0] + h[1]e^{-j\hat{\omega}} + h[2]e^{-j2\hat{\omega}} + h[3]e^{-j3\hat{\omega}} + \dots$$

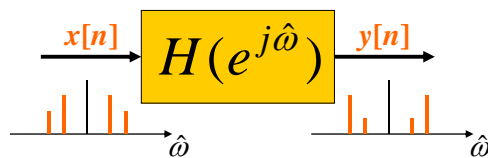
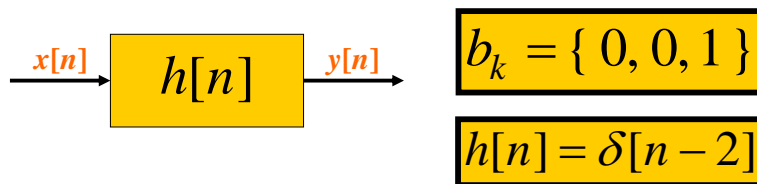
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## Ex: DELAY by 2 SYSTEM

Find  $h[n]$  and  $H(e^{j\hat{\omega}})$  for  $y[n] = x[n-2]$



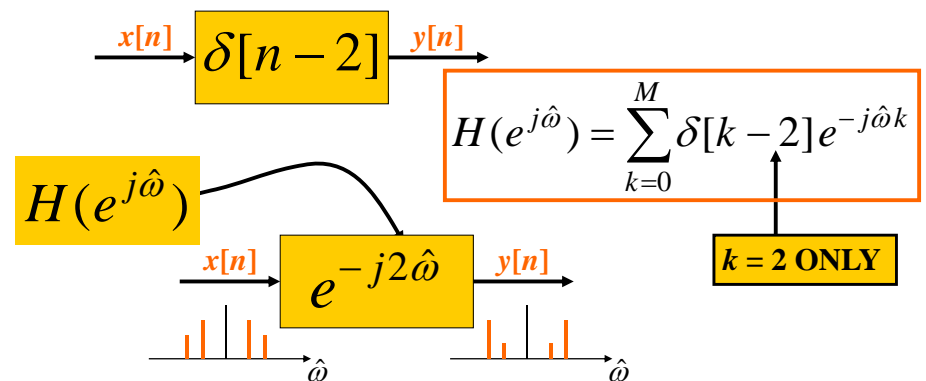
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## DELAY by 2 SYSTEM

Find  $h[n]$  and  $H(e^{j\hat{\omega}})$  for  $y[n] = x[n-2]$



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## GENERAL DELAY PROPERTY

Find  $h[n]$  and  $H(e^{j\hat{\omega}})$  for  $y[n] = x[n - n_d]$

$$h[n] = \delta[n - n_d]$$

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^M \delta[k - n_d] e^{-j\hat{\omega}k} = e^{-j\hat{\omega}n_d}$$

ONLY ONE  
non-ZERO TERM  
for  $k$  at  $k = n_d$

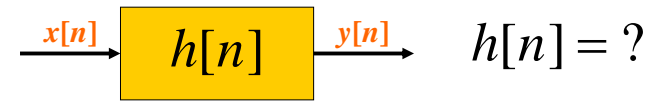
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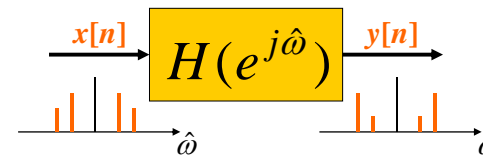
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## FREQ DOMAIN --> TIME ??

- START with  $H(e^{j\hat{\omega}})$  and find  $h[n]$  or  $b_k$



$$H(e^{j\hat{\omega}}) = 7e^{-j2\hat{\omega}} \cos(\hat{\omega})$$



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## FREQ DOMAIN --> TIME

$$H(e^{j\hat{\omega}}) = 7e^{-j2\hat{\omega}} \cos(\hat{\omega}) \quad \text{EULER's Formula}$$

$$= 7e^{-j2\hat{\omega}} (0.5e^{j\hat{\omega}} + 0.5e^{-j\hat{\omega}})$$

$$= (3.5e^{-j\hat{\omega}} + 3.5e^{-j3\hat{\omega}})$$

$$h[n] = 3.5\delta[n - 1] + 3.5\delta[n - 3]$$

$$b_k = \{ 0, 3.5, 0, 3.5 \}$$

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## PREVIOUS LECTURE REVIEW

- SINUSOIDAL INPUT SIGNAL
  - OUTPUT has SAME FREQUENCY
  - DIFFERENT Amplitude and Phase
- FREQUENCY RESPONSE of FIR
  - MAGNITUDE vs. Frequency
  - PHASE vs. Freq
  - PLOTTING

$$H(e^{j\hat{\omega}}) = |H(e^{j\hat{\omega}})| e^{j\angle H(e^{j\hat{\omega}})}$$

MAG

PHASE

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# FREQ. RESPONSE PLOTS

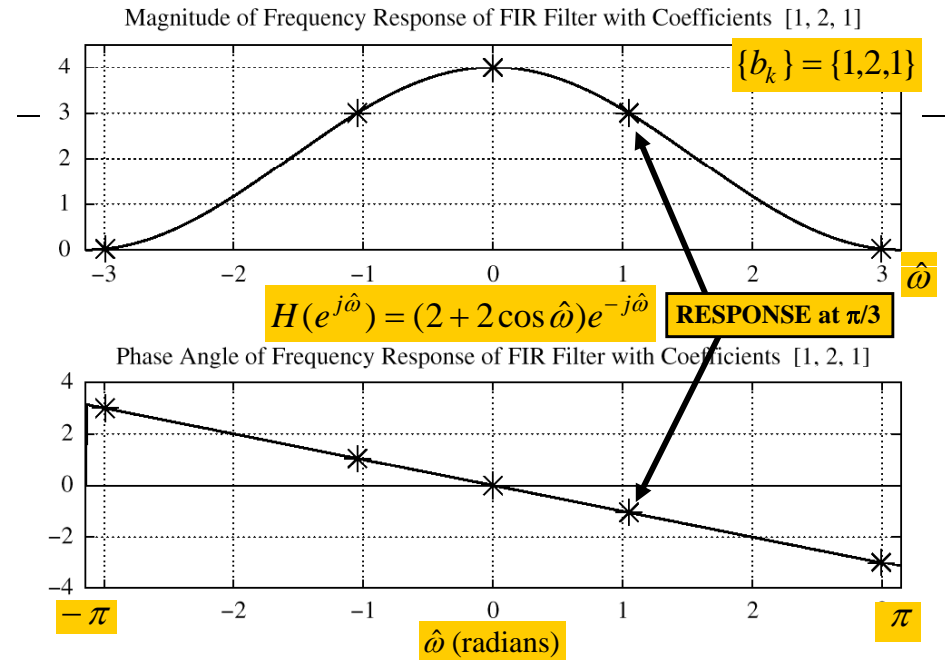
- DENSE GRID (**ww**) from  $-\pi$  to  $+\pi$ 
  - **ww** = `-pi:(pi/100):pi;`
- **HH** = `freqz(bb,1,ww)`
  - VECTOR **bb** contains Filter Coefficients
  - DSP-First: **HH** = `freakz(bb,1,ww)`

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^M b_k e^{-j\hat{\omega}k}$$

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## EX: COSINE INPUT (ans-2)

Find  $y[n]$  when  $x[n] = 2 \cos(\frac{\pi}{3}n + \frac{\pi}{4})$

$$H(e^{j\hat{\omega}}) = (2 + 2 \cos \hat{\omega})e^{-j\hat{\omega}}$$

$$y_1[n] = H(e^{j\pi/3})e^{j(\pi n/3 + \pi/4)} = 3e^{-j(\pi/3)}e^{j(\pi n/3 + \pi/4)}$$

$$y_2[n] = H(e^{-j\pi/3})e^{-j(\pi n/3 + \pi/4)} = 3e^{j(\pi/3)}e^{-j(\pi n/3 + \pi/4)}$$

$$y[n] = 3e^{j(\pi n/3 - \pi/12)} + 3e^{-j(\pi n/3 - \pi/12)}$$

$$\Rightarrow y[n] = 6 \cos(\frac{\pi}{3}n - \frac{\pi}{12})$$

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## SINUSOID thru FIR

- IF  $H^*(e^{j\hat{\omega}}) = H(e^{-j\hat{\omega}})$
- Multiply the Magnitudes
- Add the Phases

$$x[n] = A \cos(\hat{\omega}_1 n + \phi)$$

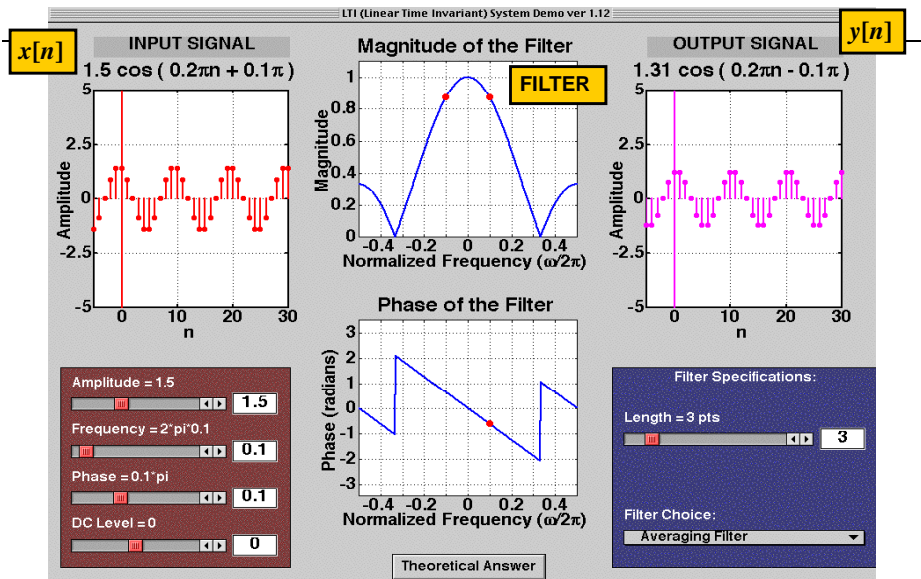
$$\Rightarrow y[n] = A |H(e^{j\hat{\omega}_1})| \cos(\hat{\omega}_1 n + \phi + \angle H(e^{j\hat{\omega}_1}))$$

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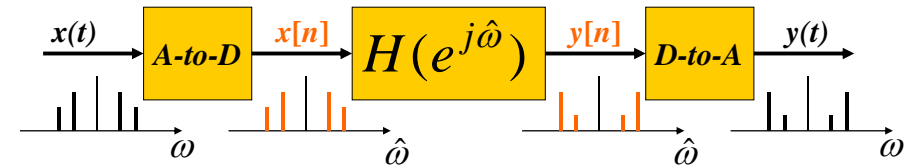
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# LTI Demo with Sinusoids

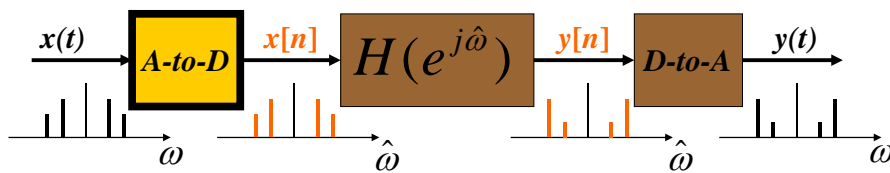


# DIGITAL "FILTERING"



- $\omega$  ■ SPECTRUM of  $x(t)$  (SUM of SINUSOIDS)
- $\hat{\omega}$  ■ SPECTRUM of  $x[n]$ 
  - Is ALIASING a PROBLEM ?
- $\hat{\omega}$  ■ SPECTRUM  $y[n]$  (FIR Gain or Nulls)
- $\omega$  ■ Then, OUTPUT  $y(t)$  = SUM of SINUSOIDS

# FREQUENCY SCALING

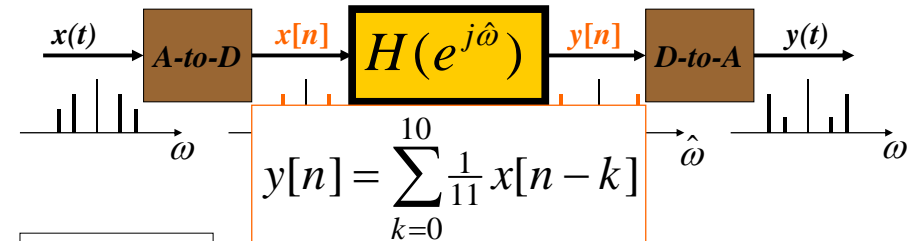


- TIME SAMPLING:
  - IF **NO** ALIASING:
  - FREQUENCY SCALING

$$t = nT_s$$

$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s}$$

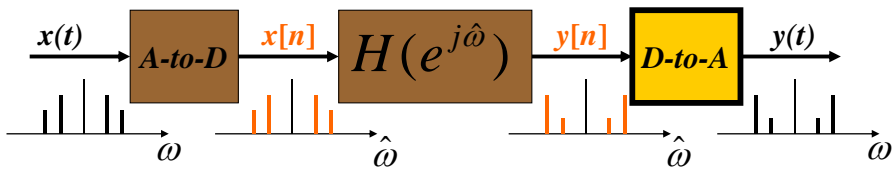
# 11-pt AVERAGER Example



$$H(e^{j\hat{\omega}}) = \frac{\sin(\frac{11}{2} \hat{\omega})}{11 \sin(\frac{1}{2} \hat{\omega})} e^{-j5\hat{\omega}} \quad ?$$

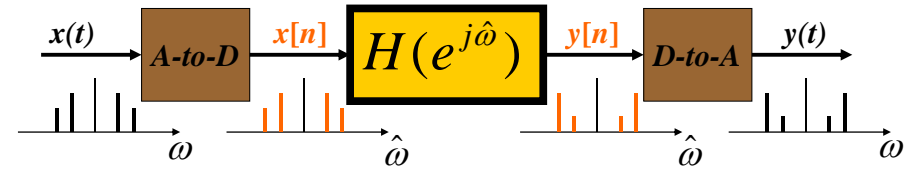
$$x(t) = \cos(2\pi(25)t) + \cos(2\pi(250)t - \frac{1}{2}\pi)$$

# D-A FREQUENCY SCALING



- TIME SAMPLING:  $t = nT_s \Rightarrow n \leftarrow tf_s$
- RECONSTRUCT up to  $0.5f_s$ 
  - FREQUENCY SCALING  $\omega = \hat{\omega} f_s$

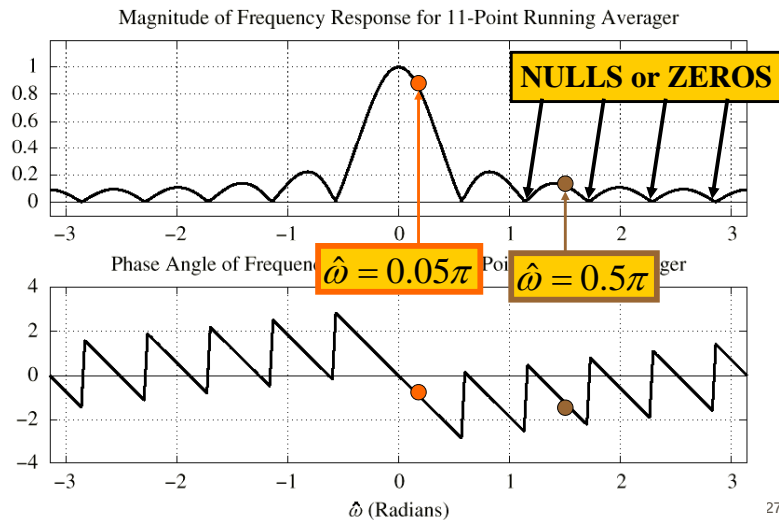
# TRACK the FREQUENCIES



250 Hz	$0.5\pi$	$H(e^{j0.5\pi})$	$0.5\pi$	250 Hz
25 Hz	$.05\pi$	$H(e^{j0.05\pi})$	$.05\pi$	25 Hz

**Fs = 1000 Hz**      **NO new freqs**

# 11-pt AVERAGER



# EVALUATE Freq. Response

$$H(e^{j\hat{\omega}}) = \frac{\sin(\frac{11}{2} \hat{\omega})}{11 \sin(\frac{1}{2} \hat{\omega})} e^{-j5\hat{\omega}}$$

At  $\hat{\omega} = 0.5\pi$

$$H(e^{j\hat{\omega}}) = \frac{\sin(\frac{11}{2} (0.5\pi))}{11 \sin(\frac{1}{2} (0.5\pi))} e^{-j5(0.5\pi)}$$

$$= \frac{\sin(2.75\pi)}{11 \sin(0.25\pi)} e^{-j2.5\pi}$$

$$= 0.0909 e^{-j0.5\pi}$$

# EVALUATE Freq. Response

$$x(t) = \cos(2\pi(25)t) + \sin(2\pi(250)t)$$

evaluating at 25 and 250 Hz.

$$H(e^{j2\pi(25)/1000}) = \frac{\sin(\pi(25)(11)/1000)}{11 \sin(\pi(25)/1000)} e^{-j2\pi(25)(5)/1000}$$

**MAG SCALE**

**$f_s = 1000$**

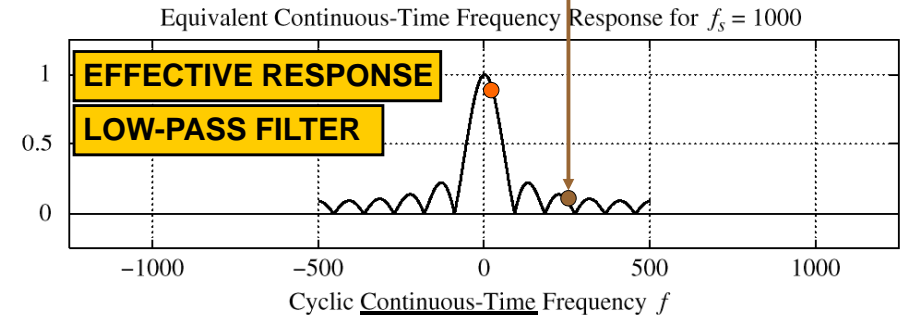
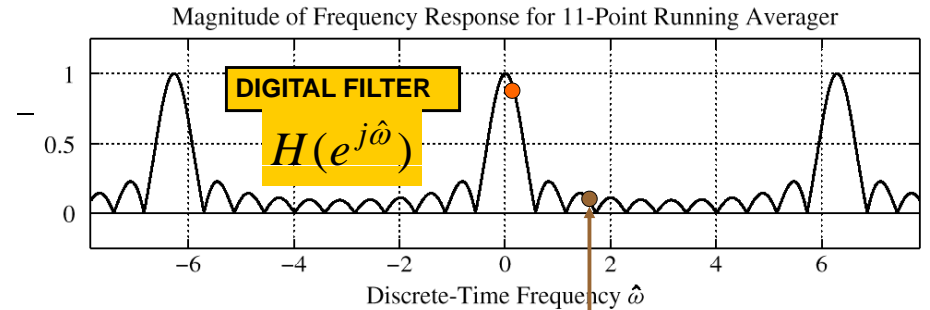
$$= 0.8811 e^{-j\pi/4}$$

**PHASE CHANGE**

$$H(e^{j2\pi(250)/1000}) = \frac{\sin(\pi(250)(11)/1000)}{11 \sin(\pi(250)/1000)} e^{-j2\pi(250)(5)/1000}$$

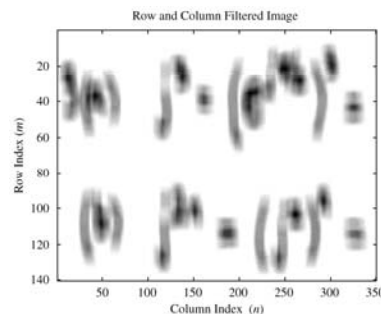
$$= 0.0909 e^{-j\pi/2}$$

$$y(t) = 0.8811 \cos(2\pi(25)t - \pi/4) + 0.0909 \sin(2\pi(250)t - \pi/2)$$



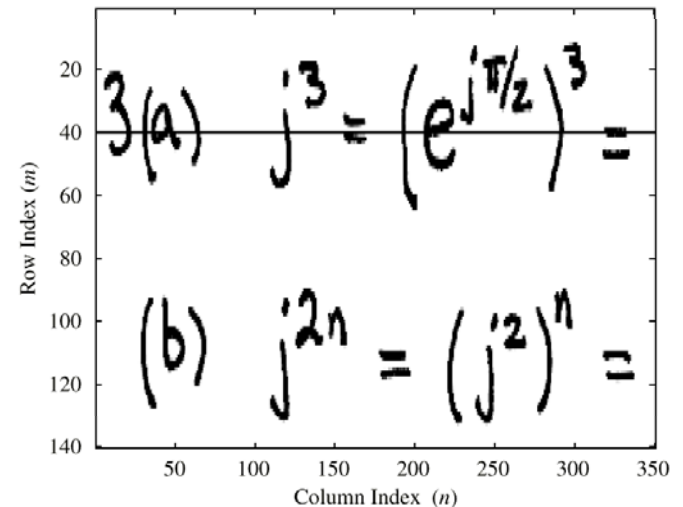
# FILTER TYPES

- LOW-PASS FILTER (**LPF**)
  - BLURRING
  - ATTENUATES HIGH FREQUENCIES
- HIGH-PASS FILTER (**HPF**)
  - SHARPENING for IMAGES
  - BOOSTS THE HIGHS
  - REMOVES DC
- BAND-PASS FILTER (**BPF**)

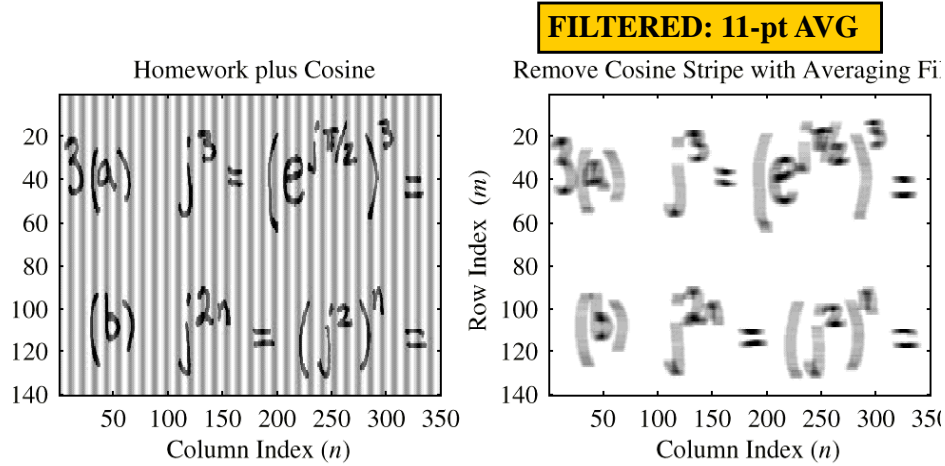


# B & W IMAGE

Original Black and White Image



# B&W IMAGE with COSINE

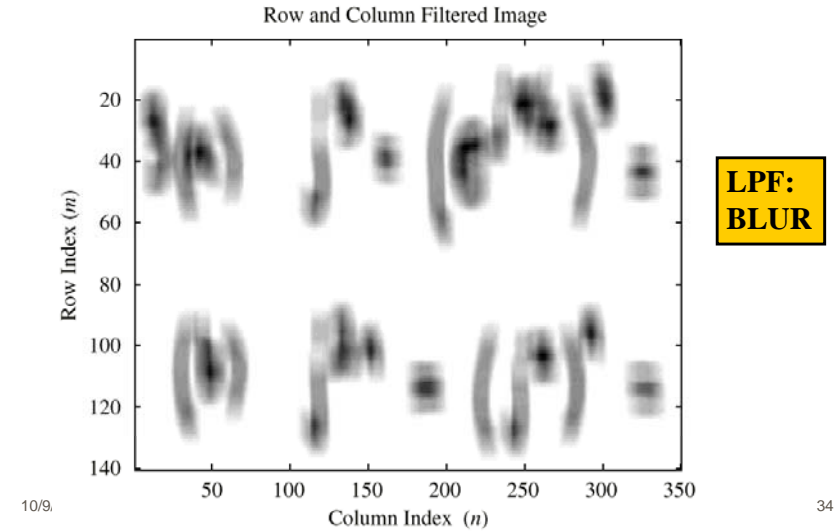


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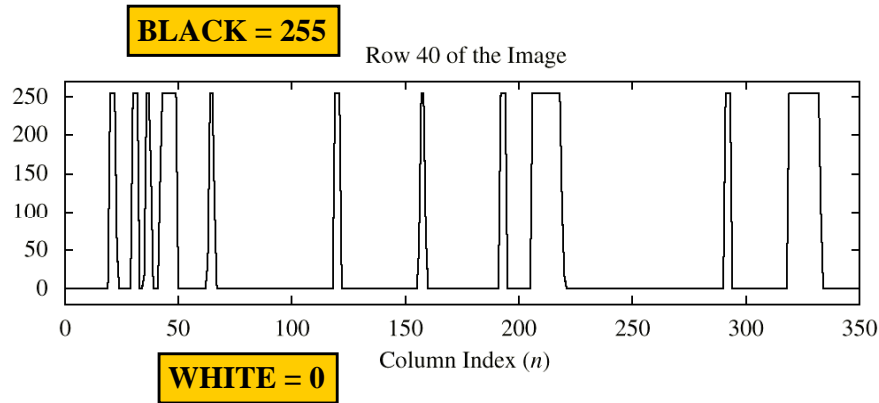
# FILTERED B&W IMAGE



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# ROW of B&W IMAGE

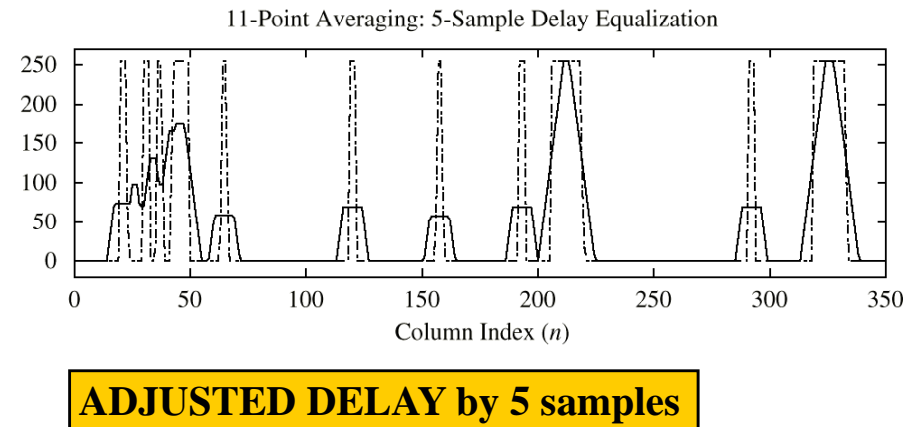


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# FILTERED ROW of IMAGE



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