ECE-2025 Fall-2009

Lecture 16

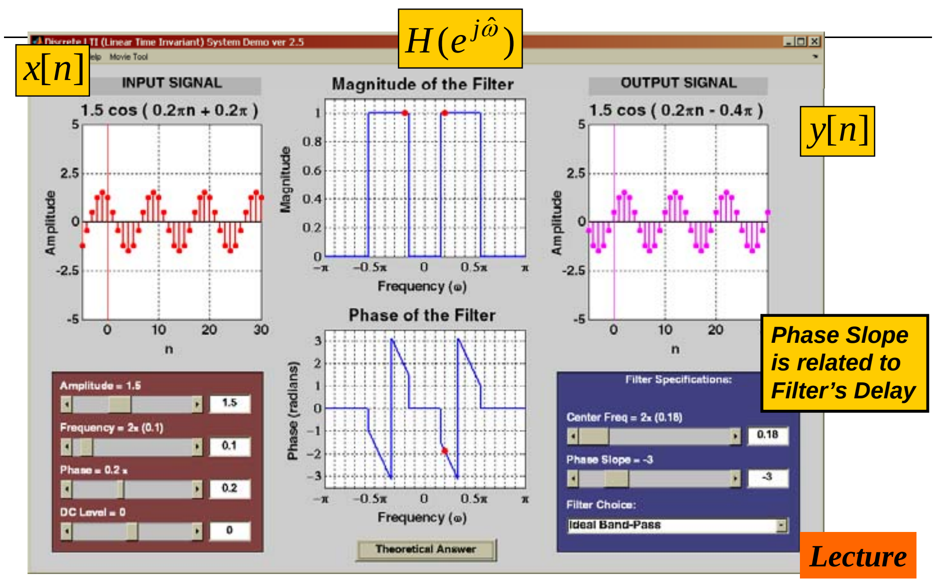
IIR Filters: H(z), Feedback, and Frequency Response

19-Oct-09

Lab & HW Info

- Quiz #2 on 23-Oct (Friday)
  - One Page HAND-WRITTEN notes; Calculator OK
  - Take test with your assigned lecture section
  - Coverage: HW #4, #5, #6, #7 and #8
    - Chapters 3(FourierSeries), 4(Sampling), 5(FIR Filters) and 6(Frequency Response)
- Quiz Review: Thurs @6pm, Klaus 2456
- Lab #8 starts 20-Oct
  - Frequency Response of your hearing
- HW #8 due this week
  - Frequency Response will be covered on Quiz #2

DLTIDEMO: Ideal Filters (Lab)



READING ASSIGNMENTS

- This Lecture:
  - Chapter 8, Sects. 8-4 8-5 & 8-6
- Other Reading:
  - Recitation: Chapter 8, all
    - POLE-ZERO PLOTS

LECTURE OBJECTIVES

- IIR: Infinite-Length Impulse Response
- SYSTEM FUNCTION: H(z)
- FEEDBACK Difference Equations
- FREQUENCY RESPONSE of IIR

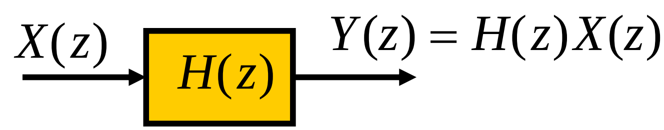
$$H(e^{j\hat{\omega}}) = H(z)\Big|_{z=e^{j\hat{\omega}}}$$

- THREE-DOMAIN APPROACH

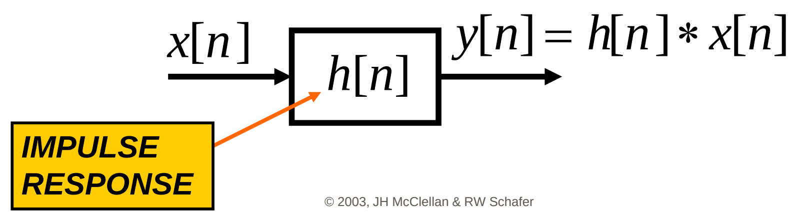
$$h[n] \leftrightarrow H(z) \leftrightarrow H(e^{j\hat{\omega}})$$

Recall: CONVOLUTION PROPERTY

- **MULTIPLICATION** of z-TRANSFORMS

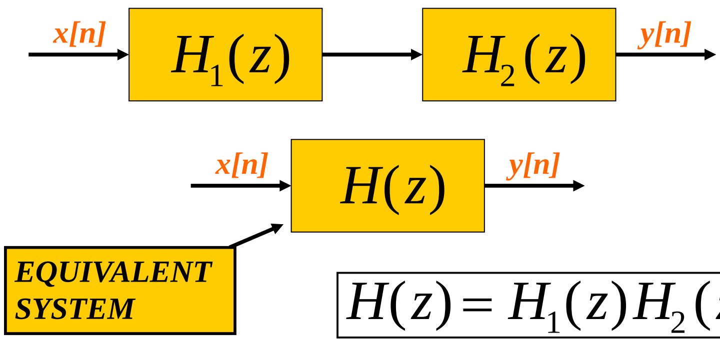


- **CONVOLUTION** in TIME-DOMAIN



Recall: CASCADE EQUIVALENT

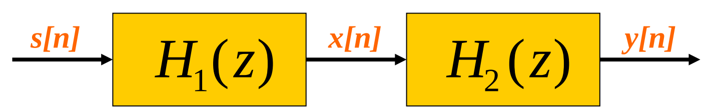
- Multiply the System Functions



$$H(z) = H_1(z)H_2(z)$$

Motivation: De-Convolution

- Can you make y[n] equal to s[n]?
  - Y(z) = H(z)S(z)

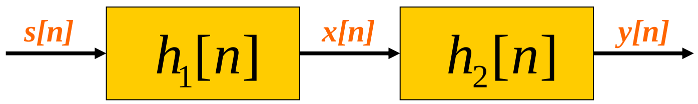


$$H(z) = 1 = H_1(z)H_2(z)$$
$$\Rightarrow H_2(z) = 1/H_1(z)$$

$$H_1(z) = 1 - az^{-1}$$
$$\Rightarrow H_2(z) = \frac{1}{1 - az^{-1}}$$

WHY DE-convolution ?

- Given $h_2[n]$, we must solve for $h_1[n]$ in a convolution equation:



$$x[n] = s[n] * h_1[n]$$
$$y[n] = x[n] * h_2[n] = s[n] * h_1[n] * h_2[n]$$
$$\Rightarrow h_1[n] * h_2[n] = \delta[n]$$

First-Order Transform Pair

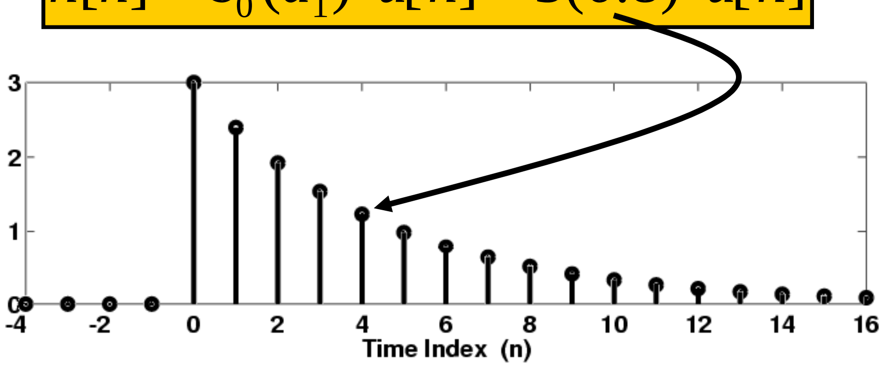
$$h[n] = b_0 (a_1)^n u[n] \quad \leftrightarrow \quad H(z) = \frac{b_0}{1 - a_1 z^{-1}}$$

- Proof: use GEOMETRIC SEQUENCE:

$$H(z) = b_0 \sum_{n=0}^{\infty} a_1^n z^{-n} = b_0 \sum_{n=0}^{\infty} (a_1 z^{-1})^n$$
$$= \frac{b_0}{1 - a_1 z^{-1}} \quad \text{if } |z| > |a_1|$$

Typical IMPULSE Response

$$h[n] = b_0 (a_1)^n u[n] = 3(0.8)^n u[n]$$

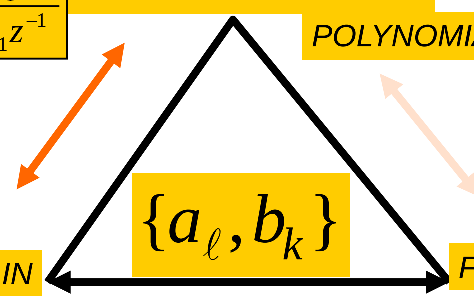


THREE DOMAINS

$$H(z) = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}}$$

Z-TRANSFORM-DOMAIN

POLYNOMIALS: H(z)



TIME-DOMAIN

FREQ-DOMAIN

$$y[n] = \sum_{\ell=1}^{N} a_\ell y[n-\ell] + \sum_{k=0}^{M} b_k x[n-k]$$

$$H(e^{j\hat{\omega}}) = ?$$

$$h[n] = \sum p^n \ \text{terms}$$

H(z) → Difference Equation

- WHAT are the FILTER COEFFICIENTS ?

$$H(z)=\frac{Y(z)}{X(z)}=\frac{b_0+b_1z^{-1}}{1-a_1z^{-1}}=\frac{B(z)}{A(z)}$$

$$(1-a_1z^{-1})Y(z)=(b_0+b_1z^{-1})X(z)$$

$$Y(z)-a_1z^{-1}Y(z)=b_0X(z)+b_1z^{-1}X(z)$$

Recall: DELAY PROPERTY of X(z)

- DELAY in TIME<-->Multiply X(z) by z^{-1}

$$x[n]\leftrightarrow X(z)$$

$$x[n-1]\leftrightarrow z^{-1}X(z)$$

Proof: $$\sum_{n=-\infty}^{\infty}x[n-1]z^{-n}=\sum_{\ell=-\infty}^{\infty}x[\ell]z^{-(\ell+1)}$$

$$=z^{-1}\sum_{\ell=-\infty}^{\infty}x[\ell]z^{-\ell}=z^{-1}X(z)$$

H(z) → Difference Equation

- DERIVE the Filter Coefficients from H(z)
  - Use **DELAY** PROPERTY

$$Y(z)=a_1z^{-1}Y(z)+b_0X(z)+b_1z^{-1}X(z)$$

$$y[n]=a_1y[n-1]+b_0x[n]+b_1x[n-1]$$

EASIER with DELAY PROPERTY

Time delay of n_0 samples multiplies the z-transform by z^{-n_0}

$$x[n-n_0]\quad\Longleftrightarrow\quad z^{-n_0}X(z)$$

H(z) = z-Transform{ h[n] }

- Summarize FIRST-ORDER IIR Filter:

$$y[n]=a_1y[n-1]+b_0x[n]+b_1x[n-1]$$

$$h[n]=b_0(a_1)^nu[n]+b_1(a_1)^{n-1}u[n-1]$$

$$H(z)=\frac{b_0+b_1z^{-1}}{1-a_1z^{-1}}=\frac{b_0}{1-a_1z^{-1}}+z^{-1}\frac{b_1}{1-a_1z^{-1}}$$

SYSTEM FUNCTION

- DIFFERENCE EQUATION:

$$y[n] = 0.8y[n-1] + 3x[n] - 2x[n-1]$$

- **READ** the FILTER COEFFS:

H(z)

$$Y(z) = \left(\frac{3 - 2z^{-1}}{1 - 0.8z^{-1}} \right) X(z)$$

ONE FEEDBACK TERM

- *ADD PREVIOUS OUTPUTS*

$$y[n] = a_1 y[n-1] + b_0 x[n] + b_1 x[n-1]$$

PREVIOUS
FEEDBACK
FIR PART of the FILTER
FEED-FORWARD

- CAUSALITY
  - NOT USING FUTURE OUTPUTS or INPUTS

FILTER COEFFICIENTS

- ADD PREVIOUS OUTPUTS

$$y[n] = 0.8y[n-1] + 3x[n] - 2x[n-1]$$

FEEDBACK COEFFICIENT
SIGN CHANGE

- MATLAB
  - `yy = filter([3,-2],[1,-0.8],xx)`

LTI? AT REST CONDITION

- y[n] = 0, for n<0
- When x[n] = 0, for n<0

INITIAL REST CONDITIONS

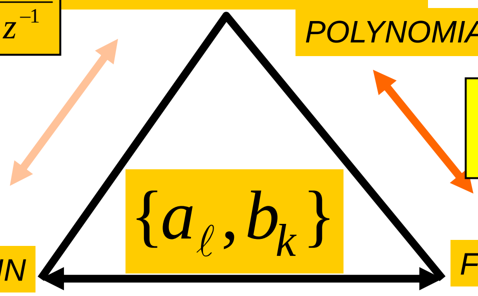
1. The input must be assumed to be zero prior to some starting time n_0, i.e., $x[n] = 0$ for $n < n_0$. We say that such inputs are *suddenly applied*.
2. The output is likewise assumed to be zero prior to the starting time of the signal, i.e., $y[n] = 0$ for $n < n_0$. We say that the system is *initially at rest* if its output is zero prior to the application of a suddenly applied input.

THREE DOMAINS: $H(e^{j\hat{\omega}})$

$H(z) = \dfrac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}}$

Z-TRANSFORM-DOMAIN

POLYNOMIALS: H(z)



$z = e^{j\hat{\omega}}$

TIME-DOMAIN

FREQ-DOMAIN

$y[n] = a_1 y[n-1] + b_0 x[n] + b_1 x[n-1]$

$H(e^{j\hat{\omega}}) = \dfrac{b_0 + b_1 e^{-j\hat{\omega}}}{1 - a_1 e^{-j\hat{\omega}}}$

FREQUENCY RESPONSE

- EVALUATE on the UNIT CIRCLE

$$H(e^{j\hat{\omega}}) = H(z)\big|_{z=e^{j\hat{\omega}}}$$

$$H(z) = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}}$$

$$H(e^{j\hat{\omega}}) = H(z)\big|_{z=e^{j\hat{\omega}}} = \frac{b_0 + b_1 e^{-j\hat{\omega}}}{1 - a_1 e^{-j\hat{\omega}}}$$

FREQ. RESPONSE FORMULA

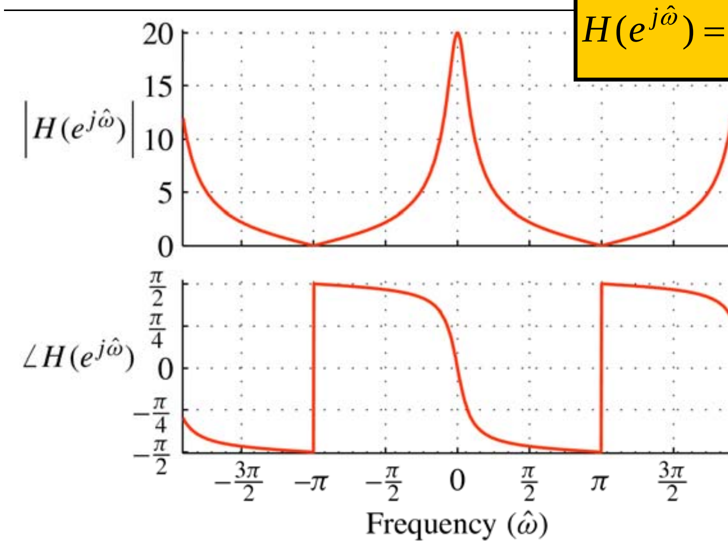
$$H(z) = \frac{2 + 2z^{-1}}{1 - 0.8z^{-1}} \rightarrow H(e^{j\hat{\omega}}) = \frac{2 + 2e^{-j\hat{\omega}}}{1 - 0.8e^{-j\hat{\omega}}}$$

$$@\ \hat{\omega} = 0, \quad |H(e^{j\hat{\omega}})| = \frac{2+2}{1-0.8} = 20$$

$$@\ \hat{\omega} = \pi, \quad |H(e^{j\hat{\omega}})| = \frac{2+2(-1)}{1+0.8} = 0$$

Frequency Response Plot

$H(e^{j\hat{\omega}}) = \dfrac{2 + 2e^{-j\hat{\omega}}}{1 - 0.8e^{-j\hat{\omega}}}$



SINUSOIDAL RESPONSE

- x[n] = SINUSOID => y[n] is SINUSOID
- Get MAGNITUDE & PHASE from H(z)

$$\text{if} \quad x[n] = e^{j\hat{\omega}n}$$

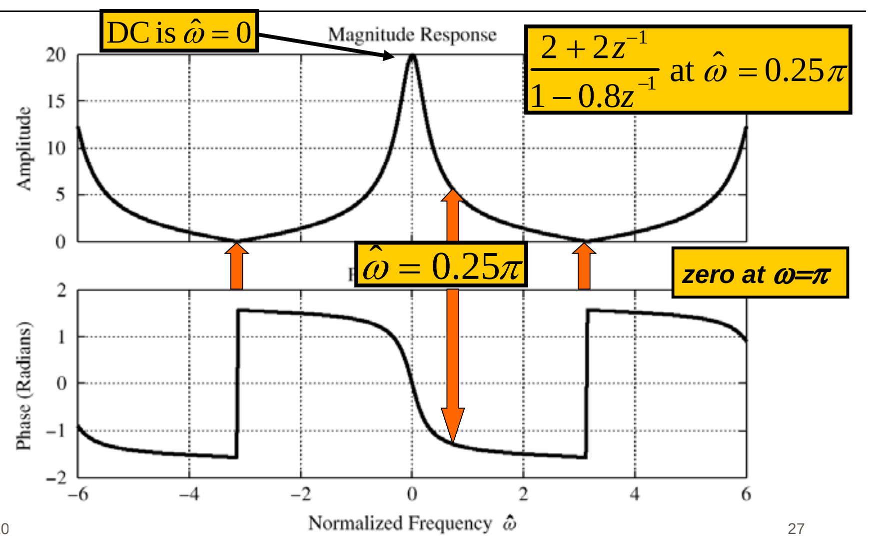
$$\text{then} \quad y[n] = H(e^{j\hat{\omega}})\, e^{j\hat{\omega}n}$$

$$\text{where} \quad H(e^{j\hat{\omega}}) = H(z)\Big|_{z=e^{j\hat{\omega}}}$$

POP QUIZ

- Given: $H(z) = \dfrac{2 + 2z^{-1}}{1 - 0.8z^{-1}}$
- Find the Impulse Response, h[n]
- Find the output, y[n]
  - When $x[n] = \cos(0.25\pi n)$

Evaluate FREQ. RESPONSE



POP QUIZ: Eval Freq. Resp.

- Given: $H(z) = \dfrac{2 + 2z^{-1}}{1 - 0.8z^{-1}}$
- Find output, y[n], when $x[n] = \cos(0.25\pi n)$
  - Evaluate at $z = e^{j0.25\pi}$

$$H(z) = \frac{2 + 2(\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2})}{1 - 0.8e^{-j0.25\pi}} = 5.182e^{-j1.309}$$

$$y[n] = 5.182\cos(0.25\pi n - 0.417\pi)$$