ECE-2025 Fall-2009

Lecture 17
Frequency Response, H(z),
Poles and Zeros
for IIR and FIR Systems
26-Oct-09

Info: Lab, HW, etc.

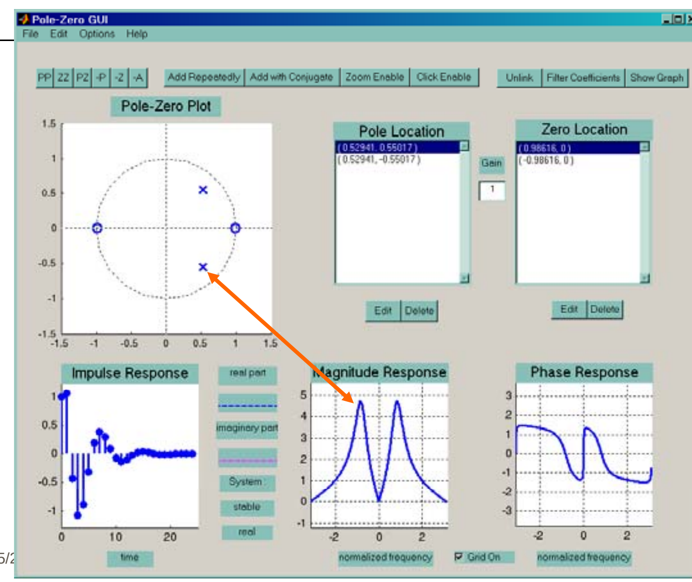
- Labs:
  - Lab #8: both partners do the hearing test.
  - Lab #9 starts on Tues, 27-Oct
  - Lab #10 will be Formal Report
- Homework:
  - #9 due NEXT week
- Quiz #3 will be 23-Nov (Monday)

Z-TRANSFORM TABLES

SHORT TABLE OF z-TRANSFORMS

	$x[n]$		$X(z)$
1.	$ax_1[n] + bx_2[n]$	$\Longleftrightarrow$	$aX_1(z) + bX_2(z)$
2.	$x[n - n_0]$	$\Longleftrightarrow$	$z^{-n_0}X(z)$
3.	$y[n] = x[n] * h[n]$	$\Longleftrightarrow$	$Y(z) = H(z)X(z)$
4.	$\delta[n]$	$\Longleftrightarrow$	1
5.	$\delta[n - n_0]$	$\Longleftrightarrow$	z^{-n_0}
6.	$a^n u[n]$	$\Longleftrightarrow$	$\dfrac{1}{1 - az^{-1}}$

PeZ Demo: Pole-Zero Placing



Lecture

READING ASSIGNMENTS

- This Lecture:
  - Chapter 7, Sections 7-6 to end
  - Chapter 8, Sections 8-4, 8-5, 8-6, 8-9, and 8-10
- Other Reading:
  - Recitation & Lab: same as above
    - ZEROS (and POLES)

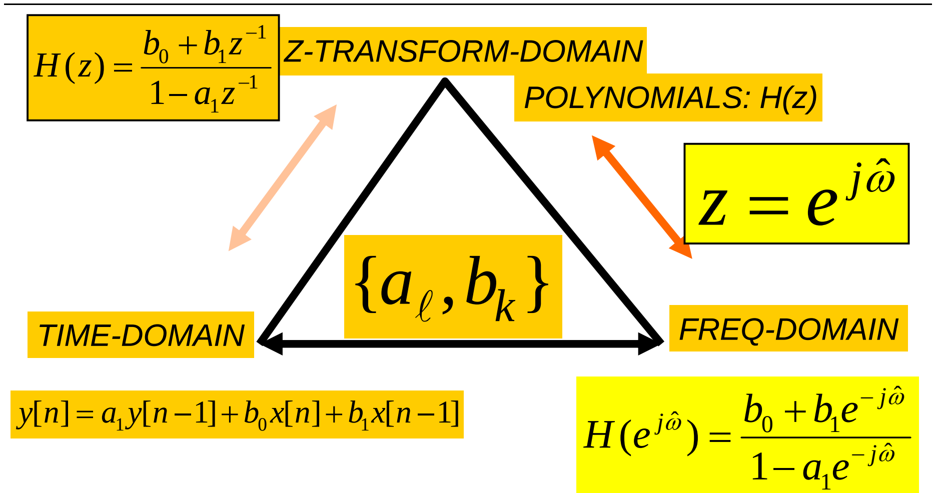
LECTURE OBJECTIVES

- ZEROS and POLES
- Relate H(z) to FREQUENCY RESPONSE

$$H(e^{j\hat{\omega}}) = H(z)\Big|_{z=e^{j\hat{\omega}}}$$

- Four demos: PeZ, movies
  - Placing Poles and Zeros
- Bandpass Filters: IIR
- Nulling Filters: FIR

THREE DOMAINS: $H(e^{j\hat{\omega}})$



H(z) = Rational Function

- First Order:

$$H(z) = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}}$$

- We can also study Second-Order Systems:

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 - a_1 z^{-1} - a_2 z^{-2}} = \frac{B(z)}{A(z)}$$

- Numerator & Denominator Polynomials

POLES & ZEROS of H(z)

- Zeros of H(z), i.e., where is H(z)=0?
  - Look for Roots of Numerator Polynomial

$$H(z)=\frac{B(z)}{A(z)},\ \text{so}\ B(z_0)=0 \Rightarrow H(z_0)=0$$

$$\text{if}\ \ A(z_0)\neq 0$$

- Poles of H(z), i.e., where is H(z)=0?
  - Look for Roots of Denominator Polynomial

$$H(z)=\frac{B(z)}{A(z)},\ \text{so}\ A(z_0)=0 \Rightarrow H(z_0)\to\infty$$

$$\text{if}\ \ B(z_0)\neq 0$$

Poles/Zeros of 1st-order H(z)

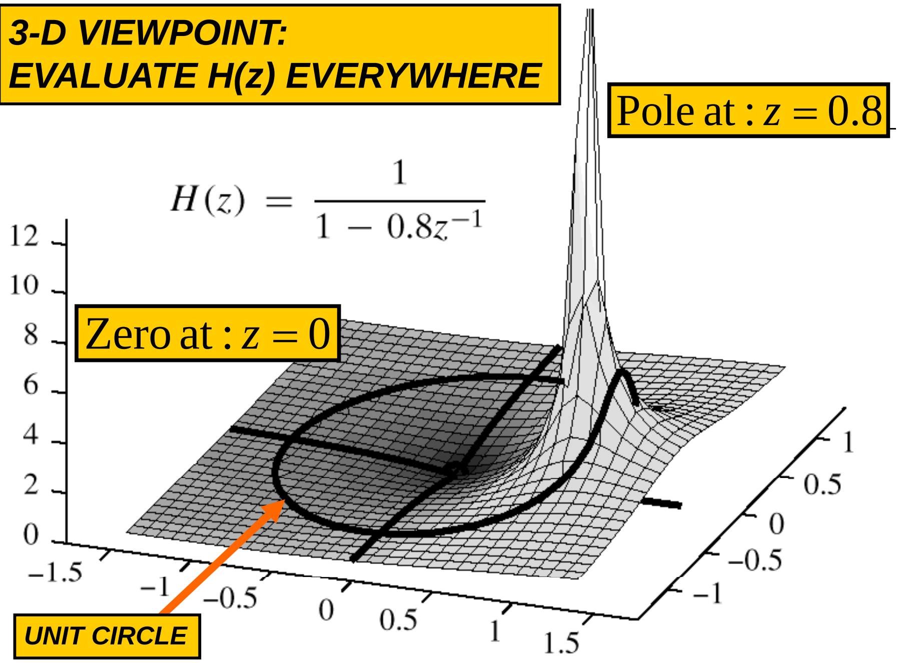
- Roots of Numerator & Denominator Polys:

$$H(z)=\frac{1+b_1z^{-1}}{1-0.8z^{-1}}$$

$$H(z)=\frac{z(1+b_1z^{-1})}{z(1-0.8z^{-1})}=\frac{z+b_1}{z-0.8}$$

$$\text{Pole at}: z=0.8 \qquad \text{Zero at}: z=-b_1$$

3-D VIEWPOINT:
EVALUATE H(z) EVERYWHERE



FREQ. RESPONSE from H(z)

$$H(e^{j\hat{\omega}})=H(z)\Big|_{z=e^{j\hat{\omega}}}$$

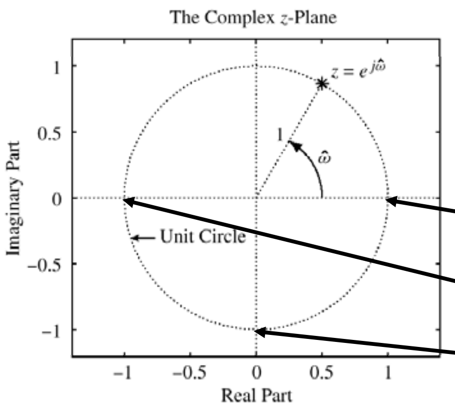
- Relate H(z) to FREQUENCY RESPONSE
- EVALUATE H(z) on the **UNIT CIRCLE**
  - ANGLE is same as FREQUENCY

$$z=e^{j\hat{\omega}} \quad (\text{as } \hat{\omega} \text{ varies})$$

$$\text{defines a CIRCLE, radius} = 1$$

UNIT CIRCLE: RECAP

- MAPPING BETWEEN z and $\hat{\omega}$



$$z = e^{j\hat{\omega}}$$

$$z = 1 \leftrightarrow \hat{\omega} = 0$$

$$z = -1 \leftrightarrow \hat{\omega} = \pm\pi$$

$$z = \pm j \leftrightarrow \hat{\omega} = \pm\tfrac{1}{2}\pi$$

IIR H(z) example: two poles

- Poles just inside the unit circle (for stability)

$$H(z) = \frac{1}{1 + 0.97z^{-1} + 0.9409z^{-2}}$$

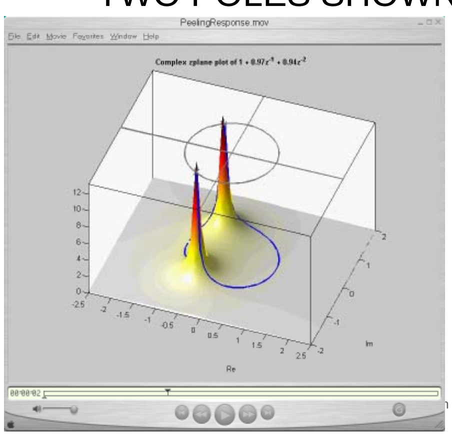
$$2 \text{ Poles}: z = 0.97e^{\pm j2\pi/3}$$

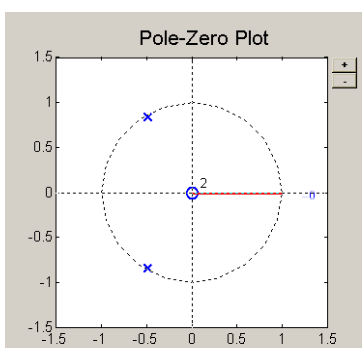
$$2 \text{ Zeros}: z = 0,0$$

- MATLAB: **`roots( )`** and **`poly( )`**
  - **`roots( [1, 0.97, 0.9409] )`**
  - **`poly( 0.97*exp(j*2*pi*[1,-1]/3) )`**

MOVIE for H(z) in 3-D

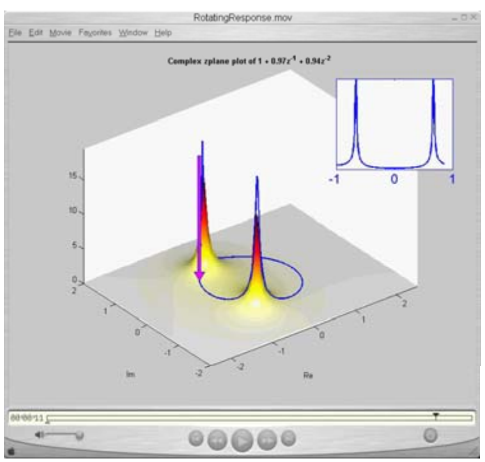
- POLES to H(z) to Frequency Reponse
  - TWO POLES SHOWN





Frequency Response from H(z)

Walking around the Unit Circle



FIR H(z) has "ONLY ZEROS"

- FIR H(z) has NO DENOMINATOR, e.g.

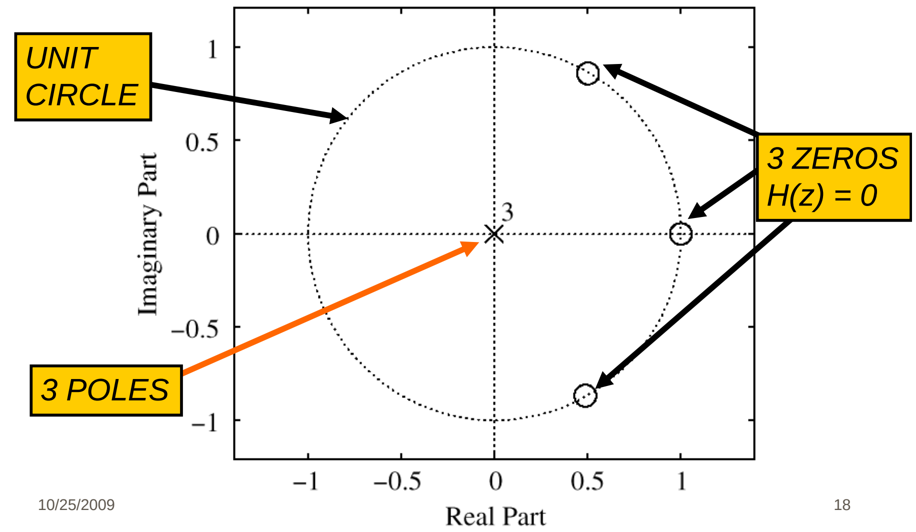
$$H(z) = 1 - 2z^{-1} + 2z^{-2} - z^{-3}$$

$$H(z) = (1 - z^{-1})(1 - z^{-1} + z^{-2}) = \frac{(z-1)(z^2 - z + 1)}{z^3}$$

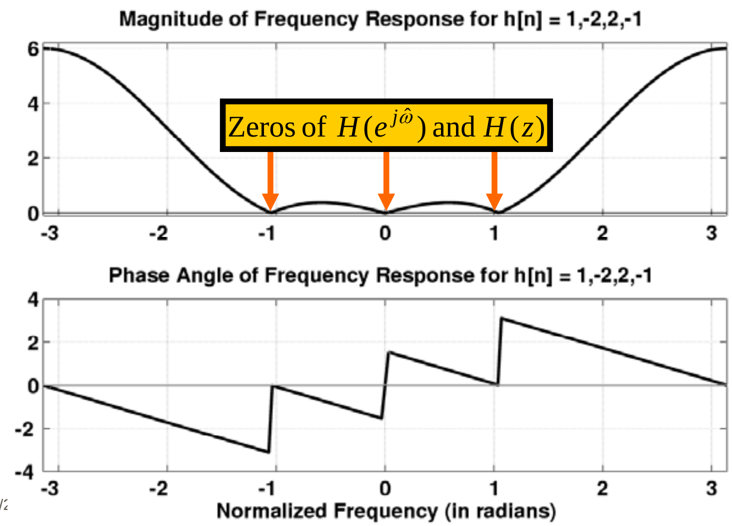
$$3\ \text{Zeros}: z = 1, \tfrac{1}{2} \pm j\tfrac{\sqrt{3}}{2} \qquad e^{\pm j\pi/3}$$

$$3\ \text{Poles}: z = 0,0,0$$

Pole-Zero Plot in z-PLANE



FIR Frequency Response

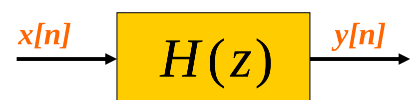


NULLING PROPERTY of H(z)

- When H(z)=0 on the unit circle.
  - Find various inputs x[n] that give zero output

$$H(z) = 1 - 2z^{-1} + 2z^{-2} - z^{-3}$$

$$H(e^{j\hat{\omega}}) = 1 - 2e^{-j\hat{\omega}} + 2e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}}$$

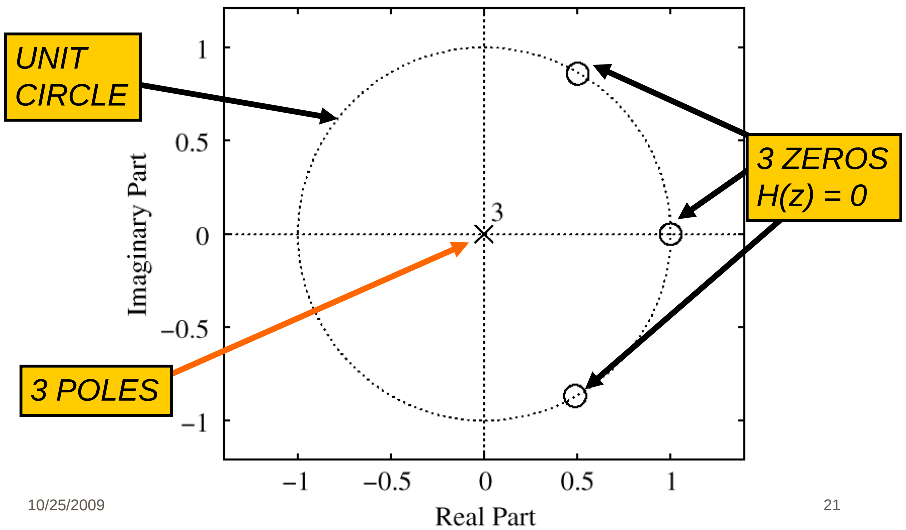


$$H(e^{j\pi/3}) = ?$$

$$x[n] = e^{j(\pi/3)n}$$

$$y[n] = H(e^{j(\pi/3)}) \cdot e^{j(\pi/3)n}$$

PLOT ZEROS in z-DOMAIN



NULLING PROPERTY of H(z)

- Evaluate H(z) at the input “frequency”

$$H(e^{j\hat{\omega}}) = 1 - 2e^{-j\hat{\omega}} + 2e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}}$$

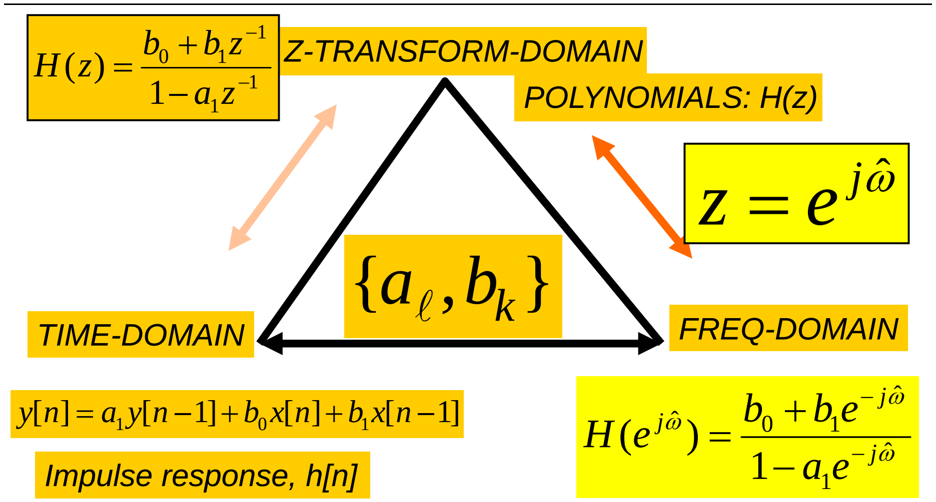
$$y[n] = H(e^{j\pi/3}) \cdot e^{j(\pi/3)n}$$

$$y[n] = (1 - 2e^{-j\pi/3} + 2e^{-j2\pi/3} - e^{-j3\pi/3}) \cdot e^{j(\pi/3)n}$$

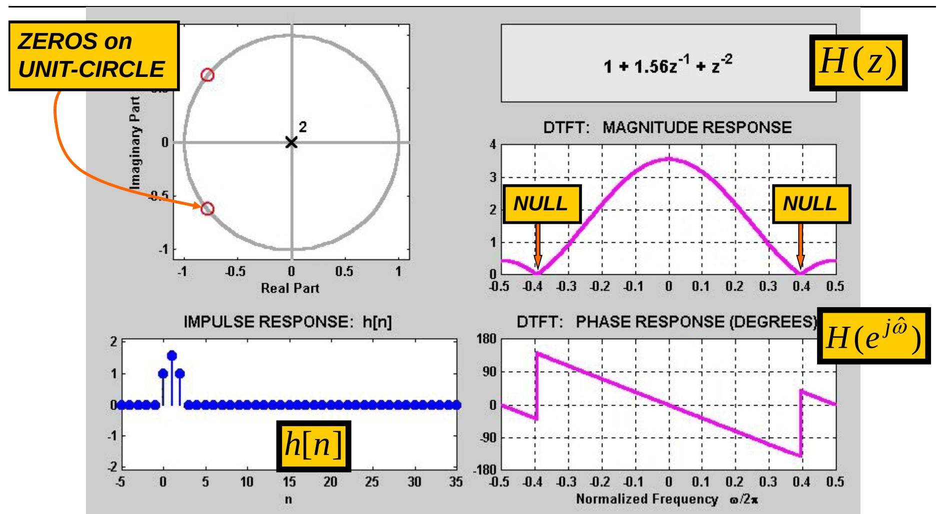
$$(1 - 2(\tfrac{1}{2} - j\tfrac{\sqrt{3}}{2}) + 2(-\tfrac{1}{2} - j\tfrac{\sqrt{3}}{2}) - (-1))$$

$$y[n] = (1 - 1 + j\sqrt{3} - 1 - j\sqrt{3} + 1) \cdot e^{j(\pi/3)n} = 0$$

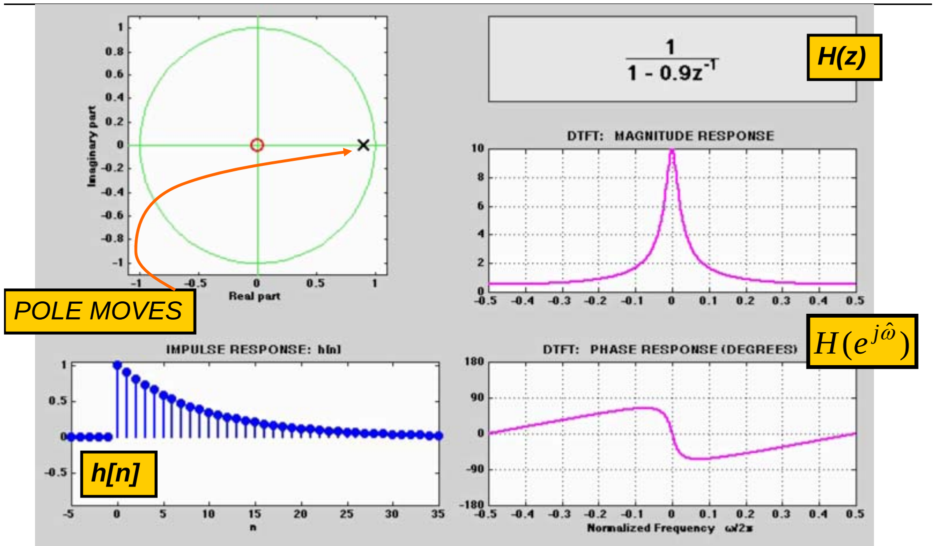
THREE DOMAINS: $H(e^{j\hat{\omega}})$



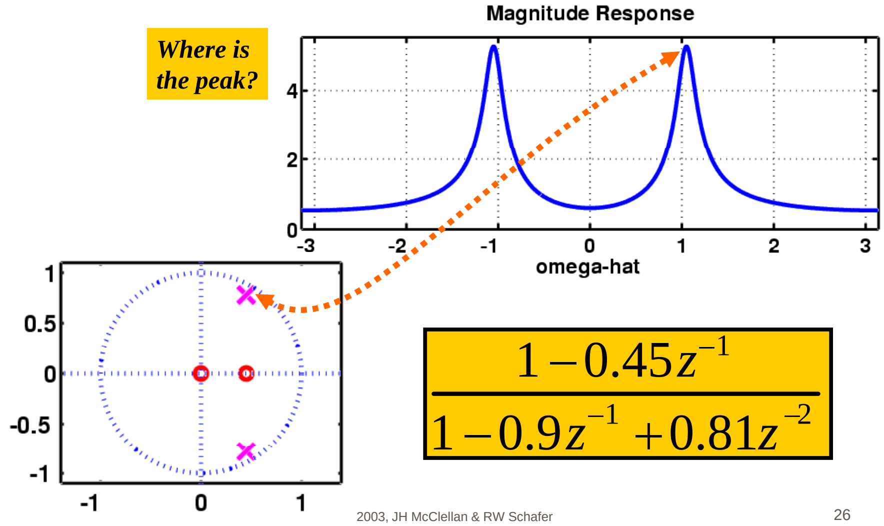
3 DOMAINS MOVIE: FIR



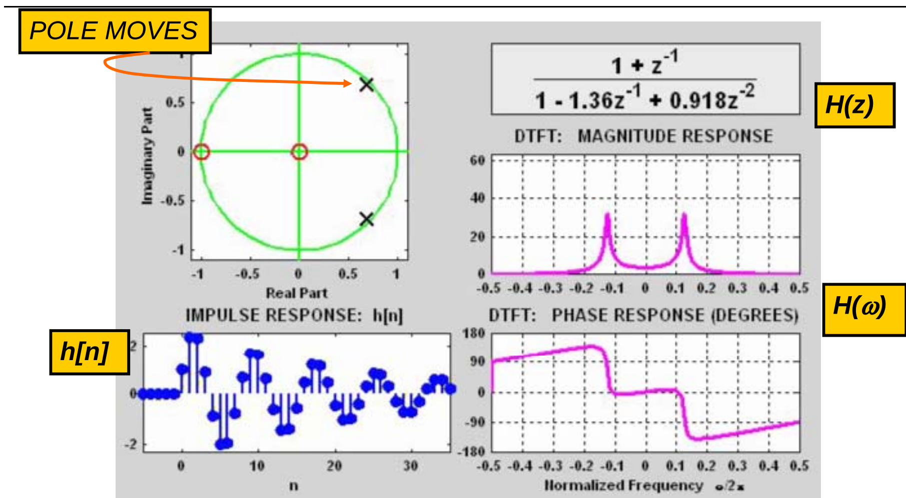
3 DOMAINS MOVIE: IIR



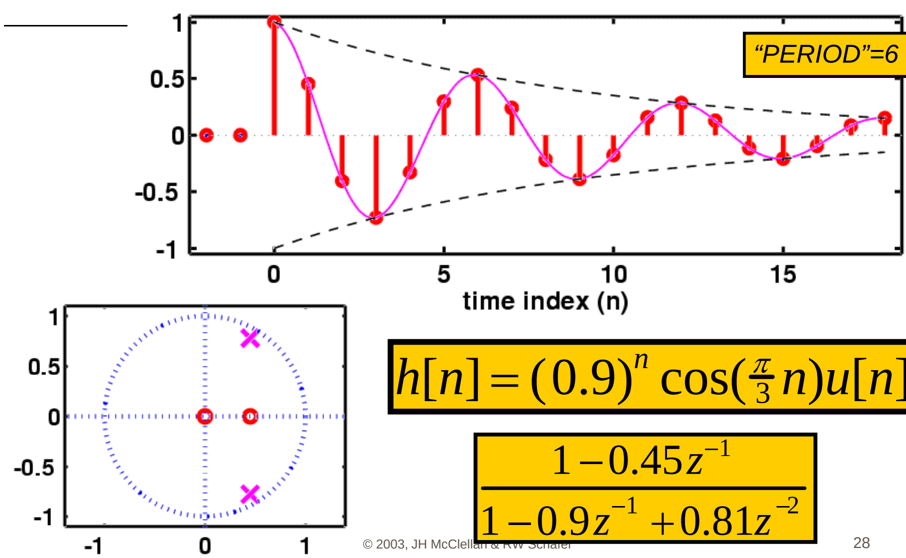
Complex POLE-ZERO PLOT



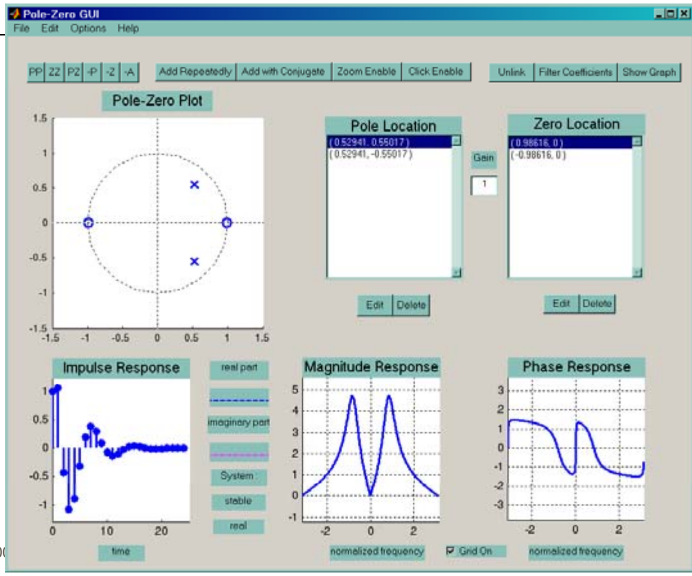
3 DOMAINS MOVIE: IIR



h[n]: Decays & Oscillates



PeZ Demo: Pole-Zero Placing



DESIGN PROBLEM

- Example:
  - Design a Lowpass FIR filter (Find b_k)
  - Reject completely 0.7π, 0.8π, and 0.9π
    - This is NULLING
  - Estimate the filter length needed to accomplish this task. How many b_k ?
- Z POLYNOMIALS provide the TOOLS

NULLING FILTER

- PLACE ZEROS to make y[n] = 0

$$H(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4} + b_5 z^{-5} + b_6 z^{-6}$$

Needs 6 ZEROS where H(z) = 0

$$H(z_k) = 0, \quad \text{for } z_k = e^{\pm j0.7\pi}, e^{\pm j0.8\pi}, e^{\pm j0.9\pi}$$

$$x[n] = e^{j0.8\pi n} \quad \Rightarrow \quad y[n] = H(e^{j0.8\pi})e^{j0.8\pi n}$$

PeZ Demo: Zero Placing