GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2009
Lab #6: A/D and D/A Conversion

Date: 29-Sept. – 12-Oct. 2009

You should read the Pre-Lab section of the lab and do all the exercises in the Pre-Lab section before your assigned lab time.

This Lab will be done entirely in-Lab. It will count 50 points.
Your score will be based on the answers that you write on the Instructor Verification sheet. During this warm-up you should work alone.

Forgeries and plagiarism are a violation of the honor code and will be referred to the Dean of Students for disciplinary action. You are allowed to discuss lab exercises with other students and you are allowed to consult old lab reports, but you cannot give or receive written material or electronic files. Your submitted work should be original and it should be your own work.

1 Introduction

The objective of this lab is to study sampling and aliasing with sinusoids. We will use a MATLAB GUI for sampling and aliasing, called **con2dis**, which tracks an input sinusoid and its spectrum through A/D and D/A converters. This demo is part of the *SP-First* Toolbox.

2 Pre-Lab

2.1 Sampling and Aliasing GUI

A primary objective of this lab is to study sampling and aliasing by using the **con2dis** GUI. If you have installed the *SP-First* Toolbox, you will already have this demo on the `matlabpath`. In Fig. 1 there are three signals: the input $x(t)$, the discrete-time signal $x[n]$ which is a sampled version of $x(t)$, and the output signal which is reconstructed from $x[n]$. Each of these time signals also has a spectrum, so there are six plots to study.

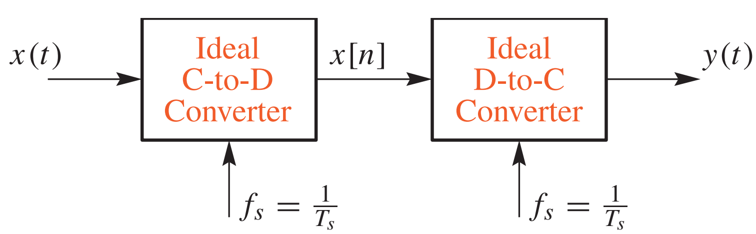

Figure 1: Ideal sampling and reconstruction systems. An ideal C-to-D converter samples $x(t)$ with a sampling period $T_s = 1/f_s$ to produce the discrete-time signal $x[n]$. The ideal D-to-C converter then forms a continuous-time signal $y(t)$ from the samples $x[n]$.

The **con2dis** GUI shows the three time signals and their spectra. Figure 2 shows the interface for the **con2dis** GUI. It has six panels: the top three are the time signals, the bottom three are the spectra. From left to right the signals are the input $x(t)$, the sampled signal $x[n]$, and the output $y(t)$. On the front-panel of the GUI, you can change the frequency of the input (sinusoidal) signal, and you can change the sampling frequency (f_s). Then the GUI will update all three time signals and their spectra. In this GUI, the sampling rate (f_s) is the same for both the C-to-D and D-to-C converters.

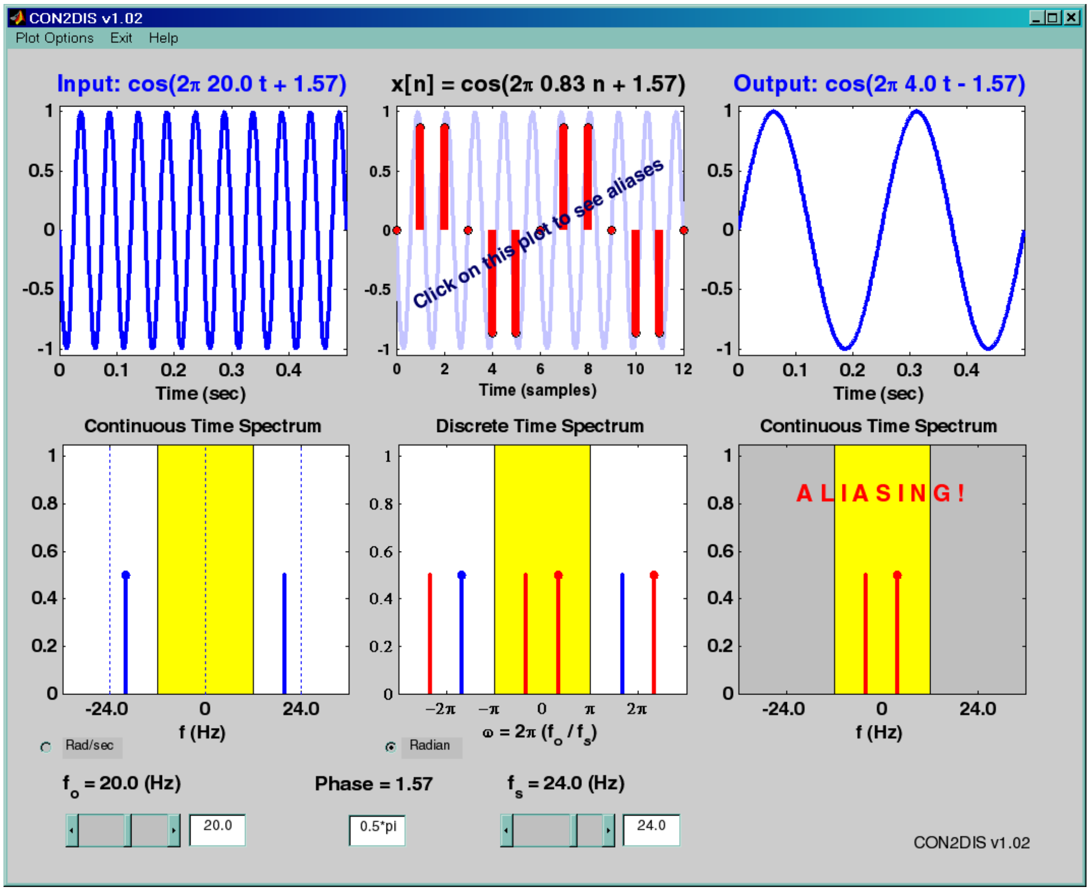

Figure 2: The **con2dis** MATLAB GUI interface. The top three panels show the time-domain signals; the bottom three the corresponding frequency-domain spectra.

2.2 Mathematical Relationships

The ideal C-to-D converter can be described in both the time domain and the frequency domain:

$$\text{Time Domain: } x[n] = x(n/f_s) \qquad \text{Frequency Domain: } \hat{\omega} = 2\pi \frac{f_0}{f_s} + 2\pi\ell$$

where f_0 is the input frequency, f_s the sampling rate, and ℓ is an integer. The $2\pi\ell$ term accounts for aliasing. Likewise, the ideal D-to-C converter can be described in both domains.

$$\text{Time Domain: } y(t) = x[nf_s] \qquad \text{Frequency Domain: } \omega = 2\pi f = \hat{\omega} f_s$$

where f is the output frequency, and $\hat{\omega}$ is the frequency of the spectral line closest to $\hat{\omega} = 0$.

2.3 Run the GUI

In the pre-Lab, you should perform the following steps with the **con2dis** GUI:

(a) Set the input to $x(t) = \cos(40\pi t + 0.5\pi)$. Determine the *Nyquist* rate for sampling this signal.

(b) Set the sampling rate to $f_s = 24$ samples/sec. Notice that this rate is too low to satisfy the Nyquist condition. Thus the output signal is not equal to the input.

(c) Determine the locations of the spectrum lines for the discrete-time signal, $x[n]$, found in the middle panels. Make sure that the **Radian** button is active so that the frequency axis for the discrete-time signal is $\hat{\omega}$.

(d) Determine the complex amplitudes for the spectral lines found in the previous part. Notice that a $*$ on top of a spectral line indicates a line that was originally a negative frequency component in the input signal.

(e) Determine the formula for the output signal, $y(t)$, shown in the rightmost panels. What is the output frequency in Hz?

3 Warm-up

The instructor verification sheet may be found at the end of this lab. ***For this lab, the verification requires that you write down your observations on the verification sheet when using the GUI. These written observations should be expressed clearly because they will be graded.***

Use ITS: For each part there is a corresponding multiple choice question in ITS that must be answered. These questions should be helpful and should guide you to the most likely answer.

3.1 Differences between the Frequency Domains: ω and $\hat{\omega}$

Here are some issues to keep in mind:

- Is the question asking about a continuous-time signal such as $x(t)$ or $y(t)$, or a discrete-time signal, $x[n]$?
- The frequency axis (ω) for the spectrum of a continuous-time signal is different from the frequency axis ($\hat{\omega}$) of a discrete-time signal, notably in the range of frequencies. The $\hat{\omega}$-axis has a primary section that goes from $\hat{\omega} = -\pi$ to $\hat{\omega} = \pi$; then the spectrum repeats every 2π.
- Thus the spectrum of a discrete-time signal will have many spectral lines separated by 2π.

Note: read Sections 4-1 and 4-2 in Chapter 4 for more information about the spectra of discrete-time signals, and for information about aliasing.

3.2 Sampling and Aliasing

Use the `con2dis` GUI to do the following exercises. The parameters of the *continuous-time input* signal are its frequency f_0 in Hz, and its phase φ in rads; the amplitude of $x(t)$ is always one. The sampling rate of the A/D converter and the D/A converter are identical: f_s in samples/sec (or hertz).

In all cases, write a concise explanation of your answer. "Trial and error" is not a legitimate justification, so try to write something based on the theory you have learned.

(a) Set the input frequency to $f_0 = 13.4$ Hz. Determine the smallest integer value of the sampling rate f_s so that no aliasing occurs. The units of f_s are samples per second. You must justify your response by citing a theorem or property about sampling.

(b) Set the input frequency to $f_0 = 13$ Hz and the input phase to $\varphi = -1.3$ rads. Determine the locations of the spectral lines in the spectrum of the discrete-time signal (middle plot of the bottom row) when the sampling rate is $f_s = 29$ Hz. Calculate $\hat{\omega}$ for the two spectral lines that lie in the range $-\pi \leq \hat{\omega} \leq \pi$. Explain how you calculated the two values for $\hat{\omega}$, i.e., give the formula for $\hat{\omega}$ in terms of f_0 and f_s.

(c) *Folded Alias:* Set the input frequency to $f_0 = 13$ Hz, the input phase to $\varphi = -1.3$ rads, and $f_s = 20$ Hz. Write down the formula for the output signal, and then write a justification consisting of three steps:

(1) calculating the values of $\hat{\omega}$ for the blue spectral lines in the spectrum of the discrete-time signal $x[n]$ shown in the middle plots,
(2) aliasing $\hat{\omega}$ (blue to red), and
(3) transforming $x[n]$ into $y(t)$ using an equation that describes the ideal D-to-C converter and uses the *principal alias*.

(d) Set the sampling rate to $f_s = 23$ Hz, and assume that the output signal has a frequency of 9 Hz, and a phase of $+1.3$ rads. Determine ***three different values of the input frequency*** that will give this output signal, and which lie in the range of the **`con2dis`** GUI, i.e., $[\,0, 34.2\,]$ Hz. In addition, determine the corresponding value of the input phase φ for each input frequency. Assume that the amplitudes are always one.

(e) Set the input frequency to $f_0 = 13$ Hz and the input phase to $\varphi = -1.3$ rads. ***Determine the sampling rate*** f_s so that the output signal has a frequency of 9 Hz, and a phase of $+1.3$ rads. Trial and error helps in getting the answer, but you must write an explanation based on theory.

Lab #6

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WORKSHEET & VERIFICATION PAGE

For each verification, be prepared to explain your answer and respond to other related questions that the lab TA's or professors might ask. Turn this page in at the end of your lab period.

Name: ____ Date of Lab: __________

Part	Observations, Explanations, and Justifications
3.2(a)	Minimum sampling rate for no aliasing is $f_s =$
3.2(a)	Justification:
3.2(b)	$\hat{\omega} =$ Explain:
3.2(c)	Output signal: $y(t) =$
3.2(c)	Explain 3 steps:
3.2(d)	Three different input signals (frequency and phase):
3.2(e)	$f_s =$ Explain:

ITS completed: Verified:____________________ Date/Time:__________