GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

**ECE 2025 Fall 2009**

**Lab #9: Filter Design: IIR Bandpass Filters**

Date: 2 Oct. – 2-Nov. 2009

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**You should read the Pre-Lab section of the lab and do all the exercises in the Pre-Lab section before your assigned lab time.**

**ITS:** When you come to the lab, you **must** answer the online ITS questions. You can use MATLAB or any notes you might have but you cannot discuss the exercises with any other students.

**Verification:** The Warm-up section of each lab must be completed **during your assigned Lab time** and the steps marked *Instructor Verification* must also be signed off **during the lab time**. When you have completed a step that requires verification, simply raise your hand and demonstrate the step to the TA or instructor. After completing the warm-up section, turn in the verification sheet to your TA *before leaving the lab*.

It is only necessary to turn in Section 4 as the lab report for this lab. More information on the lab report format can be found on `t-square` under the **INFO** link. Please ***label*** the axes of your plots and include a title and Figure number for every plot. In order to reduce *orphan plots*, include each plot as a figure *embedded* within your report. This can be done easily with MATLAB's `notebook` capability. For more information on how to include figures and plots from MATLAB in your report file, consult the **INFO** link on `t-square`, or ask your TA for details.

***Forgeries and plagiarism are a violation of the honor code and will be referred to the Dean of Students for disciplinary action. You are allowed to discuss lab exercises with other students and you are allowed to consult old lab reports, but you cannot give or receive written material or electronic files. Your submitted work should be original and it should be your own work.***

The report will be **due during the period 3–9-Nov. at the start of your lab.**

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## 1 PeZ: Introduction

In order to build an intuitive understanding of the relationship between the location of poles and zeros in the $z$-domain, the impulse response $h[n]$ in the $n$-domain, and the frequency response $H(e^{j\hat{\omega}})$ (the $\hat{\omega}$-domain), a graphical user interface (GUI) program called **PeZ** was written in MATLAB for doing interactive explorations of the three domains.[1] **PeZ** is based on the system function, represented as a ratio of polynomials in $z^{-1}$, which can be expressed in either factored or expanded form as:

$$H(z) = \frac{B(z)}{A(z)} = G\,\frac{\prod_{k=1}^{M}(1 - z_k z^{-1})}{\prod_{\ell=1}^{N}(1 - p_\ell z^{-1})} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{\ell=1}^{N} a_\ell z^{-\ell}} \tag{1}$$

where $M$ is the number of zeros and $N$ the number of poles.

The **PeZ** GUI is contained in the *SP-First* toolbox. To run **PeZ**, type `pezdemo` at the command prompt and you will see the GUI shown in Fig. 1.[2]

[1]The original **PeZ** was written by Craig Ulmer; a later version by Koon Kong is the one that we will use in this lab. Recent modifications by Greg Krudysz have added new features such as movie-making capability.

[2]The command `pez` will invoke the older version of **PeZ** which is distinguished by a black background in all the plot regions.

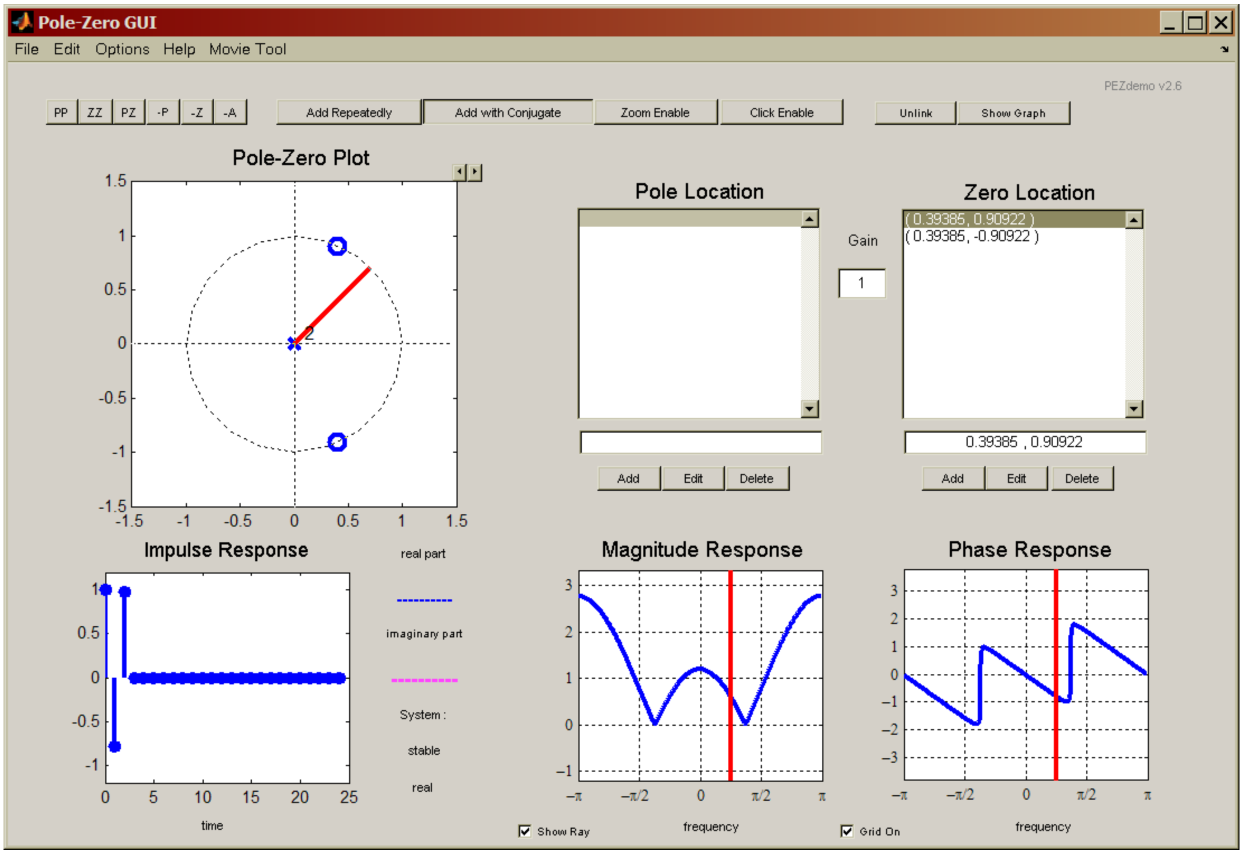


Figure 1: GUI interface for pezdemo running in MATLAB version 6. A length-3 FIR filter is shown. Zero locations are given in rectangular coordinates.

### 1.1 Controls for PeZ using `pezdemo`

The **PeZ** GUI is controlled by the `Pole-Zero Plot` where the user can add (or delete) poles and zeros, as well as move them around with the pointing device. For example, Fig. 1 shows a second-order FIR filter with two zeros, while Fig. 2 shows a case where two (complex-conjugate) poles have been added, along with two zeros on the unit circle. The buttons named `PP` and `ZZ` were used to add these poles and zeros. By default, the `Add with Conjugate` property is turned on, so poles and zeros are typically added in pairs to satisfy the complex-conjugate property:

A polynomial with real coefficients has roots that are real, or occur in complex-conjugate pairs.

To learn about the other controls in pezdemo, access the menu item called "Help" for extensive information about all the **PeZ** controls and menus. Here are a few things to try. You can use the Pole-Zero Plot to selectively place poles and zeros in the $z$-plane, and then observe (in the other plots) how their placement affects the impulse and frequency responses. In **PeZ** an individual pole/zero pair can be moved around and the corresponding $H(e^{j\hat{\omega}})$ and $h[n]$ plots will be updated as you drag the pole (or zero). The red ray in the $z$-domain window is tied to the red vertical lines on the frequency responses, and they move together. This helps identify frequency domain features that are caused by pole locations or zero locations, because the angle around the unit circle corresponds to frequency $\hat{\omega}$. Since exact placement of poles and zeros with the mouse is difficult, an `Edit` button is provided for numerical entry of the real and imaginary parts. Before you can edit a pole or zero, however, you must first select it in the list of `Pole Locations` or `Zero Locations`. Removal of individual poles or zeros can also be performed by using the `-P` or `-Z` buttons, or with the `Delete` button. Note that all poles and/or zeros can be easily cleared by clicking on the `-A` button.

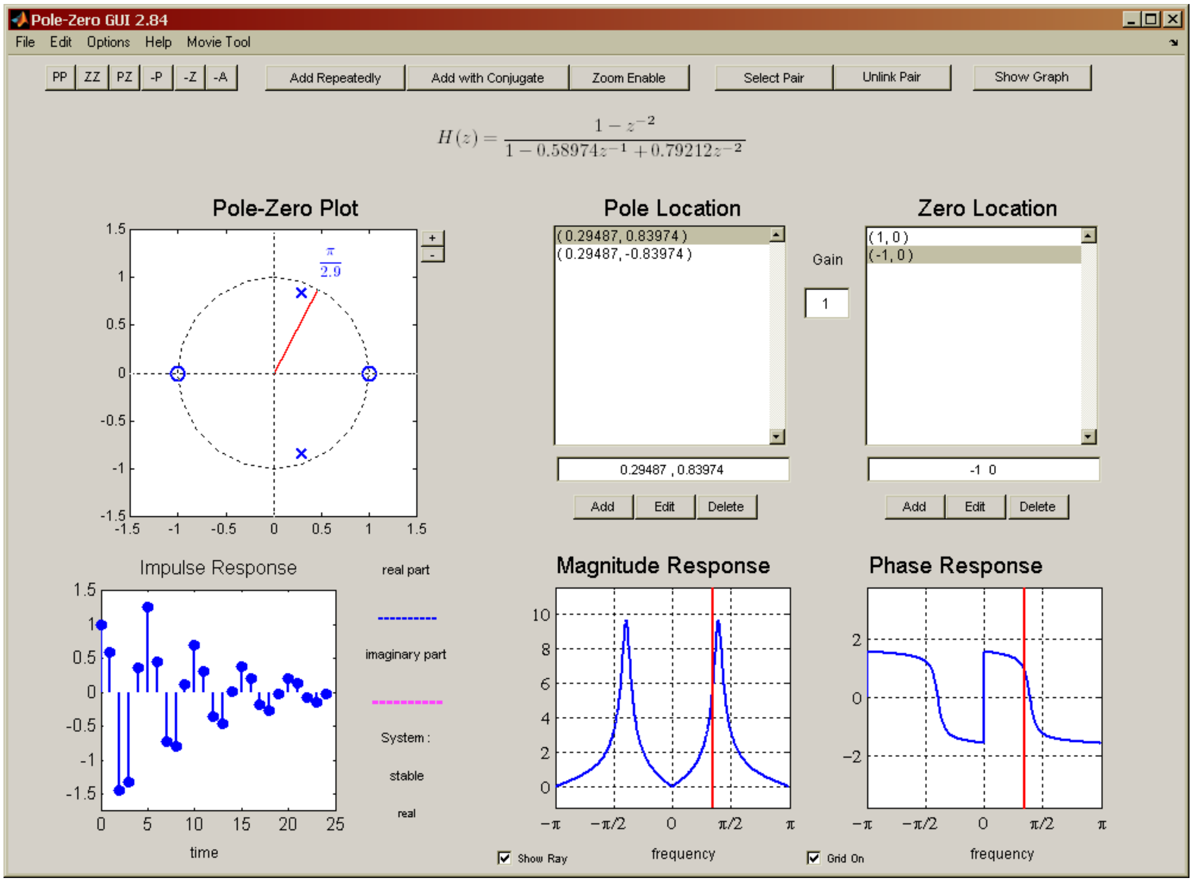


Figure 2: GUI interface for pezdemo showing a second-order filter. Pole and zero locations are given in rectangular coordinates. The "Gain" could be adjusted to make the peak of the frequency response equal to one in the passbands.

## 2 Pre-Lab

Try various operations with **PeZ** to gain some familiarity with the interface. Here are two suggested filters that you can create.

### 2.1 Create an FIR Filter with PeZ

Implement the following FIR system:

$$H(z) = 1 - z^{-1} + z^{-2}$$

by factoring the polynomial and placing the two zeros correctly. Observe the following two facts:

- The impulse response $h[n]$ values (for FIR) are equal to the polynomial coefficients of $H(z)$.
- The frequency response has nulls because the zeros of $H(z)$ lie exactly on the unit circle. Compare the frequencies of the nulls to the angles of the zeros

Move the zero-pair around the unit circle and observe that the location of the null also moves.

### 2.2 Create an IIR Filter with PeZ

Implement the following first-order IIR system:

$$H(z) = \frac{1 - z^{-1}}{1 + 0.9z^{-1}}$$

by placing its pole and zero at the correct locations in the $z$-plane. First try placing the pole and zero with the mouse, and then use the `Edit` feature to get exact locations. Since **PeZ** wants to add complex-conjugate pairs, you might have to delete one of the poles/zeros that were added; or you can turn off the `Add with Conjugate` feature. Look at the frequency response and determine what kind of filter you have.

Now, use the mouse to "grab" the pole and move it from $z = -0.9$ to $z = +0.8$. To move along the real axis, you can use `Options -> Move on Real Line` from the GUI menu. Observe how the frequency response changes. Describe the type of filter that you have created (i.e., HPF, LPF, or BPF).

### 2.3 Create a Second-Order IIR Filter with PeZ

Use the **PeZ** interface to implement the following second-order system:

$$H(z) = \frac{1 - z^{-2}}{1 + 0.8z^{-1} + 0.64z^{-2}}$$

by determining where the two poles and two zeros are located and then placing the poles and zeros at the correct locations in the $z$-plane. First try placing the poles and zeros with the mouse, and then use the `Edit` feature to get exact locations. Since **PeZ** wants to add complex-conjugate pairs, you should only have to add one of the poles; for the zeros, the `Add with Conjugate` feature should be turned off because you will be adding two real-valued zeros.

Look at the frequency response and determine what kind of filter you have.

### 2.4 Not Always Bandpass Filters

It is tempting to think that with two poles the frequency response ends up always having a peak, but there are two interesting cases where that doesn't happen: (1) all-pass filters where $|H(e^{j\hat{\omega}})| = \text{constant}$, and (2) IIR notch filters that null out one frequency, but are relatively flat across the rest of the frequency band.

• Implement the following second-order system:

$$H(z) = \frac{64 + 80z^{-1} + 100z^{-2}}{1 + 0.8z^{-1} + 0.64z^{-2}}$$

by determining where the two poles and two zeros are located and then placing the poles and zeros at the correct locations in the $z$-plane.
⇒ Look at the frequency response and determine what kind of filter you have.[3]

• Now, use the mouse to "grab" the zero-pair and move the zeros to be exactly on the unit-circle at the same angle as the poles. Observe how the frequency response changes. In addition, determine the $H(z)$ for this filter.

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + 0.8z^{-1} + 0.64z^{-2}}$$

⇒ Describe the type of filter that you have now created.

## 3 Warm-up

In this part of the lab, you will use **PeZ** to create filters with complex conjugate poles and zeros. These are called *second-order filters* because the numerator and/or denominator polynomial is a quadratic with two roots.

***The lab verification requires that you write down your observations on the verification sheet when using the PeZ GUI. These written observations will be graded.***

[3] The relationship between the poles and zeros of an all-pass filter is zero = 1/(pole)*; this situation where two poles and two zeros are linked together can be done with the `PZ` option in **PeZ**.

## 3.1 Relationships between $z$, $n$, and $\hat{\omega}$ domains

Work through the following exercises and keep track of your observations by filling in the worksheet at the end of this assignment. In general, you want to make note of the following quantities:

- Is the length of $h[n]$ finite or infinite, i.e., FIR or IIR?
- *Frequency Domain:* How does $H(e^{j\hat{\omega}})$ change with respect to null location, peak location and/or peak width?
- *Time Domain:* How does $h[n]$ change with respect to its rate of decay for IIR filters? For example, when the impulse response is of the form $h[n] = a^n u[n]$, the impulse response will fall off more rapidly when $a$ is smaller; and if $|a| > 1$ the impulse response blows up.
- *Time Domain:* For second-order IIR filters, $h[n]$ will exhibit an oscillating component, what is the period of oscillation? Also, estimate the decay rate of the "envelope" that overlays the oscillation.

Note: review the "Three-Domains - FIR" under the Demos link for Chapter 7 and "Three-Domains - IIR" under the Demos link for Chapter 8 for movies and examples of these relationships.

### 3.1.1 Real Poles

(a) Use **PeZ** to place a single pole at $z = -\frac{1}{2}$. You may have to use the `Edit` button to get the location exactly right. Describe the important features of the impulse response $h[n]$, and also the important features of the frequency response $H(e^{j\hat{\omega}})$. Furthermore, use the plots in **PeZ** for this case as the reference for answering the next two parts.

(b) Move the pole slowly from a location close to the origin out to $z = -\frac{1}{2}$, and then to out to $z = -0.99$. Stay on the real axis by using `Options -> Move on Real Line` from the GUI menu. Observe the changes in the impulse response $h[n]$ and the frequency response $H(e^{j\hat{\omega}})$. Record your observations on the Verification Sheet.

When you move poles and zeros, the impulse response and frequency response plots are updated continually in **PeZ**. Select the pole you want to move and start to drag it slowly. At the same time, watch for the update of the plots in the impulse response and frequency response panels.

(c) Move the pole outside the unit circle. Describe the changes in $h[n]$. Explain how the appearance of $h[n]$ validates the statement that the system is not stable. In this case, the frequency response $H(e^{j\hat{\omega}})$ is not legitimate because the system is no longer stable.

(d) In general, where should poles be placed to guarantee system stability? By stability we mean that the system's output does not blow up as $n \to \infty$.

**Instructor Verification** (separate page)

## 3.2 Complex Poles and Zeros

**PeZ** assumes real coefficients for the numerator and denominator polynomials when the `Add with Conjugate` mode is on (which it is by default). Therefore, if we enter a complex pole or zero, **PeZ** will automatically insert a second root at the conjugate location, i.e., $z = \frac{1}{3} + j\frac{1}{2}$ would be accompanied by $z = \frac{1}{3} - j\frac{1}{2}$.

(a) State the property of the polynomial coefficients of $A(z) = 1 - a_1 z^{-1} - a_2 z^{-2}$ that will guarantee that the two roots of $A(z)$ are either both real, or are complex conjugates of each other.

(b) Clear all the poles and zeros from **PeZ**. Now place a zero pair at $z = \pm j$. Determine the radius and angle of the zeros. Note that **PeZ** automatically places a conjugate pole in the $z$-domain. The frequency response has a null—record the frequency (location) of this null.

(c) Grab the zero and move it around the unit circle; notice how the frequency response changes. Suppose that you want a null in the frequency response at $\hat{\omega} = \pm\pi/4$, determine the zero locations.

(d) Clear all the poles and zeros from **PeZ**. Now place a pole pair at $-0.3 \pm j0.8$, and zeros at $z = \pm 1$. Note that **PeZ** automatically places a conjugate pole in the $z$-domain. Determine the radius and angle of the poles. The frequency response has a peak—record the frequency (location) of this peak.

(e) Write out the expression for $H(z)$ created in the previous part. *Hint:* use MATLAB's `poly` function.

(f) Change the angle of the pole and observe $H(e^{j\hat{\omega}})$: move the pole-pair angles to $(\pm 90°)$, then to $(\pm 60°)$ and $(\pm 45°)$. Describe the changes in $|H(e^{j\hat{\omega}})|$. Concentrate on the height and location of the peak versus frequency $\hat{\omega}$.

(g) Start again with the pole pair at $-0.3 \pm j0.8$, and zeros at $z = \pm 1$. Decrease the radial distance of the poles from the origin (by dragging), e.g., try $-0.2 \pm j0.533$, and then $-0.1 \pm j0.266$. If you use the **`Pole Location`** edit window to change the values, the two poles will be "unlinked" and you will have to edit them separately. Therefore, dragging is a more informative way to do this even though it's less precise. Describe the changes in both $h[n]$ and $|H(e^{j\hat{\omega}})|$, as you reduce the pole radius.

(h) Increase the magnitude of the poles by pushing them closer to the unit circle, and then move the poles outside the unit circle. When the pole-pair is outside the unit circle, describe what happens to $h[n]$.

**Instructor Verification** (separate page)

# 4 Lab Exercise: Bandpass Filter Design for IIR

It is easy to design a narrow passband IIR filter by putting a complex pole-pair near the unit circle.

## 4.1 Complex Poles

The first exercise is to move one pole-pair around and obtain formulas for how the frequency response changes as a function of the pole-pair radius and angle.

(a) Place a single pole-pair at $z = 0.92e^{\pm j0.3\pi}$, and zeros at $z = \pm 1$. Then determine the coefficients of the numerator and denominator of the resulting $H(z)$.

(b) Make a plot of the frequency response (magnitude only) with `freqz` and measure the width of the peak versus frequency. This presents a problem because we must define how to measure width. The usual definition is to measure the width at the "3-dB level." In order to do this, the measurement must be made with respect to the peak value of the frequency response. If the peak value is $H_{\max}$, then the "3-dB level" is at $0.707H_{\max}$.[4]

(c) Move the pole-pair so that the angles remain fixed at $\pm 0.3\pi$, but the radius moves to $r = 0.96$, $r = 0.98$ and $r = 0.99$. In each case, measure the 3-db width of the peak. Using these measured values, create a formula for the width that is proportional to $(1-r)$, e.g., the following works quite well

$$\text{PeakWidth} \approx K(1-r)/\sqrt{r}$$

where $K$ is a constant of proportionality.

(d) Move the pole-pair so that its radius remains fixed and the angles change from $\pm 0.3\pi$ to $\pm 0.2\pi$ and then to $\pm 0.4\pi$. State a formula for the peak location as a function of the pole location.

[4] The frequency response is often plotted on a logarithmic scale using decibels, i.e., $20\log_{10}|H(e^{j\hat{\omega}})|$. If you compute $20\log_{10}(1/\sqrt{2})$ you get $-3.01$ dB, and $1/\sqrt{2} \approx 0.707$.

## 4.2 Passband and Stopband

We can characterize general bandpass filters if we define the passband width to be equal to the 3-dB width (as in the previous section). We also need a a definition for the stopband, and we will arbitrarily define the stopbands of a BPF to be those regions where the frequency response (magnitude) is below −20 db, which is equivalent to 10% of the peak value.

(a) Determine the stopband regions for three of the filters designed in the previous section. Use the cases where the pole angles are $\pm 0.3\pi$ and the radii are $r = 0.92, 0.96, 0.98$ and $0.99$. In each case, measure the frequency regions of the two stopbands. There is one lower stopband for $0 \le \hat{\omega} \le \hat{\omega}_{s1}$ and one upper stopband for $\hat{\omega}_{s2} \le \hat{\omega} \le \pi$.

(b) For the same four filters, record the passband edges. The passband will be the peak width at the 3-dB level, so it will occupy a region such as $\hat{\omega}_{p1} \le \hat{\omega} \le \hat{\omega}_{p2}$, where $\hat{\omega}_{p1}$ and $\hat{\omega}_{p2}$ are the band edges.

(c) Usually, filter design becomes difficult when we want the passband and stoband edges to be very close to one another. The difference between neighboring passband and stopband edges is called the *Transition Width.* Therefore, summarize the measurements of the previous two parts in a table that lists the two transition widths for each filter versus $r$. Derive a simple **approximate** formula for the transition width versus $r$. Try to get a formula that depends directly (or inversely) on $r$ or $(1-r)$.

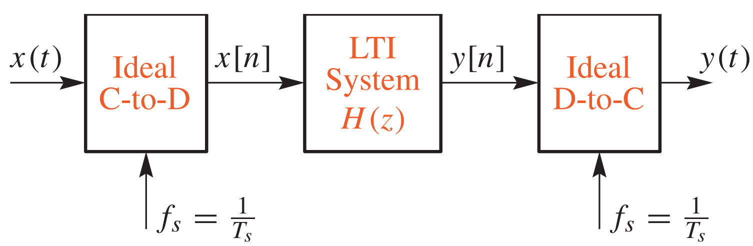


Figure 3: Filtering the analog signal $x(t)$ with a digital filter.

### 4.2.1 Design a Bandpass Filter Based on Analog Frequencies

One last question that relates to your understanding of sampling as well as digital filtering. Design a BPF whose passband is $3100 \le f \le 3500$ Hz when the sampling rate is 22050 Hz using the IIR method above, i.e., determine the value of $r$, and the angles of the poles. In addition, use two zeros: one at DC and the other at $z = -1$. As a reminder, you are designing a digital filter to be used as the system $H(z)$ in Fig. 3. Once you have the filter, determine its stopbands and give the stopband edges in hertz.

# Lab #9

# ECE-2025 Fall-2009
# WORKSHEET & VERIFICATION PAGE

*For each verification, be prepared to explain your answer and respond to other related questions that the lab TA's or professors might ask. Turn this page in at the end of your lab period.*

Name: ______________________________ Date of Lab: __________

| Part | Observations from **PeZ** |
|---|---|
| 3.1.1(a) | $h[n]$ oscillates, but decays exponentially, $H(e^{j\hat{\omega}})$ has a hump at $\hat{\omega} =$? |
| 3.1.1(b) | |
| 3.1.1(c) | |
| 3.1.1(d) | Stability Condition: |

Verified:____________________ Date/Time:__________

| | |
|---|---|
| 3.2(a) | |
| 3.2(b) | |
| 3.2(c) | |
| 3.2(d) | |
| 3.2(e) | |
| 3.2(f) | |
| 3.2(g) | |
| 3.2(h) | |

$\Longrightarrow$ Answered ITS questions? Verified:____________________ Date/Time:__________

Completed Peer Evlauation?