**GEORGIA INSTITUTE OF TECHNOLOGY**

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

# FINAL EXAM

DATE: 1-May-08 COURSE: ECE-2025

NAME: ______________________ GT username: ______________
LAST, FIRST (ex: gpburdell3)

3 points 3 points 3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

L05:Tues-Noon (Chang)
L07:Tues-1:30pm (Chang) L08:Thurs-1:30pm (Coyle)
L01:M-3pm (McClellan) L09:Tues-3pm (Lanterman) L02:W-3pm (Clements) L10:Thur-3pm (Coyle)
L11:Tues-4:30pm (Lanterman) L04:W-4:30pm (Clements)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning clearly to receive partial credit.
  Explanations are also required to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

| *Problem* | *Value* | *Score* |
|---|---|---|
| 1 | 25 | |
| 2 | 25 | |
| 3 | 25 | |
| 4 | 25 | |
| 5 | 25 | |
| 6 | 25 | |
| 7 | 25 | |
| 8 | 25 | |
| No/Wrong Rec | −3 | |

**PROBLEM sp-08-F.1:**

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems, i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

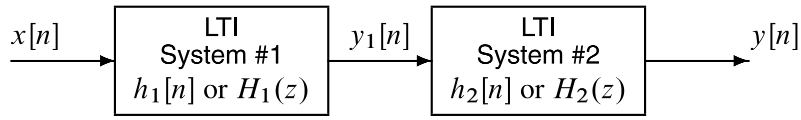


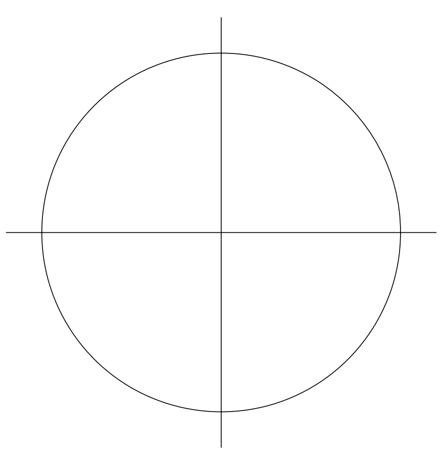

Figure 1: Cascade connection of two discrete-time LTI systems.

Suppose that $H_1(z) = 3 - 3z^{-5}$ is the system function for System #1, and

System #2 is an FIR filter described by a difference equation: $y[n] = 5y_1[n-1] - 5y_1[n-6]$

(a) Determine the impulse response $h[n]$ of the overall system. ***Give your answer as a stem plot.***


(b) Make a pole-zero plot for the first system. Account for ***all*** poles and zeros.



**PROBLEM sp-08-F.2:**
Circle the correct answer where applicable.

(a) A sinusoidal signal $x(t)$ is defined by: $x(t) = \Re e\{(1+j)e^{j\pi t}\}$. When $x(t)$ is plotted versus time $(t)$, its maximum value will be:

(a) $A = 1$
(b) $A = 1 + j$
(c) $A = \sqrt{2}$
(d) $A = 0$
(e) none of the above

(b) Determine the amplitude $(A)$ and phase $(\phi)$ of the sinusoid that is the sum of the following three sinusoids: $10\cos(6t + \pi/2) + 7\cos(6t - \pi/6) + 7\cos(6t + 7\pi/6)$,

(a) $A = 10$ and $\phi = \pi/2$.
(b) $A = 7$ and $\phi = \pi/2$.
(c) $A = 0$ and $\phi = 0$.
(d) $A = 3$ and $\phi = \pi/2$.
(e) $A = 24$ and $\phi = \pi/2$.

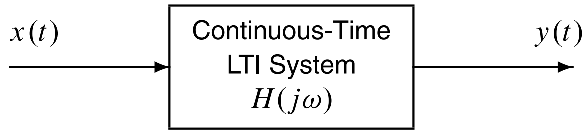
(c) In the DTMF Lab, the row and column frequency components were filtered and then downsampled by two before using `onefreq.p` to estimate the $\hat{\omega}$ frequency in the range $0 \le \hat{\omega} \le \pi$. The MATLAB code below does similar operations.

```
fsamp = 3000;
tt = 0:(1/fsamp):1;
xcol = cos(2*pi*1200*tt);   %<-- typical signal after the column BPF
xdn = xcol(1:2:end)
omegaHat = onefreq(xdn);    %<-- get one estimate by using all of xdn
```

Determine the frequency `omegaHat` that will be returned by the `onefreq` function on the last line.

| `omegaHat` = | rads |
|---|---|

**PROBLEM sp-08-F.3:**

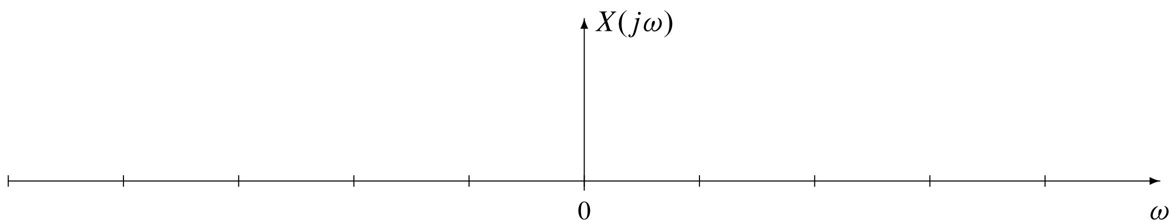



The periodic input to the above system is defined by the equation:

$$x(t) = \sum_{k=-2}^{2} a_k e^{j4kt}, \quad \text{where} \quad a_k = \frac{2}{\pi(4+k^2)} + \frac{1}{\pi}\delta[k]$$

(a) Determine the Fourier transform of the periodic signal $x(t)$. Give a formula and then plot it on the graph below. Label your plot with numerical values to receive full credit.

$X(j\omega) =$




(b) The frequency response of the LTI system is given by the following equation:

$$H(j\omega) = \frac{80 + j5\omega}{20 + j5\omega}$$

For $x(t)$ given above, the output signal can be written as $y(t) = \sum_{k=0}^{2} B_k \cos(4kt + \psi_k)$

Determine the numerical values of the parameters $B_0$, $B_2$ and $\psi_2$.

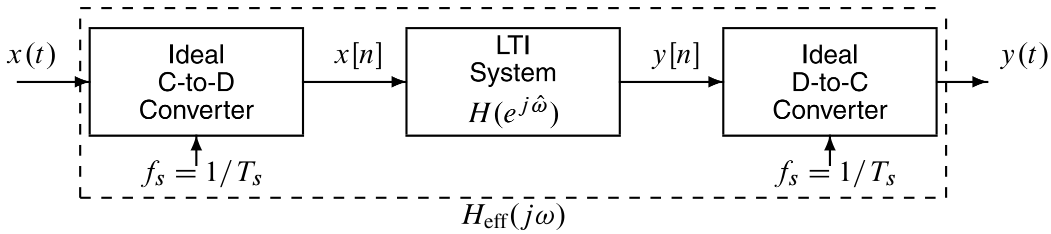
$B_0 =$

$B_2 =$

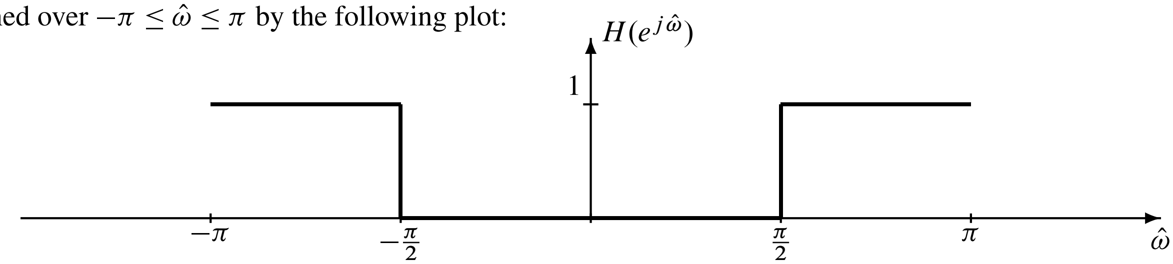
$\psi_2 =$

**PROBLEM sp-08-F.4:**

Consider the following system for discrete-time filtering of a continuous-time signal:



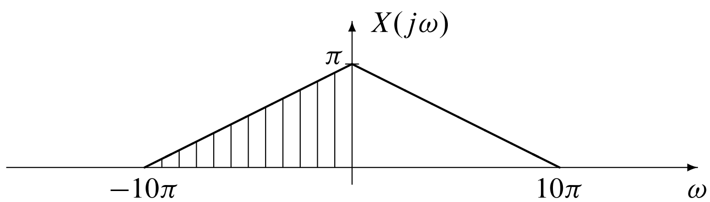

The sampling rate is $f_s$ **= 12 samples/sec**, and the discrete-time system has frequency response $H(e^{j\hat{\omega}})$ defined over $-\pi \le \hat{\omega} \le \pi$ by the following plot:

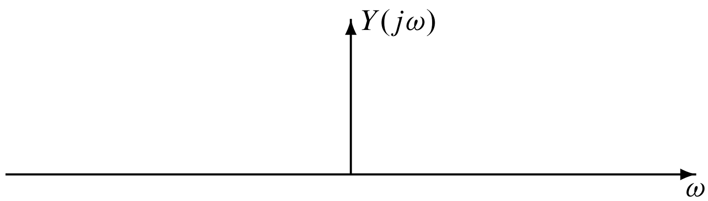



(a) The effective frequency response of the overall system will be an ideal filter. Determine the cutoff frequency (in rad/s) of the effective analog system.
In addition, **state the frequency range** where $H_{\text{eff}}(j\omega)$ is valid.

| Cutoff Frequency = | rad/s |
|---|---|
| Frequency range = | rad/s |

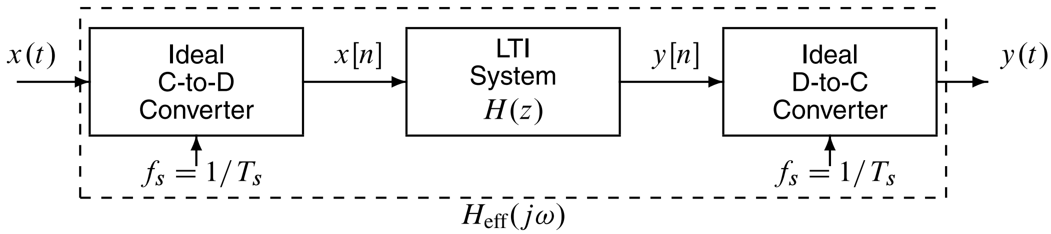
(b) Assume that the input signal $x(t)$ has a bandlimited Fourier transform $X(j\omega)$ as depicted below. For this input, determine $Y(j\omega)$, the Fourier transform of the output $y(t)$, and make a plot of $Y(j\omega)$.



**PROBLEM sp-08-F.5:**

Consider the following system for discrete-time filtering of a continuous-time signal:



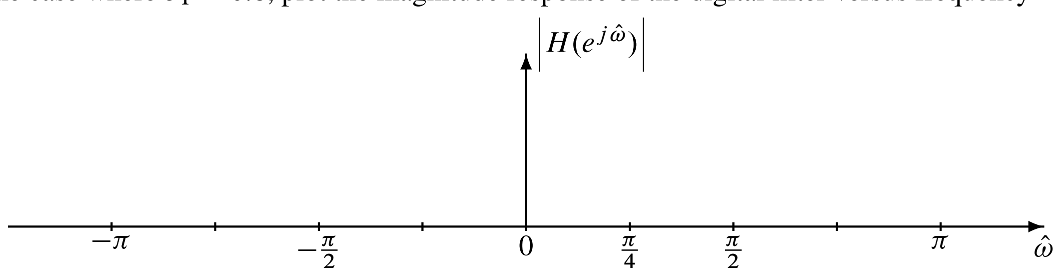

Assume that the discrete-time system has a system function $H(z)$ defined as: $H(z) = 1 + b_1 z^{-1} + z^{-2}$

(a) For the case where $b_1 = 0.8$, determine a formula for the frequency response of the discrete-time filter. Express your answer in the following form by finding $\alpha$ and $\beta$:

$$H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}\,(\alpha + \beta\cos(\hat{\omega}))$$

| |
|---|
| $\alpha =$ |
| $\beta =$ |

(b) For the case where $b_1 = 0.8$, plot the magnitude response of the digital filter versus frequency




(c) The effective frequency response of this system is able to null out one sinusoid; it is similar to the system used in the lab to remove a sinusoidal interference from an EKG signal. The value of the filter coefficient $b_1$ controls the (frequency) location of the null. If the sampling rate is 8000 Hz, determine the value of $b_1$ so that the overall effective frequency response has a null at 1000 Hz.

| |
|---|
| $b_1 =$ |

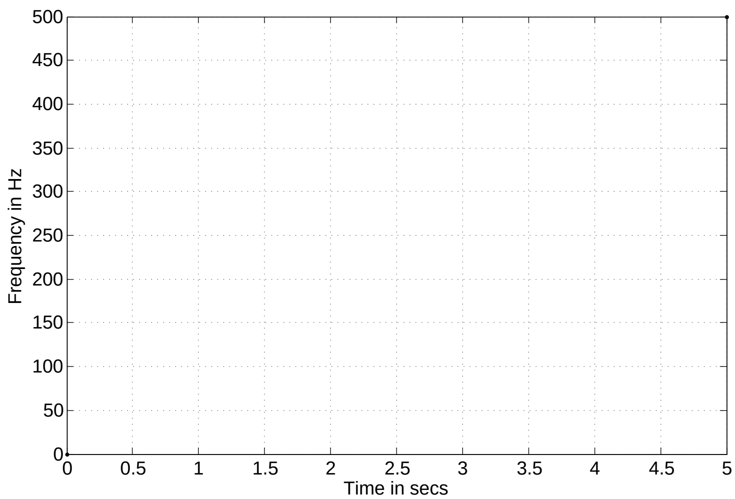
**PROBLEM sp-08-F.6:**

Consider the following snippet of code:

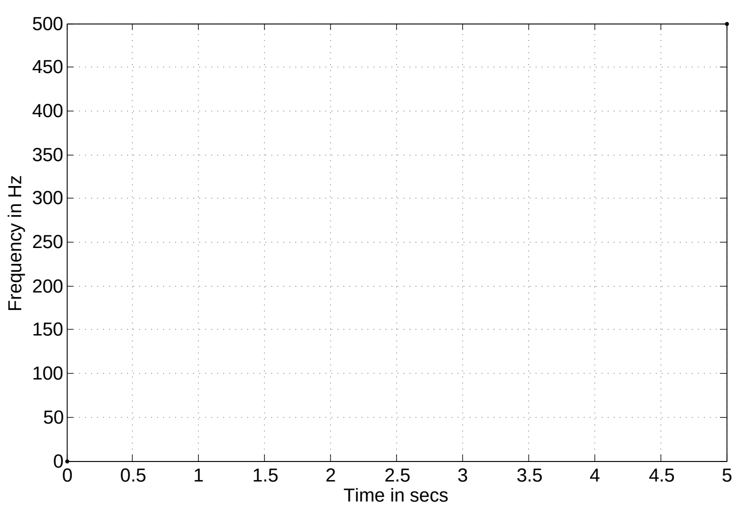
```
fs = 1000;
tt = 0:(1/fs):5,
xx = ... %<-- this line is given below for each part
specgram(xx,128,fs);
```

For two variations of the MATLAB code on the third line, sketch the resulting spectrogram.

(a) `xx = cos(2*pi*100*tt) .* cos(2*pi*300*tt);`




(b) `xx = cos(2*pi*(100*tt.*(tt + 1)));`



**PROBLEM sp-08-F.7:**

For each of the following time-domain signals, select the correct match from the list of Fourier transforms below. ***Write each answer in the box provided.*** (The operator $*$ denotes convolution.)

(a) $x(t) = \dfrac{\sin(5\pi t)}{\pi t} * \delta(t-4)$

$\boxed{\phantom{00}}$

(b) $x(t) = 2\dfrac{\sin(\pi t)}{\pi t} * \cos(4\pi t)$

$\boxed{\phantom{00}}$

(c) $x(t) = \dfrac{\sin(5\pi t)}{\pi t} * \dfrac{\sin(3\pi t)}{\pi t}$

$\boxed{\phantom{00}}$

(d) $x(t) = \displaystyle\int_{-\infty}^{\infty} \delta(\lambda+3)\delta(t-\lambda-7)\,d\lambda$

$\boxed{\phantom{00}}$

(e) $x(t) = 2\dfrac{\sin(\pi t)}{\pi t}\cos(4\pi t)$

$\boxed{\phantom{00}}$

(f) $x(t) = \dfrac{d}{dt}\left\{\dfrac{\sin(5\pi t)}{\pi t}\right\}$

$\boxed{\phantom{00}}$

---

Each of the time signals above has a Fourier transform that can be found in the list below.

**[0]** $X(j\omega) = u(\omega+5\pi) - u(\omega-5\pi)$

**[1]** $X(j\omega) = e^{-j4\omega}[u(\omega+5\pi) - u(\omega-5\pi)]$

**[2]** $X(j\omega) = u(\omega+3\pi) - u(\omega-3\pi)$

**[3]** $X(j\omega) = j\omega[u(\omega+5\pi) - u(\omega-5\pi)]$

**[4]** $X(j\omega) = e^{-j4\omega}$

**[5]** $X(j\omega) = 0$

**[6]** $X(j\omega) = e^{-j4\omega}\,[j\pi\delta(\omega+\pi) - j\pi\delta(\omega-\pi)]$

**[7]** $X(j\omega) = [j\pi u(\omega+\pi) - j\pi u(\omega-\pi)] * \delta(\omega-4\pi)$

**[8]** $X(j\omega) = 2u(\omega+\pi) - 2u(\omega-\pi) + \pi\delta(\omega+4\pi) + \pi\delta(\omega-4\pi)$

**[9]** $X(j\omega) = u(\omega+5\pi) - u(\omega+3\pi) + u(\omega-3\pi) - u(\omega-5\pi)$

**PROBLEM sp-08-F.8:**

The two subparts of this problem are completely independent of one another.

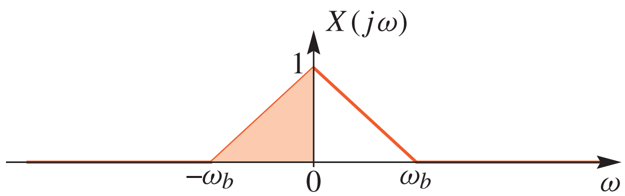
(a) When two finite-duration signals are convolved, the result is a finite-duration signal. In this subpart,

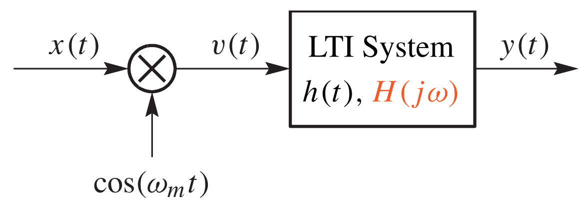
$$h(t) = t^2[u(t-25) - u(t)] \quad \text{and} \quad x(t) = 9[u(t-5) - u(t-10)]$$

Determine starting and ending times of output signal $y(t) = x(t) * h(t)$, i.e., find $T_1$ and $T_2$ so that $y(t) = 0$ for $t < T_1$ and for $t > T_2$.

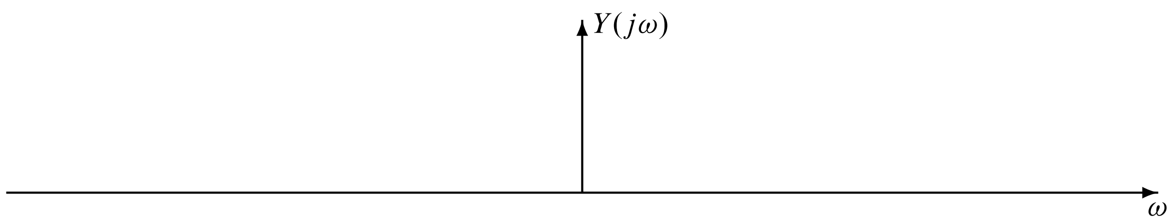
| $T_1 =$ | sec. | $T_2 =$ | sec. |
|---|---|---|---|

(b) The system below involves a multiplier followed by a filter:




The Fourier transform of the input is bandlimited to $\omega_b = 20\pi$ rad/s, and the frequency of the cosine multiplier is $\omega_m = 100\pi$ rad/s.
The filter is an ideal LPF defined by $H(j\omega) = 2[u(\omega + 100\pi) - u(\omega - 100\pi)]$.
Make a sketch of $Y(j\omega)$, the Fourier transform of the output $y(t)$ when the input is $X(j\omega)$.



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# FINAL EXAM

DATE: 1-May-08 COURSE: ECE-2025

NAME: Answer Key
LAST, FIRST

GT username: Version-1
(ex: gpburdell13)

3 points 3 points 3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

| | | | |
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| 8 | 25 | |
| No/Wrong Rec | −3 | |

**PROBLEM sp-08-F.1:**

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems, i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

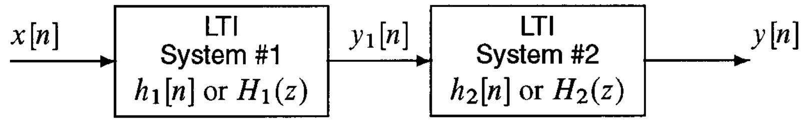
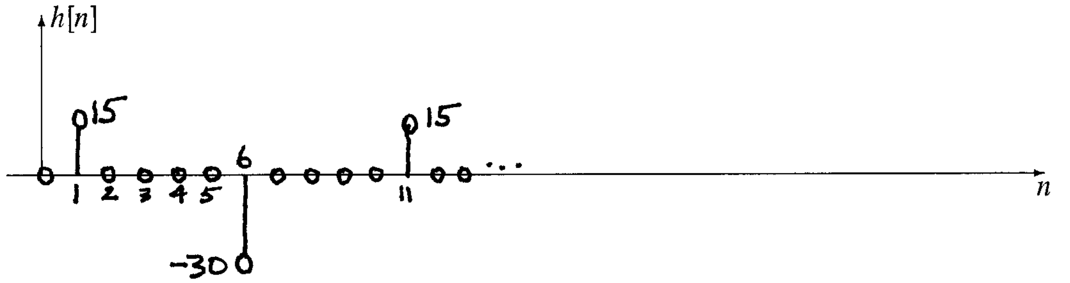



Figure 1: Cascade connection of two discrete-time LTI systems.

Suppose that $H_1(z) = 3 - 3z^{-5}$ is the system function for System #1, and

System #2 is an FIR filter described by a difference equation: $y[n] = 5y_1[n-1] - 5y_1[n-6]$

(a) Determine the impulse response $h[n]$ of the overall system. ***Give your answer as a stem plot.***



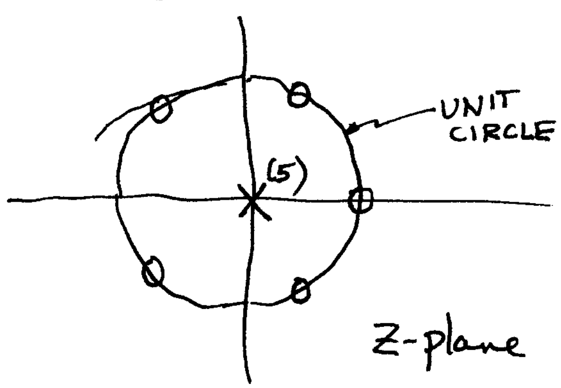

$$H(z) = (3 - 3z^{-5})(5z^{-1} - 5z^{-6})$$

$$= 15z^{-1} \underbrace{-15z^{-6} - 15z^{-6}}_{-30z^{-6}} + 15z^{-11}$$

Inverse z-Trans:

$$h[n] = 15\delta[n-1] - 30\delta[n-6] + 15\delta[n-11]$$

(b) Make a pole-zero plot for the first system. Account for ***all*** poles and zeros.




$$H_1(z) = 3(1 - z^{-5}) \stackrel{?}{=} 0$$

$\Rightarrow z^5 - 1 = 0$ or $z^5 = 1$

Solution is 5 zeros at

$z = e^{j\frac{2\pi}{5}k}$ $\quad k = 0, 1, 2, 3, 4$

$\frac{2\pi}{5}$ rads $\longrightarrow 72°$

Also, 5 poles at $z = 0$

because $H_1(z) = 3\,\dfrac{z^5 - 1}{z^5}$

**PROBLEM sp-08-F.2:**

Circle the correct answer where applicable.

(a) A sinusoidal signal $x(t)$ is defined by: $x(t) = \Re e\{(1+j)e^{j\pi t}\}$. When $x(t)$ is plotted versus time $(t)$, its maximum value will be:

(a) $A = 1$
(b) $A = 1 + j$
(c) $A = \sqrt{2}$
(d) $A = 0$
(e) none of the above

$1 + j = \sqrt{2}\, e^{j\pi/4}$

$x(t) = \Re e\{\sqrt{2}\, e^{j\pi/4} e^{j\pi t}\}$

$= \sqrt{2} \cos(\pi t + \pi/4)$

amp: this is the max value.

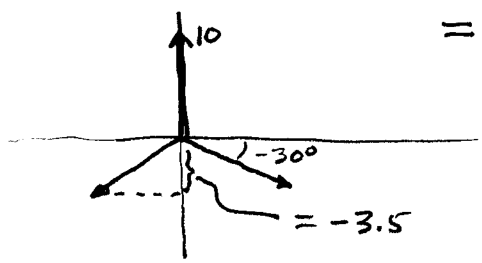
(b) Determine the amplitude $(A)$ and phase $(\phi)$ of the sinusoid that is the sum of the following three sinusoids: $10\cos(6t + \pi/2) + 7\cos(6t - \pi/6) + 7\cos(6t + 7\pi/6)$,

(a) $A = 10$ and $\phi = \pi/2$.
(b) $A = 7$ and $\phi = \pi/2$.
(c) $A = 0$ and $\phi = 0$.
(d) $A = 3$ and $\phi = \pi/2$.
(e) $A = 24$ and $\phi = \pi/2$.

Use phasors, i.e., complex amps

$10e^{j\pi/2} + 7e^{-j\pi/6} + 7e^{j7\pi/6}$

$= 3e^{j\pi/2}$




(c) In the DTMF Lab, the row and column frequency components were filtered and then downsampled by two before using `onefreq.p` to estimate the $\hat{\omega}$ frequency in the range $0 \le \hat{\omega} \le \pi$. The MATLAB code below does similar operations.

```
fsamp = 3000;
tt = 0:(1/fsamp):1;
xcol = cos(2*pi*1200*tt);     %<-- typical signal after the column BPF
xdn = xcol(1:2:end)
omegaHat = onefreq(xdn);      %<-- get one estimate by using all of xdn
```

Determine the frequency `omegaHat` that will be returned by the `onefreq` function on the last line.

| omegaHat = $0.4\pi$ | rads |
|---|---|

$= 1.26$ rads

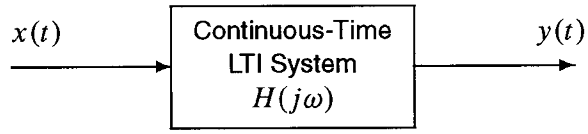
After downsampling the cosine is effectively sampled at $f_s = 1500$ Hz.

Thus, $\hat{\omega} = \dfrac{2\pi(1200)}{1500} = 1.6\pi$

which aliases to $-0.4\pi$

But there is also a negative freq component at $-1.6\pi$ which aliases to $+0.4\pi$

**PROBLEM sp-08-F.3:**

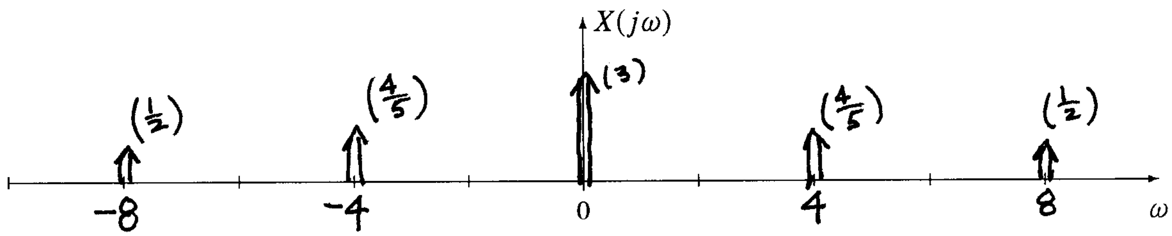



The periodic input to the above system is defined by the equation:

$$x(t)=\sum_{k=-2}^{2} a_k e^{j4kt}, \quad \text{where} \quad a_k = \frac{2}{\pi(4+k^2)} + \frac{1}{\pi}\delta[k] \Rightarrow \begin{cases} a_0 = \frac{2}{4\pi} + \frac{1}{\pi} = \frac{3}{2\pi} \\ a_1 = a_{-1} = 2/5\pi \\ a_2 = a_{-2} = 1/4\pi \end{cases}$$

(a) Determine the Fourier transform of the periodic signal $x(t)$. Give a formula and then plot it on the graph below. Label your plot with numerical values to receive full credit. $X(j\omega) = \sum 2\pi a_k \delta(\omega - 4k)$

$$\boxed{X(j\omega) = 3\delta(\omega) + \frac{4}{5}\delta(\omega-4) + \frac{4}{5}\delta(\omega+4) + \frac{1}{2}\delta(\omega-8) + \frac{1}{2}\delta(\omega+8)}$$




(b) The frequency response of the LTI system is given by the following equation:

$$H(j\omega) = \frac{80 + j5\omega}{20 + j5\omega}$$

For $x(t)$ given above, the output signal can be written as $y(t) = \sum_{k=0}^{2} B_k \cos(4kt + \psi_k)$ $= \sum_{-2}^{2} b_k e^{j4kt}$

Determine the values of the parameters $B_0$, $B_2$ and $\psi_2$.

$\boxed{B_0 = 1.91, \text{ or } 6/\pi}$

$\boxed{B_2 = 0.318, \text{ or } 1/\pi}$

$\boxed{\psi_2 = -0.644 \text{ rad}}$

$= -0.205\pi$

$$b_0 = a_0 H(j0) = \left(\frac{3}{2\pi}\right)\left(\frac{80}{20}\right) = 6/\pi = 1.91$$

$$B_0 = b_0$$

$$b_2 = a_2 H(j8) = \left(\frac{1}{4\pi}\right)\left(\frac{80+j40}{20+j40}\right)$$

$-0.205\pi$

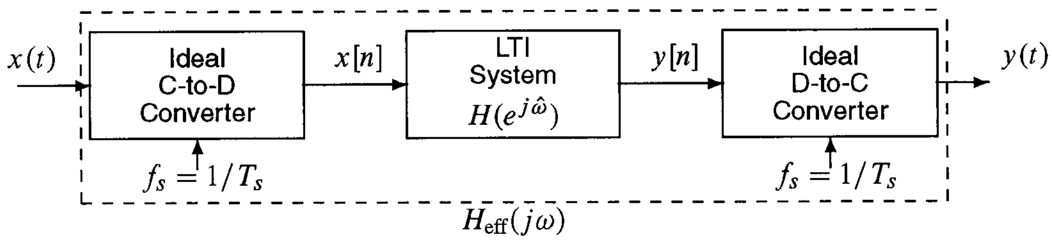
$$= \frac{1}{4\pi}\left(2e^{-j0.644}\right) = 0.159e^{-j0.644}$$

$$B_2 = 2|b_2| = 0.318$$

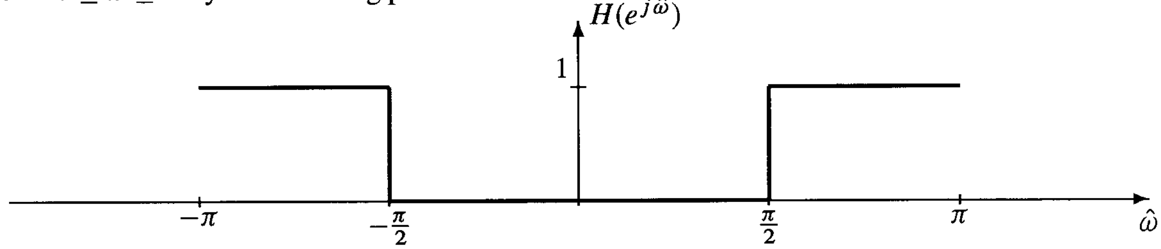
$$\psi_2 = \angle b_2 = -0.644$$

**PROBLEM sp-08-F.4:**

Consider the following system for discrete-time filtering of a continuous-time signal:




The sampling rate is $f_s$ = **12 samples/sec**, and the discrete-time system has frequency response $H(e^{j\hat{\omega}})$ defined over $-\pi \le \hat{\omega} \le \pi$ by the following plot:



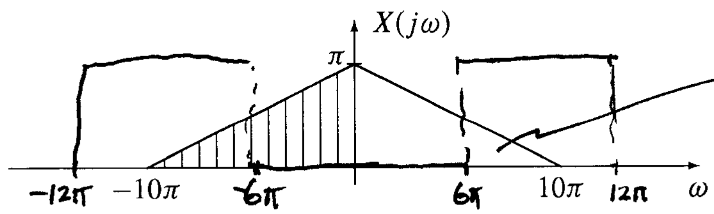

(a) The effective frequency response of the overall system will be an ideal filter. Determine the cutoff frequency (in rad/s) of the effective analog system.
In addition, **state the frequency range** where $H_{\text{eff}}(j\omega)$ is valid.

$\hat{\omega} = \omega / f_s$, so
$\hat{\omega} = \pi/2$ maps to
$\omega = (\pi/2)(12) = 6\pi$ rad/s

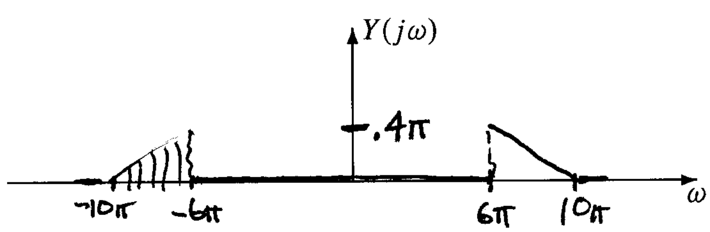
| | |
|---|---|
| Cutoff Frequency = $6\pi$ | rad/s |
| Frequency range = $-12\pi$ to $+12\pi$ | rad/s |

Valid for $-\frac{1}{2}f_s$ to $+\frac{1}{2}f_s$ which is $\omega = 2\pi(-\frac{12}{2}) = -12\pi$ to $+12\pi$

(b) Assume that the input signal $x(t)$ has a bandlimited Fourier transform $X(j\omega)$ as depicted below. For this input, determine $Y(j\omega)$, the Fourier transform of the output $y(t)$, and make a plot of $Y(j\omega)$.

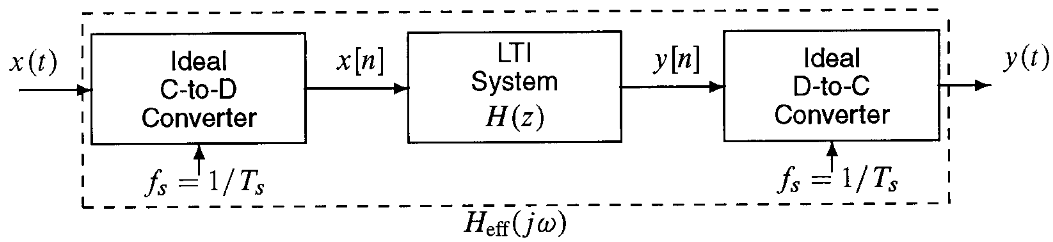



This region is passed



**PROBLEM sp-08-F.5:**

Consider the following system for discrete-time filtering of a continuous-time signal:




Assume that the discrete-time system has a system function $H(z)$ defined as: $H(z) = 1 + b_1 z^{-1} + z^{-2}$

$H(e^{j\hat{\omega}}) = 1 + b_1 e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$

(a) For the case where $b_1 = 0.8$, determine a formula for the frequency response of the discrete-time filter. Express your answer in the following form by finding $\alpha$ and $\beta$:

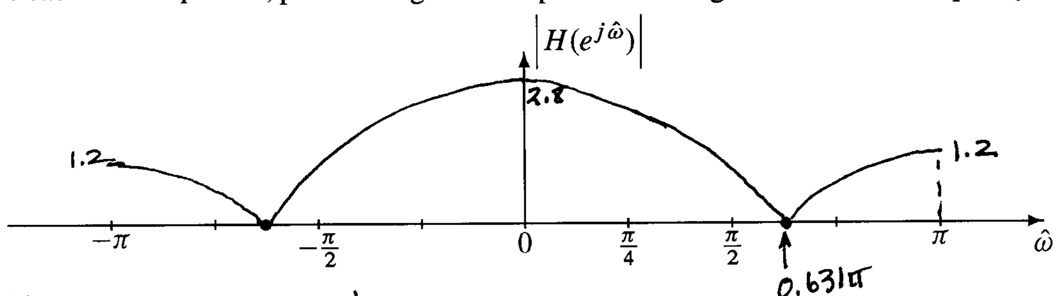
$$H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}(\alpha + \beta\cos(\hat{\omega}))$$

$\boxed{\alpha = 0.8}$

$\boxed{\beta = 2}$

$$H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}(e^{j\hat{\omega}} + b_1 + e^{-j\hat{\omega}})$$
$$= e^{-j\hat{\omega}}(b_1 + 2\cos\hat{\omega})$$
$$= e^{-j\omega}(+0.8 + 2\cos\hat{\omega})$$

(b) For the case where $b_1 = 0.8$, plot the magnitude response of the digital filter versus frequency




$|H(e^{j\hat{\omega}})| = |+0.8 + 2\cos\hat{\omega}|$

at $\hat{\omega} = 0$, $|H| = 2.8$

at $\hat{\omega} = \pi/2$, $|H| = 0.8$

at $\hat{\omega} = \pi$, $|H| = 1.2$

$|H(e^{j\hat{\omega}})| = 0$ when $+0.8 + 2\cos\hat{\omega} = 0$

$\Rightarrow \hat{\omega} = 1.98$ rad $= 0.631\pi$ rad

(c) The effective frequency response of this system is able to null out one sinusoid; it is similar to the system used in the lab to remove a sinusoidal interference from an EKG signal. The value of the filter coefficient $b_1$ controls the (frequency) location of the null. If the sampling rate is 8000 Hz, determine the value of $b_1$ so that the overall effective frequency response has a null at 1000 Hz.

$H(e^{j\hat{\omega}}) = 0$ when $b_1 + 2\cos(\hat{\omega}) = 0 \Rightarrow b_1 = -2\cos(\hat{\omega})$

To null 1000 Hz when $f_s = 8000$ Hz we need a null at: $\hat{\omega} = 2\pi\left(\frac{1000}{8000}\right) = \pi/4$

$\Rightarrow b_1 = -2\cos(\pi/4) = -\sqrt{2} = -1.414$

$$\boxed{\begin{aligned} b_1 &= -\sqrt{2} \\ &= -1.414 \end{aligned}}$$

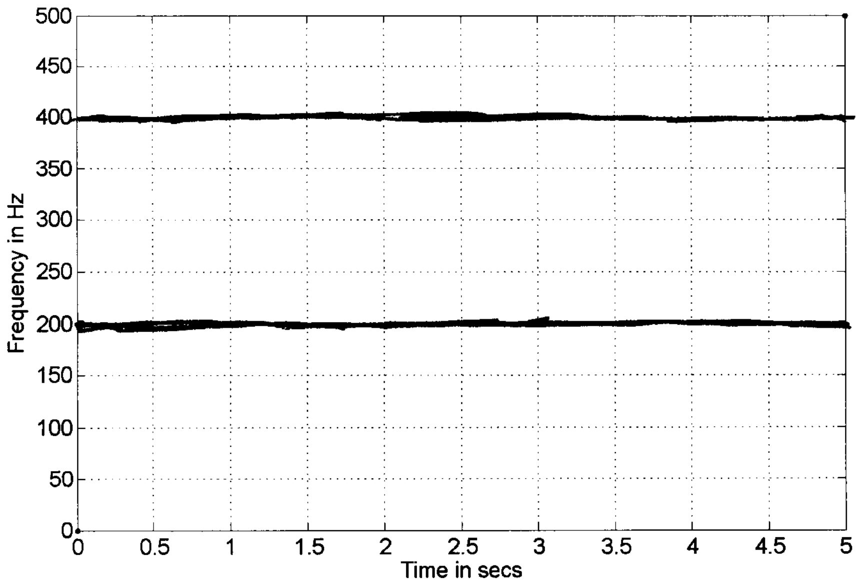
**PROBLEM sp-08-F.6:**

Consider the following snippet of code:

```
fs = 1000;
tt = 0:(1/fs):5,
xx = ... %<-- this line is given below for each part
specgram(xx,128,fs);
```

For two variations of the MATLAB code on the third line, sketch the resulting spectrogram.

(a) `xx = cos(2*pi*100*tt) .* cos(2*pi*300*tt);`




Product:

$\cos(2\pi(100)t)\cos(2\pi(300)t)$

Must be converted to a sum:

$\left[\frac{1}{2}e^{j200\pi t} + \frac{1}{2}e^{-j200\pi t}\right]$

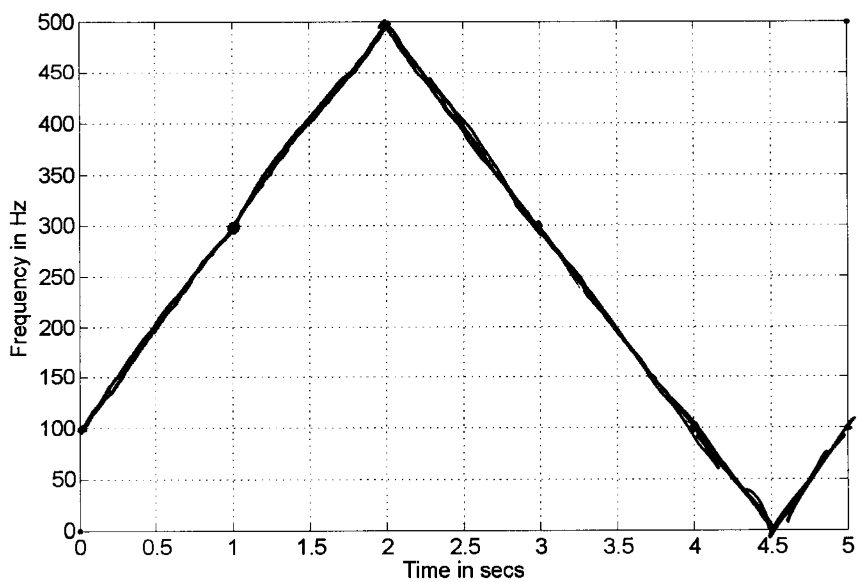
$\times\left[\frac{1}{2}e^{j600\pi t} + \frac{1}{2}e^{-j600\pi t}\right]$

$\Rightarrow$ terms at

$\omega = 800\pi, 400\pi, -800\pi, -400\pi$

$f = 400, 200, -400, -200$

(b) `xx = cos(2*pi*(100*tt.*(tt + 1)));`




$f_i(t) = \frac{1}{\cancel{2\pi}}\frac{d}{dt}\left\{\cancel{2\pi}(100)t(1+t)\right\}$

$= 100 + 200t$

slope

When $f_i(t)$ tries to go higher than $f_s/2$ it aliases, actually folding.

**PROBLEM sp-08-F.7:**

For each of the following time-domain signals, select the correct match from the list of Fourier transforms below. ***Write each answer in the box provided.*** (The operator $*$ denotes convolution.)

(a) $x(t) = \dfrac{\sin(5\pi t)}{\pi t} * \delta(t-4)$

$\boxed{1}$

(b) $x(t) = 2\dfrac{\sin(\pi t)}{\pi t} * \cos(4\pi t)$

$\boxed{5}$

(c) $x(t) = \dfrac{\sin(5\pi t)}{\pi t} * \dfrac{\sin(3\pi t)}{\pi t}$

$\boxed{2}$

(d) $x(t) = \displaystyle\int_{-\infty}^{\infty} \delta(\lambda+3)\delta(t-\lambda-7)d\lambda$

$\boxed{4}$

(e) $x(t) = 2\dfrac{\sin(\pi t)}{\pi t}\cos(4\pi t)$

$\boxed{9}$

(f) $x(t) = \dfrac{d}{dt}\left\{\dfrac{\sin(5\pi t)}{\pi t}\right\}$

$\boxed{3}$

---

Each of the time signals above has a Fourier transform that can be found in the list below.

[0] $X(j\omega) = u(\omega+5\pi) - u(\omega-5\pi)$

[1] $X(j\omega) = e^{-j4\omega}[u(\omega+5\pi) - u(\omega-5\pi)]$

[2] $X(j\omega) = u(\omega+3\pi) - u(\omega-3\pi)$

[3] $X(j\omega) = j\omega[u(\omega+5\pi) - u(\omega-5\pi)]$

[4] $X(j\omega) = e^{-j4\omega}$

[5] $X(j\omega) = 0$

[6] $X(j\omega) = e^{-j4\omega}[j\pi\delta(\omega+\pi) - j\pi\delta(\omega-\pi)]$

[7] $X(j\omega) = [j\pi u(\omega+\pi) - j\pi u(\omega-\pi)] * \delta(\omega-4\pi)$

[8] $X(j\omega) = 2u(\omega+\pi) - 2u(\omega-\pi) + \pi\delta(\omega+4\pi) + \pi\delta(\omega-4\pi)$

[9] $X(j\omega) = u(\omega+5\pi) - u(\omega+3\pi) + u(\omega-3\pi) - u(\omega-5\pi)$

**PROBLEM sp-08-F.8:**

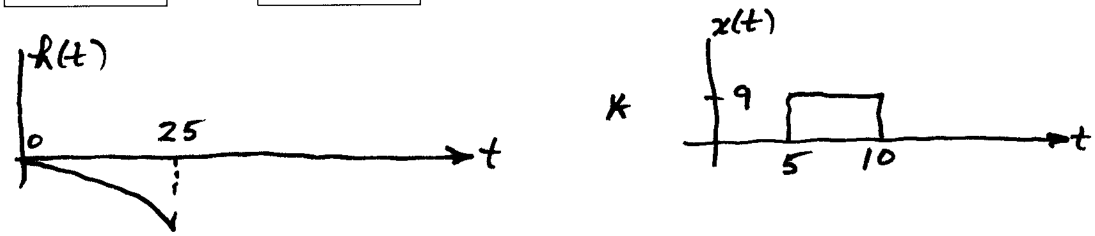
The two subparts of this problem are completely independent of one another.

(a) When two finite-duration signals are convolved, the result is a finite-duration signal. In this subpart,

$$h(t) = t^2[u(t-25) - u(t)] \quad \text{and} \quad x(t) = 9[u(t-5) - u(t-10)]$$

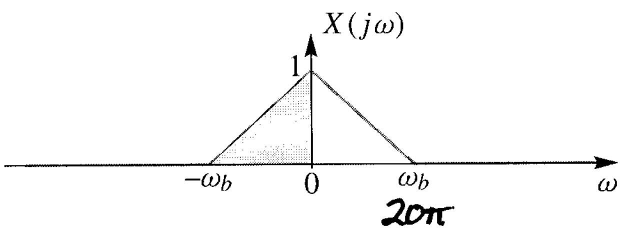
Determine starting and ending times of output signal $y(t) = x(t) * h(t)$, i.e., find $T_1$ and $T_2$ so that $y(t) = 0$ for $t < T_1$ and for $t > T_2$.

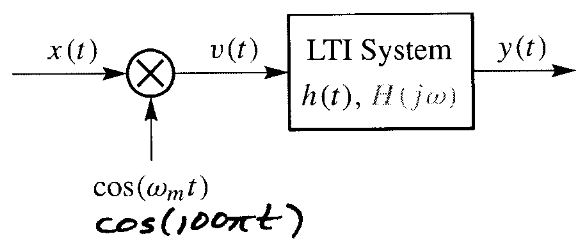
$\boxed{T_1 = 5}$ sec. $\boxed{T_2 = 35}$ sec.




Total Duration for y(t) is 25+5 = 30s
Since x(t) is delayed by 5, so is y(t).
y(t) starts at t=5 and ends at 5+30 = 35s.

(b) The system below involves a multiplier followed by a filter:




The Fourier transform of the input is bandlimited to $\omega_b = 20\pi$ rad/s, and the frequency of the cosine multiplier is $\omega_m = 100\pi$ rad/s.
The filter is an ideal LPF defined by $H(j\omega) = 2[u(j(\omega + 100\pi)) - u(j(\omega - 100\pi))]$.
Make a sketch of $Y(j\omega)$, the Fourier transform of the output $y(t)$ when the input is $X(j\omega)$.