GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #2

DATE: 14-Mar-08 COURSE: ECE-2025

NAME: ______________________________ GT username: ________________

LAST, FIRST (ex: gpburdell3)

3 points 3 points 3 points

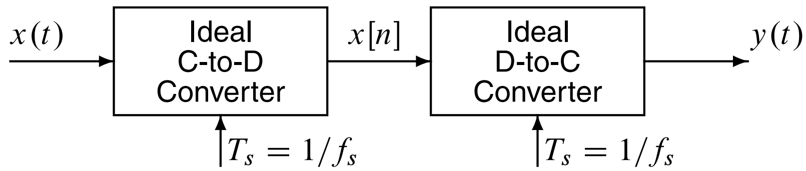
Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

	L05:Tues-Noon (Chang)		
	L07:Tues-1:30pm (Chang)		L08:Thurs-1:30pm (Coyle)
L01:M-3pm (McClellan)	L09:Tues-3pm (Lanterman)	L02:W-3pm (Clements)	L10:Thur-3pm (Coyle)
	L11:Tues-4:30pm (Lanterman)	L04:W-4:30pm (Clements)	

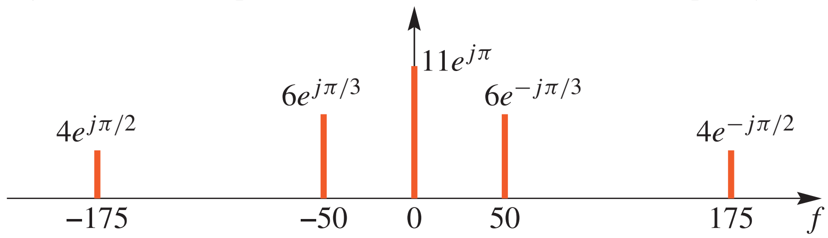
- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning clearly to receive partial credit.
  Explanations are also required to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	25	
2	25	
3	25	
4	25	
No/Wrong Rec	–3	

PROBLEM sp-08-Q.2.1:



In all parts below, the sampling rates of the C/D and D/C converters are **equal,** and the input to the Ideal C/D converter is a signal $x(t)$ whose spectrum is shown below, where the frequency f is in hertz.

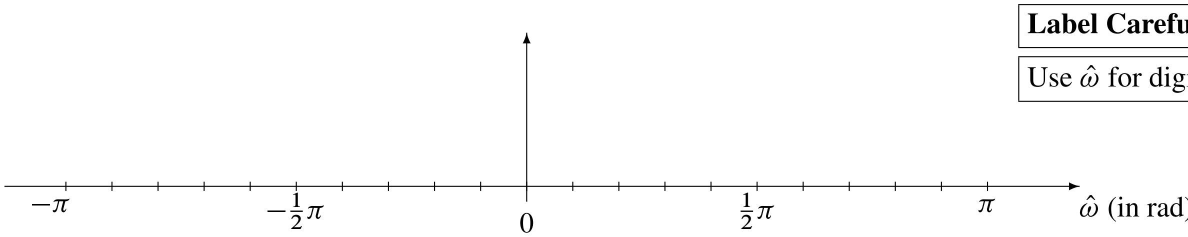


(a) Determine the Nyquist rate (in hertz) for sampling the signal $x(t)$. $f_{\text{Nyquist}} =$ ☐ Hz

(b) If the sampling rate is $f_s = 200$ samples/sec., plot the spectrum of the discrete-time signal $x[n]$ over the range of frequencies $-\pi \leq \hat{\omega} \leq \pi$. Make sure to label the frequency, amplitude and phase of each spectral component.

Label Carefully

Use $\hat{\omega}$ for digital freq.



(c) If the sampling rate is $f_s = 200$ samples/sec., list ***all*** frequencies (positive and negative) that will be present in the spectrum of the output signal, $y(t)$. Give your answer in ***hertz***.

PROBLEM sp-08-Q.2.2:

Pick the correct output signal (from the list on the right) and enter the number in the answer box:

System Description and Input Signal

(a) $x[n] = \delta[n-1] - \delta[n-2]$

and $y[n] = x[n] + x[n-1]$

ANS =

(b) $x[n] = \delta[n-8]$

and $y[n] = \begin{cases} x[2^n] & n \geq 0 \\ 0 & n < 0 \end{cases}$

ANS =

(c) `yy = conv([0,1,0,-1], [0,1,0,0,0])`

ANS =

(d) $x[n] = 1 + \cos(2\pi n/3)$ for all n

and $h[n] = \delta[n] + \delta[n-1] + \delta[n-2]$

ANS =

(e) $y[n] = \delta[n-3] * (\delta[n] - \delta[n-2])$

ANS =

Output Signal

1 $y[n] = \delta[n-3] - \delta[n-5]$

2 $y[n] = 3\cos(2\pi n/3 + 2\pi/3)$ for all n

3 $y[n] = \delta[n-2] - \delta[n-4]$

4 $y[n] = \delta[n-1] - \delta[n-3]$

5 $y[n] = 3$ for all n

6 $y[n] = 0$ for all n

7 $y[n] = \delta[n-3]$

8 None of the above

Hint: one of the systems on the left is NOT a Linear, Time-Invariant, System.

PROBLEM sp-08-Q.2.3:

Pick the correct frequency response (from the list on the right) and enter the number in the answer box:

Time-Domain Description

(a) $y[n] = x[n] + x[n-1] + x[n-2]$

ANS =

(b) $h[n] = u[n] - u[n-2]$

ANS =

(c) $h[n] = \delta[n-1] - \delta[n-3]$

ANS =

(d) $\{b_k\} = \{\,1,\ 0,\ -1\,\}$

ANS =

(e) Select **one** system[1] (from the list on the right) that will **null out** DC.

ANS =

Frequency Response

1 $H(e^{j\hat{\omega}}) = 1 - e^{-j2\hat{\omega}}$

2 $H(e^{j\hat{\omega}}) = 2e^{-j2\hat{\omega}}\cos(\hat{\omega})$

3 $H(e^{j\hat{\omega}}) = 2je^{-j2\hat{\omega}}\sin(\hat{\omega})$

4 $H(e^{j\hat{\omega}}) = \dfrac{\sin(\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})}\, e^{-j\hat{\omega}/2}$

5 $H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}$

6 $H(e^{j\hat{\omega}}) = \dfrac{\sin(2\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})}\, e^{-j3\hat{\omega}/2}$

7 $H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}(1 + 2\cos(\hat{\omega}))$

8 None of the above

[1] There might be several systems that null the sinusoid, but finding one is sufficient.

PROBLEM sp-08-Q.2.4:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems, i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

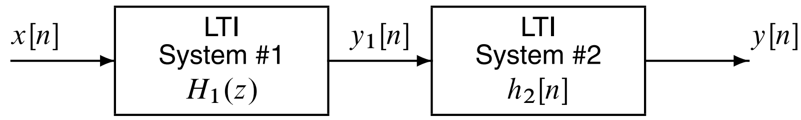


Figure 1: Cascade connection of two LTI systems.

(a) Suppose that System #1 is an FIR filter described by the system function:

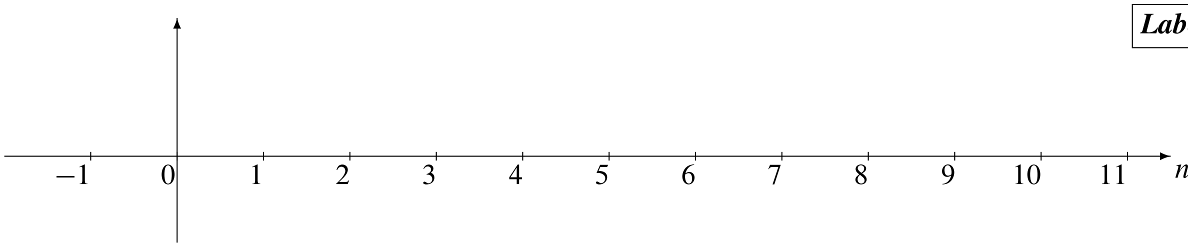
$$H_1(z) = 3z^{-2} - 2z^{-3} - z^{-4} + 4z^{-5}$$

and System #2 is described by the impulse response

$$h_2[n] = \delta[n-1] - \delta[n-3]$$

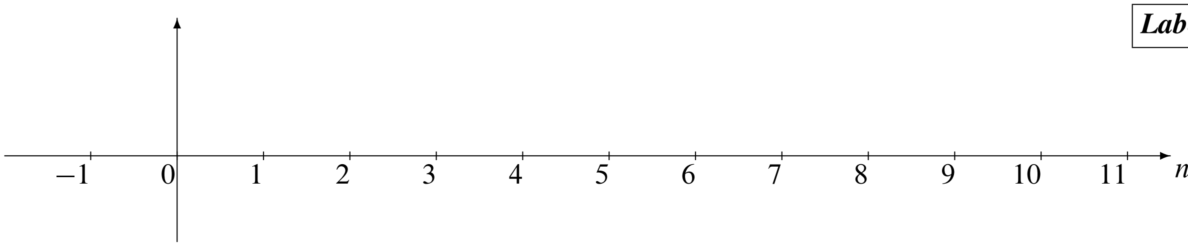
If the input signal is an impulse, i.e., $x[n] = \delta[n]$, determine the output signal, $y[n]$. Give your answer as a ***plot.***

Label Carefully



(b) Determine the impulse response sequence, $h_1[n]$, of the first system. Give your answer as a ***plot.***

Label Carefully



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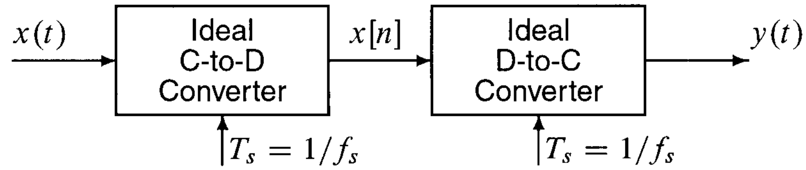
PROBLEM sp-08-Q.2.1:



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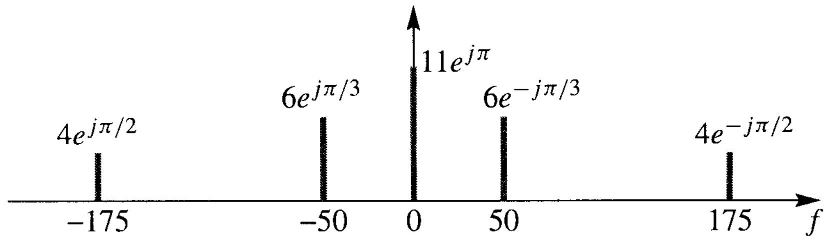


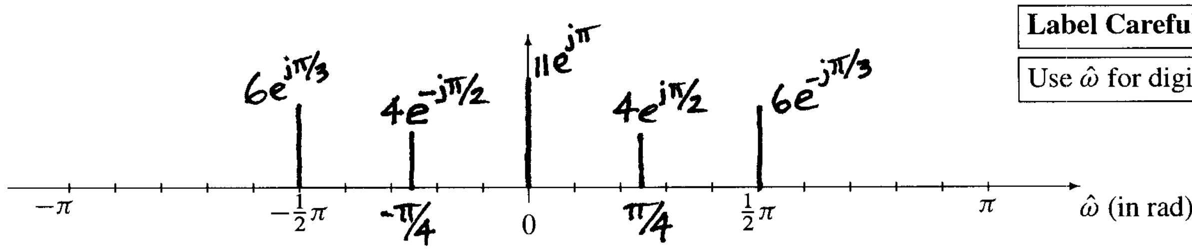
(a) Determine the Nyquist rate (in hertz) for sampling the signal $x(t)$. $f_{\text{Nyquist}} =$ **350** Hz

$2 \times f_{MAX}$ and $f_{MAX} = 175$ Hz

(b) If the sampling rate is $f_s = 200$ samples/sec., plot the spectrum of the discrete-time signal $x[n]$ over the range of frequencies $-\pi \le \hat{\omega} \le \pi$. Make sure to label the frequency, amplitude and phase of each spectral component.

Label Carefully

Use $\hat{\omega}$ for digital freq.



$\hat{\omega} = \omega/f_s + 2\pi\ell$

$\omega = 0 \rightarrow \hat{\omega} = 0 + 2\pi\ell$

$\omega = 2\pi(50) \rightarrow 2\pi(50)/200 = \pi/2$

$\omega = 2\pi(175) \rightarrow \hat{\omega} = 2\pi(175)/200 + 2\pi\ell$ $\quad \frac{7\pi}{4} - 2\pi = -\pi/4$

$= 7\pi/4 + 2\pi\ell \leftarrow$ use $\ell = -1$ to get between $-\pi$ & π

(c) If the sampling rate is $f_s = 200$ samples/sec., list *all* frequencies (positive and negative) that will be present in the spectrum of the output signal, $y(t)$.

$\omega = \hat{\omega} f_s = 200\hat{\omega}$

$\hat{\omega} = 0 \rightarrow \omega = 0$

$\hat{\omega} = \pi/4 \rightarrow \omega = 200(\pi/4) = 50\pi = 2\pi(25)$

$\hat{\omega} = \pi/2 \rightarrow \omega = 200(\pi/2) = 100\pi = 2\pi(50)$

$y(t)$ has $0, \pm 50\pi$ rad/s, $\pm 100\pi$ rad/s

or $0, \pm 25$ Hz, ± 50 Hz

PROBLEM sp-08-Q.2.2:

Pick the correct output signal (from the list on the right) and enter the number in the answer box:

System Description and Input Signal

(a) $x[n] = \delta[n-1] - \delta[n-2]$

and $y[n] = x[n] + x[n-1]$

ANS = 4

$y[n] = \delta[n-1] - \delta[n-2] + \delta[n-2] - \delta[n-3]$

$= \delta[n-1] - \delta[n-3]$

(b) $x[n] = \delta[n-8]$

and $y[n] = \begin{cases} x[2^n] & n \ge 0 \\ 0 & n < 0 \end{cases}$

ANS = 7

$y[0] = x[1] = 0$

$y[1] = x[2] = 0$

$y[2] = x[4] = 0$

$y[3] = x[8] = 1$

$y[4] = x[16] = 0 \ldots$

so $y[n]$ is an impulse at $n=3$

(c) `yy = conv([0,1,0,-1], [0,1,0,0,0])`

ANS = 3

```
  0 1 0 -1
  0 1 0  0  0
 ---------------
  0 0 0  0  0 ...
    0 1  0 -1 ....
 ---------------
  0 0 1  0 -1 ....
      ↑     ↑
     n=2   n=4
```

(d) $x[n] = 1 + \cos(2\pi n/3)$ for all n

and $h[n] = \delta[n] + \delta[n-1] + \delta[n-2]$

ANS = 5

$H(e^{j\hat{\omega}}) = 1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$

at $\hat{\omega} = 0$, $H(e^{j0}) = 3$

at $\hat{\omega} = 2\pi/3$, $H(e^{j2\pi/3}) = 0$

$\therefore y[n] = 3$

(e) $y[n] = \delta[n-3] * (\delta[n] - \delta[n-2])$

ANS = 1

shift by 3 to the right, i.e., delay

$y[n] = \delta[n-3] - \delta[n-5]$

Output Signal

1. $y[n] = \delta[n-3] - \delta[n-5]$
2. $y[n] = 3\cos(2\pi n/3 + 2\pi/3)$ for all n
3. $y[n] = \delta[n-2] - \delta[n-4]$
4. $y[n] = \delta[n-1] - \delta[n-3]$
5. $y[n] = 3$ for all n
6. $y[n] = 0$ for all n
7. $y[n] = \delta[n-3]$
8. None of the above

Hint: one of the systems on the left is NOT a Linear, Time-Invariant, System.

PROBLEM sp-08-Q.2.3:

Pick the correct frequency response (from the list on the right) and enter the number in the answer box:

Time-Domain Description

(a) $y[n] = x[n] + x[n-1] + x[n-2]$

ANS = 7

$H(e^{j\hat{\omega}}) = 1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$

$= e^{-j\hat{\omega}}(e^{j\hat{\omega}} + 1 + e^{-j\hat{\omega}})$

$= e^{-j\hat{\omega}}(1 + 2\cos\hat{\omega})$

(b) $h[n] = u[n] - u[n-2] = \delta[n] + \delta[n-1]$

ANS = 4 2-pt running sum

$H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}/2} \frac{\sin(\hat{\omega})}{\sin(\hat{\omega}/2)}$

Use Dirichlet with L=2

(c) $h[n] = \delta[n-1] - \delta[n-3]$

ANS = 3

$H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}} - e^{-j3\hat{\omega}}$

$= e^{-j2\hat{\omega}}\underbrace{(e^{j\hat{\omega}} - e^{-j\hat{\omega}})}_{2j\sin(\hat{\omega})}$

(d) $\{b_k\} = \{1, 0, -1\}$

ANS = 1

$H(e^{j\hat{\omega}}) = 1 - e^{-j2\hat{\omega}}$

(e) Select <u>**one**</u> system[1] (from the list on the right) that will **null out** DC.

ANS = 1 or 3

Which ones are zero at $\hat{\omega} = 0$?

Frequency Response

[1] $H(e^{j\hat{\omega}}) = 1 - e^{-j2\hat{\omega}}$

$H(e^{j0}) = 1 - 1 = 0$

[2] $H(e^{j\hat{\omega}}) = 2e^{-j2\hat{\omega}}\cos(\hat{\omega})$

$H(e^{j0}) = 2$

[3] $H(e^{j\hat{\omega}}) = 2je^{-j2\hat{\omega}}\sin(\hat{\omega})$

$H(e^{j0}) = 2j(0) = 0$

[4] $H(e^{j\hat{\omega}}) = \frac{\sin(\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})} e^{-j\hat{\omega}/2}$

$H(e^{j0}) = 2$

[5] $H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}$

$H(e^{j0}) = 1$

[6] $H(e^{j\hat{\omega}}) = \frac{\sin(2\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})} e^{-j3\hat{\omega}/2}$

$H(e^{j0}) = 4$

[7] $H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}(1 + 2\cos(\hat{\omega}))$

$H(e^{j0}) = 1 + 2 = 3$

[8] None of the above

[1] There might be several systems that null the sinusoid, but finding one is sufficient.

PROBLEM sp-08-Q.2.4:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems, i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

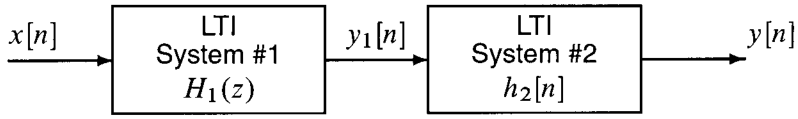


Figure 1: Cascade connection of two LTI systems.

(a) Suppose that System #1 is an FIR filter described by the system function:

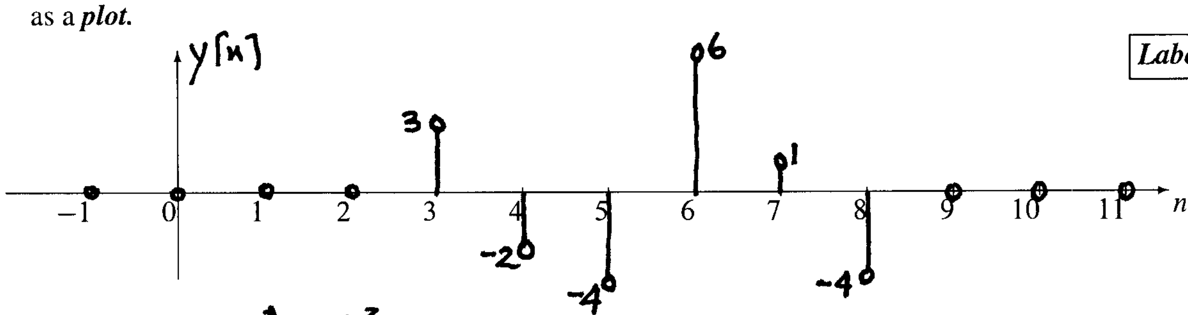
$$H_1(z) = 3z^{-2} - 2z^{-3} - z^{-4} + 4z^{-5}$$

and System #2 is described by the impulse response

$$h_2[n] = \delta[n-1] - \delta[n-3]$$

If the input signal is an impulse, i.e., $x[n] = \delta[n]$, determine the output signal, $y[n]$. Give your answer as a ***plot.***

Label Carefully



$H_2(z) = z^{-1} - z^{-3}$

$H_1(z)H_2(z) = (3z^{-2} - 2z^{-3} - z^{-4} + 4z^{-5})(z^{-1} - z^{-3})$

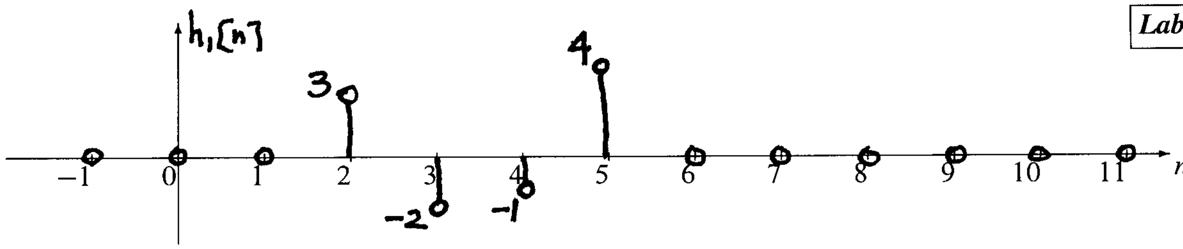
$= 3z^{-3} - 2z^{-4} - 4z^{-5} + 6z^{-6} + z^{-7} - 4z^{-8}$

$y[n] = 3\delta[n-3] - 2\delta[n-4] - 4\delta[n-5] + 6\delta[n-6] + \delta[n-7] - 4\delta[n-8]$

$Y(z) = H_1(z)H_2(z)X(z)$ and $X(z) = 1$ when $x[n] = \delta[n]$.

(b) Determine the impulse response sequence, $h_1[n]$, of the first system. Give your answer as a ***plot.***

Label Carefully



The coefficients of $H_1(z)$ are the $h[n]$ values.

$h_1[n] = 3\delta[n-2] - 2\delta[n-3] - \delta[n-4] + 4\delta[n-5]$