ECE-2025 Spring-2011

Lecture 22
Amplitude Modulation (AM)
18-Apr-11

Info: Lab & HW

- Lab #12 starts on Wed, 20-Apr; ends 26-Apr
  - LAST Lab, In Lab again
- HW #10 this week
- HW #11 on the last day of classes.
  - One HW will be dropped
- **Quiz #4 will be 22-Apr (Friday)**
  - **Covers HW #9 and #10**
  - **Labs #9, #10 and #11**
  - Bring your own FT Tables **HANDWRITTEN**

Pop Quiz

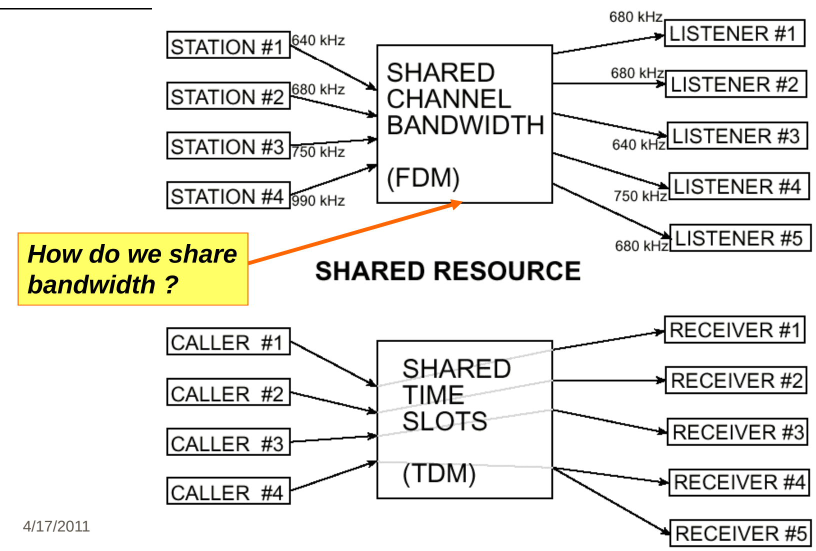
$$e^{-2t}u(t) * \delta(t+1) = ?$$

$$\left(e^{-2t}u(t)\right)\delta(t+1) = ?$$

$$\int_{-\infty}^{\infty}\left(e^{-2t}u(t)\right)\delta(t-1)dt = ?$$

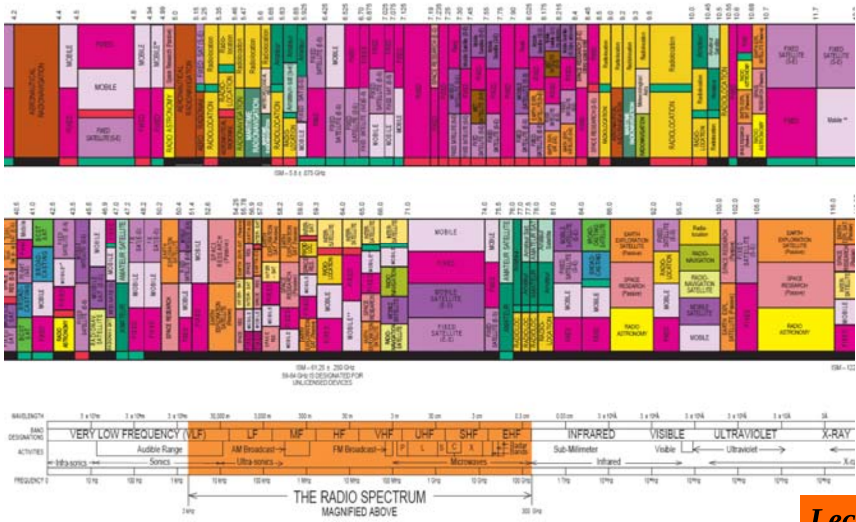
Lecture

The way communication systems work



Frequency Allocation (Govt)

http://www.ntia.doc.gov/osmhome/allochrt.pdf



Lecture

LECTURE OBJECTIVES

- Review of FT properties
  - Convolution <--> multiplication
  - Frequency shifting
- Sinewave Amplitude Modulation
  - AM radio
- Frequency-division multiplexing
  - FDM
- Reading: Chapter 12, Section 12-2

Table of Easy FT Properties

Linearity Property

$$a x_1(t) + b x_2(t) \Leftrightarrow a X_1(j\omega) + b X_2(j\omega)$$

Delay Property

$$x(t - t_d) \Leftrightarrow e^{-j\omega t_d} X(j\omega)$$

Frequency Shifting

$$x(t) e^{j\omega_0 t} \Leftrightarrow X(j(\omega - \omega_0))$$

Scaling

$$x(at) \Leftrightarrow \frac{1}{|a|} X(j\tfrac{\omega}{a})$$

Significant FT Properties

$$x(t) * h(t) \Leftrightarrow X(j\omega) H(j\omega)$$

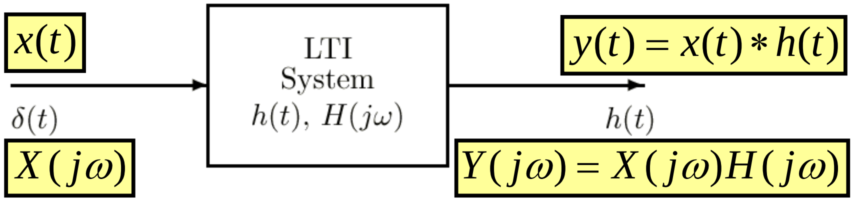
$$x(t) p(t) \Leftrightarrow \frac{1}{2\pi} X(j\omega) * P(j\omega)$$

$$x(t) e^{j\omega_0 t} \Leftrightarrow X(j(\omega - \omega_0))$$

Differentiation Property

$$\frac{d x(t)}{dt} \Leftrightarrow (j\omega) X(j\omega)$$

Convolution Property



- Convolution in the time-domain

$$y(t)=\int_{-\infty}^{\infty}x(\tau)h(t-\tau)d\tau = x(t)*h(t)$$

corresponds to **MULTIPLICATION** in the frequency-domain $Y(j\omega)=X(j\omega)H(j\omega)$

Delta in Frequency as Input to LTI System

$$X(j\omega)=A\delta(\omega-\omega_0)$$

$$Y(j\omega)=H(j\omega)X(j\omega)=H(j\omega)A\delta(\omega-\omega_0)$$

$$Y(j\omega)=AH(j\omega_0)\delta(\omega-\omega_0)$$

$$y(t)=AH(j\omega_0)(\tfrac{1}{2\pi})e^{j\omega_0 t}$$

Convolution or Multiplication

$$y(t)=\cos(at)*e^{-at}u(t)=?$$

$$\cos(at)\leftrightarrow \pi\delta(\omega+a)+\pi\delta(\omega-a)$$

$$e^{-at}u(t)\leftrightarrow \frac{1}{a+j\omega}$$

$$Y(j\omega)=\pi\delta(\omega+a)\frac{1}{a+j\omega}+\pi\delta(\omega-a)\frac{1}{a+j\omega}$$

$$Y(j\omega)=\pi\delta(\omega+a)\frac{1}{a-ja}+\pi\delta(\omega-a)\frac{1}{a+ja}$$

Multiply Transforms

Convolution or Multiplication (2)

$$y(t)=\cos(at)*e^{-at}u(t)=?$$

$$Y(j\omega)=\pi\delta(\omega+a)\frac{1}{a-ja}+\pi\delta(\omega-a)\frac{1}{a+ja}$$

$$Y(j\omega)=\pi\delta(\omega+a)\tfrac{1}{a\sqrt{2}}e^{j\pi/4}+\pi\delta(\omega-a)\tfrac{1}{a\sqrt{2}}e^{-j\pi/4}$$

$$\Rightarrow y(t)=\tfrac{1}{a\sqrt{2}}e^{j\pi/4}\left(\tfrac{\pi}{2\pi}e^{-jat}\right)+\tfrac{1}{a\sqrt{2}}e^{-j\pi/4}\left(\tfrac{1}{2}e^{jat}\right)$$

$$\Rightarrow y(t)=\tfrac{1}{a\sqrt{2}}\cos(at-\pi/4)$$

Multiply "Mags" Add Phases

Frequency Shifting Property

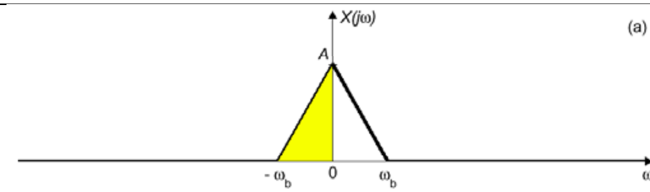
$$x(t)e^{j\omega_0 t} \Leftrightarrow X(j(\omega-\omega_0))$$

$$\int_{-\infty}^{\infty} e^{j\omega_0 t} x(t) e^{-j\omega t} dt = \int_{-\infty}^{\infty} x(t) e^{-j(\omega-\omega_0)t} dt$$

$$= X(j(\omega-\omega_0))$$

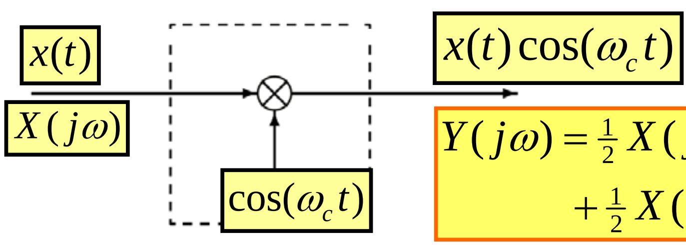
$$y(t) = \frac{\sin(7t)}{\pi t} e^{j\omega_0 t} \Leftrightarrow Y(j\omega) = \begin{cases} 1 & \omega_0 - 7 < \omega < \omega_0 + 7 \\ 0 & \text{elsewhere} \end{cases}$$

Communication at Baseband



- Why would we need to shift away from baseband to a higher frequency?
- Inefficient ($$)!
  - Huge antennas
  - Poor propagation
  - Only one transmitter at a time

Amplitude Modulator (AM)

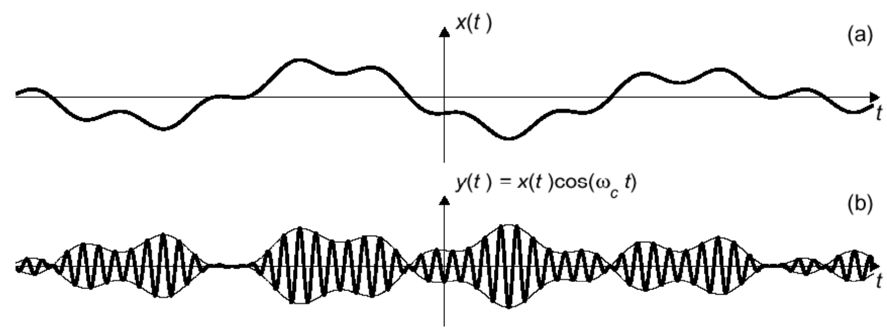


$$Y(j\omega) = \tfrac{1}{2} X(j(\omega-\omega_c)) + \tfrac{1}{2} X(j(\omega+\omega_c))$$

- *x(t)* modulates the amplitude of the cosine wave. The result in the frequency-domain is two shifted copies of *X(jω).*

Typical AM Waveform

- In the time-domain, the "envelope" of sine-wave peaks follows |*x(t)*|

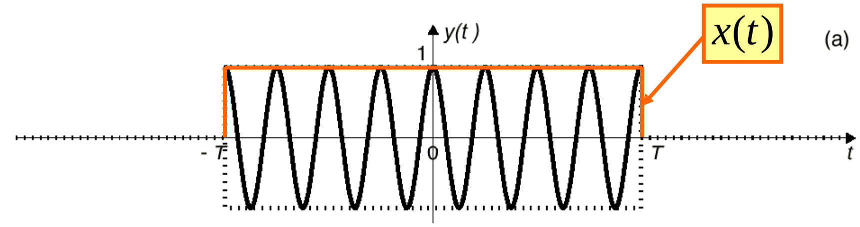


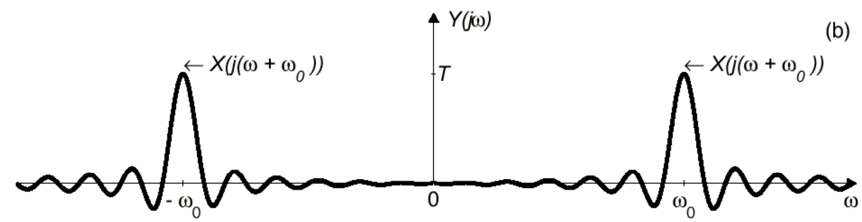
$$y(t) = x(t)\cos(\omega_c t) \Leftrightarrow$$
$$Y(j\omega) = \tfrac{1}{2}X(j(\omega-\omega_c)) + \tfrac{1}{2}X(j(\omega+\omega_c))$$

$$x(t) = \begin{cases} 1 & |t| < T \\ 0 & |t| > T \end{cases} \quad \Leftrightarrow \quad X(j\omega) = 2\frac{\sin(\omega T)}{(\omega)}$$

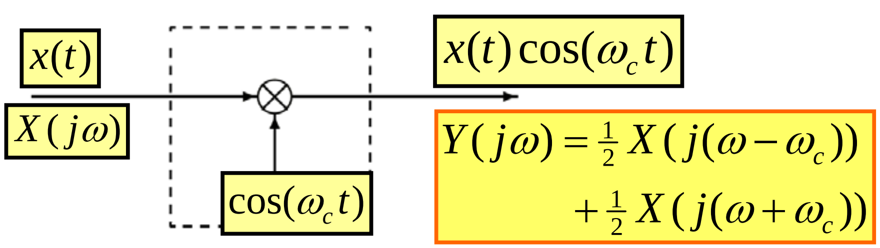
$$y(t) = x(t)\cos(\omega_c t) \Leftrightarrow$$
$$Y(j\omega) = \frac{\sin((\omega-\omega_c)T)}{(\omega-\omega_c)} + \frac{\sin((\omega+\omega_c)T)}{(\omega+\omega_c)}$$

$$y(t) = x(t)\cos(\omega_0 t) \Leftrightarrow$$
$$Y(j\omega) = \tfrac{1}{2}X(j(\omega-\omega_0)) + \tfrac{1}{2}X(j(\omega+\omega_0))$$



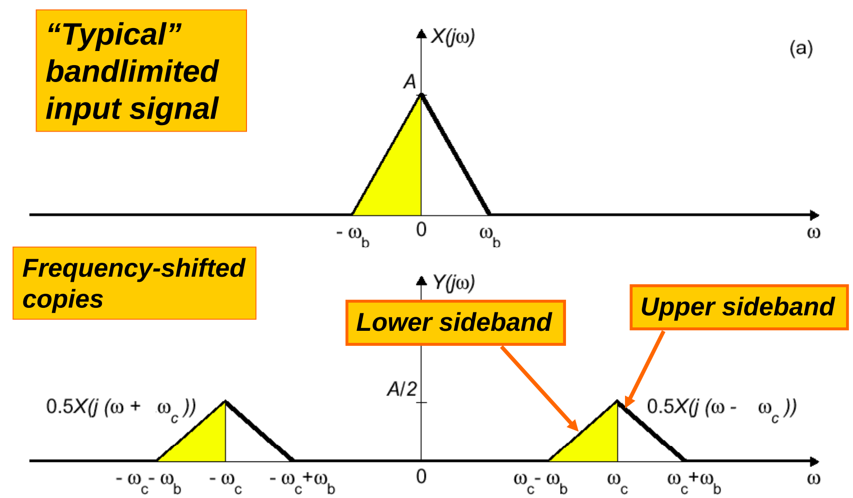


DSBAM Modulator

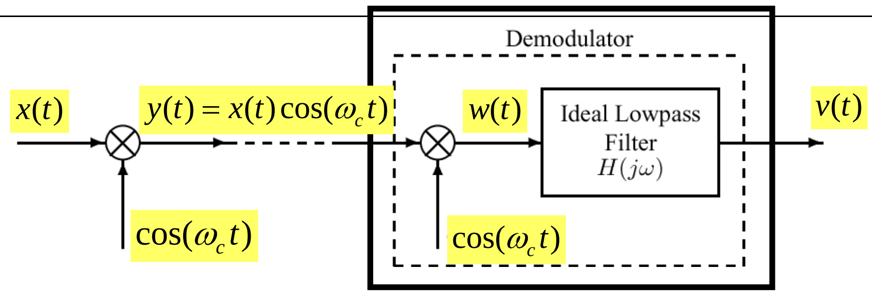


- If $X(j\omega)=0$ for $|\omega|>\omega_b$ and $\omega_c>\omega_b$, the result in the frequency-domain is two shifted and scaled **exact copies** of $X(j\omega)$.

Double Sideband AM (DSBAM)



DSBAM DEmodulator

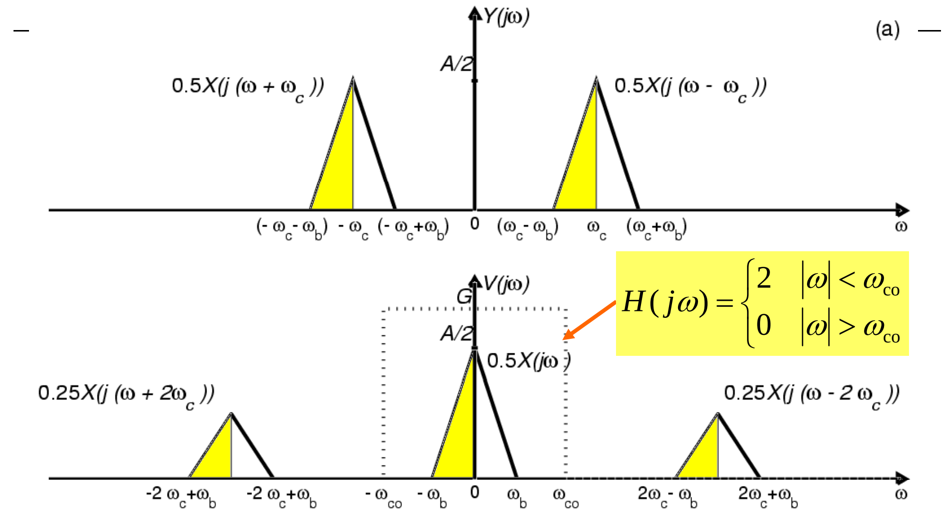


$$w(t) = x(t)[\cos(\omega_c t)]^2 = \tfrac{1}{2}x(t) + \tfrac{1}{2}x(t)\cos(2\omega_c t)$$

$$W(j\omega) = \tfrac{1}{2}X(j\omega) + \tfrac{1}{4}X(j(\omega - 2\omega_c)) + \tfrac{1}{4}X(j(\omega + 2\omega_c))$$

$$V(j\omega) = H(j\omega)W(j\omega)$$

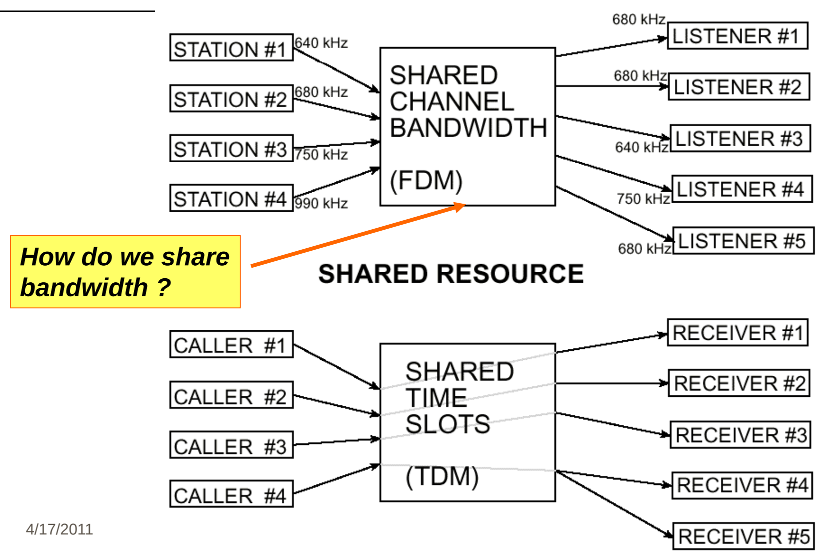
DSBAM Demodulation



$$H(j\omega) = \begin{cases} 2 & |\omega| < \omega_{co} \\ 0 & |\omega| > \omega_{co} \end{cases}$$

$$V(j\omega) = H(j\omega)W(j\omega) = X(j\omega) \text{ if } \omega_b < \omega_{co} < 2\omega_c - \omega_b$$

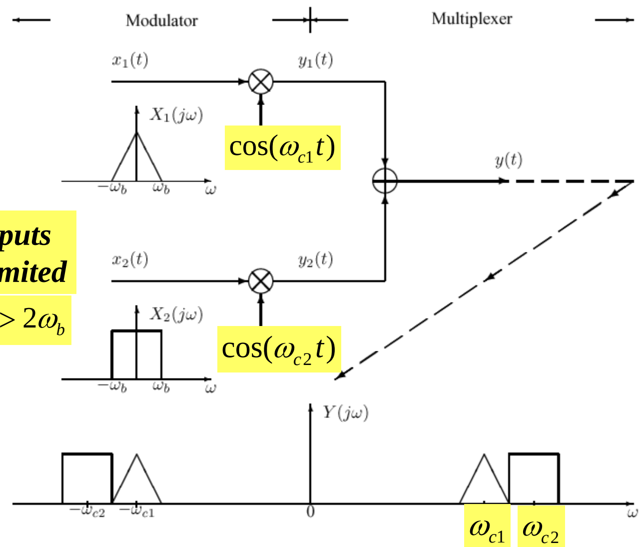
The way communication systems work



Frequency-Division Multiplexing (FDM)

- Shifting spectrum of signal to higher frequency:
  - Permits transmission of low-frequency signals with high-frequency EM waves, which can use smaller antennas more efficiently
  - By allocating a frequency band to each signal, multiple bandlimited signals can share the same spectrum region.
  - AM radio: 530-1620 kHz (10 kHz bands)
  - FM radio: 88.1-107.9 MHz (200 kHz bands)

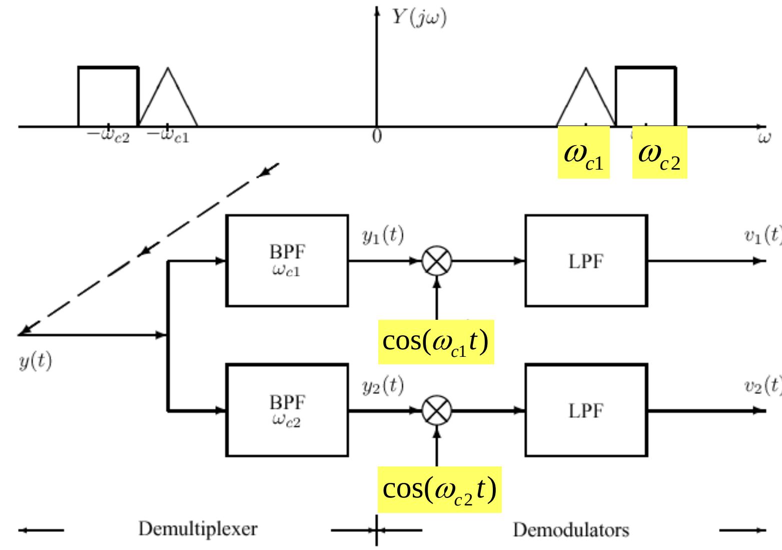
FDM Block Diagram (Xmitter)



Spectrum of inputs must be bandlimited

Need $|\omega_{c2}-\omega_{c1}|>2\omega_b$

Frequency-Division De-Mux



Bandpass Filters for De-Mux