

**Problem 1.1:**

(a) A negative real number has a phase (or angle) of  $\pi$ .

(b, d, f) Convert to polar first, and then raise to the power by multiplying the angle and raising the angle to the power.

$$(b) -33 - j33 = 33\sqrt{2}e^{-j5\pi/4} \Rightarrow (33\sqrt{2}e^{-j5\pi/4})^2 = 33^2(2)e^{-j5\pi/2} = 2178e^{j\pi/2}$$

$$(d) (-j8)^3 = (8e^{-j\pi/2})^3 = 8^3 e^{-j3\pi/2} = 512e^{j\pi/2}$$

(f) Use the same strategy.

MATLAB verification is given below:

```
%- Problem 1.1
disp('Problem 1.1')
zprint([-2*pi, (-33-33i).^2, 1-j*sqrt(3), (-8i).^3, 3-4i, (-3+4i).^7])
```

Problem 1.1

Z =	X	+ jY	Magnitude	Phase	Ph/pi	Ph(deg)
	-6.283	0	6.283	3.142	1.000	180.00
	0	2178	2178	1.571	0.500	90.00
	1	-1.732	2	-1.047	-0.333	-60.00
	0	512	512	1.571	0.500	90.00
	3	-4	5	-0.927	-0.295	-53.13
-7.644e+004	1.612e+004	7.813e+004	2.934	0.934	168.09	

**Problem 1.2:**

(a) The angle  $-5\pi/6$  lies in the 3<sup>rd</sup> quadrant, so both the real and imaginary parts will be negative.

(b) The sum in the exponent can be expanded to a product of exponentials:  $e^\pi e^{-j5\pi/4}$

(c)  $71\pi/2 = 2\pi(36) - 0.5\pi$ , so the angle is  $-0.5\pi$ , which means the number has a real part equal to zero (i.e., is purely imaginary), and its imaginary part is negative.

(c)  $-41\pi = 2\pi(-20) - \pi$ , so the angle is  $-\pi$ , which means the number is a negative real number with no imaginary part.

```
%- Problem 1.2
disp('Problem 1.2')
zprint([8*exp(-5i*pi/6), exp(pi-5i*pi/4), pi*exp(71i*pi/2), (pi^exp(1))*exp(-41i*pi)])
```

Problem 1.2

Z =	X	+ jY	Magnitude	Phase	Ph/pi	Ph(deg)
	-6.928	-4	8	-2.618	-0.833	-150.00
	-16.36	16.36	23.14	2.356	0.750	135.00
3.084e-015	-3.142		3.142	-1.571	-0.500	-90.00
-22.46	-3.521e-013		22.46	-3.142	-1.000	-180.00

### **Problem 1.3:**

(a) Conjugate means "change the sign of all j's", thus we get  $-1 + j$  which lies in the 2<sup>nd</sup> quadrant.

(b) Since  $j = e^{j\pi/2}$ , multiplying by  $j$  will add  $\pi/2$  to the phase, so the new phase is  $\pi/2 - 17\pi/6 = -14\pi/6 = -7\pi/3 = -2\pi - \pi/3$

(c) Division is easiest in polar form, so convert  $z_1 = -1 - j = \sqrt{2} e^{-j3\pi/4}$  and then divide the magnitudes and subtract the phases. The magnitude will be  $\sqrt{3/2}$ , and the phase will be  $-17\pi/6 - (-3\pi/4) = -34\pi/12 + 9\pi/12 = -25\pi/12 = -2\pi - \pi/12$

(d) Squaring is easiest in polar form: square the magnitude and double the phase.

(e) Do the inverse in polar form: negate the phase and invert the magnitude.

(f) Multiply in polar form: multiply the magnitudes and add the phases.

(g) Addition is easiest in rectangular form:  $z_2 = \sqrt{3} e^{-j17\pi/6} = -1.5 - j0.866$

(h) Magnitude squared is just that:  $|z_2|^2 = (\sqrt{3})^2 = 3$

(i) A number plus its conjugate equals twice the real part;  $z_2 + z_2^* = 2\Re(z_2) = -3$

**%- Problem 1.3**

```
disp('Problem 1.3')
```

```
z1 = -1-j;
```

```
z2 = sqrt(3)*exp(-17i*pi/6);
```

```
zprint([conj(z1), j*z2, z2/z1, z2*z2, 1/z1, z1*z2, z1+conj(z2), abs(z2).^2  
2*real(z2)])
```

Problem 1.3

Z =	X	+	jY	Magnitude	Phase	Ph/pi	Ph(deg)
	-1		1	1.414	2.356	0.750	135.00
	0.866		-1.5	1.732	-1.047	-0.333	-60.00
	1.183		-0.317	1.225	-0.262	-0.083	-15.00
	1.5		2.598	3	1.047	0.333	60.00
	-0.5		0.5	0.7071	2.356	0.750	135.00
	0.634		2.366	2.449	1.309	0.417	75.00
	-2.5		-0.134	2.504	-3.088	-0.983	-176.93
	3		0	3	0.000	0.000	0.00
	-3		0	3	3.142	1.000	180.00

### **Problem 1.4:**

The MATLAB code at the bottom makes all the plots. It would be best if students sketched the plots because that would be good practice for quizzes.

(a) **Concepts:** Sinusoid, Period, Frequency, Phase

**Approach-1:** One way is to get a formula and make the plot with MATLAB. In that case, the period must be used to calculate the frequency. Then you have  $A$ ,  $\omega$ , and  $\varphi$ , so you can write  $A \cos(\omega t + \varphi)$ .

**Approach-2:** The second approach is better for sketching. Use the period and phase to calculate the time of a positive maximum. This requires a slight modification of the formula for  $t_m = -\varphi / \omega = -\varphi T / 2\pi$ .

Since this time is negative, add  $T$  to get a positive time for a peak. The sketch the sine wave can be made by putting a positive peak at the desired location, and labeling the  $x$ -axis so that the distance to the next positive peak is  $T$ . Label the vertical  $y$ -axis to have the correct amplitude.

**Answer:** The phase is 0.7854 rads which is very close to  $0.7854 = \pi / 4$  radians, so the the time location of a positive peak is  $t_m = -\varphi / \omega = -\varphi T / 2\pi = -(\pi / 4)(500\pi) / (2\pi) = -62.5\pi \approx -196.4$  secs. The period is about  $T = 500\pi \approx 1570.8$  secs. The MATLAB plot starts at  $t = 0$ , so the first positive peak is at  $t_m = T - 196.4 = 1374.4$  s. It's hard to see these exact values on the MATLAB plot below, but they seem to be correct. The length of two periods is 3141.6 s.

If the direct MATLAB evaluation approach is used, then the frequency is

$$\omega = 2\pi / T = 2\pi / 500\pi = 0.004 \text{ rad/s, and the sinusoidal formula is } 3 \cos(0.004t + \pi / 4)$$

(b) **Concepts:** Sinusoid, Period, Frequency, Phase

**Approach-1:** The mathematical formula is given for the sinusoid, so that formula could be programmed directly into MATLAB—this is the first approach.

**Approach-2:** A second approach is to make a sketch by hand. In this case, the period must be found from the frequency,  $T = 2\pi / \omega$ , and the time location of a positive peak must be found from the phase and the frequency, i.e.,  $t_m = -\varphi / \omega$ .

**Answer:** The phase is  $-\pi / 4$  radians, so the time location of a positive peak is

$$t_m = -\varphi / \omega = -(\pi / 4) / (7\pi / 50) = 1.786 \text{ secs. The period is } T = 2\pi / \omega = 2\pi / (7\pi / 50) \approx 14.286 \text{ secs.}$$

It's hard to see these exact values on the MATLAB plot below, but the plot seems to be correct. If the direct MATLAB evaluation approach is used, then the sinusoidal formula is used as given. The range of times to give two periods is  $2T = 28.572$  s.

(c) **Concepts:** Sinusoid, Period, Frequency, Phase, Time-shift

**Approach-1:** The mathematical formula is given for the sinusoid, so that formula could be programmed directly into MATLAB—this is the first approach.

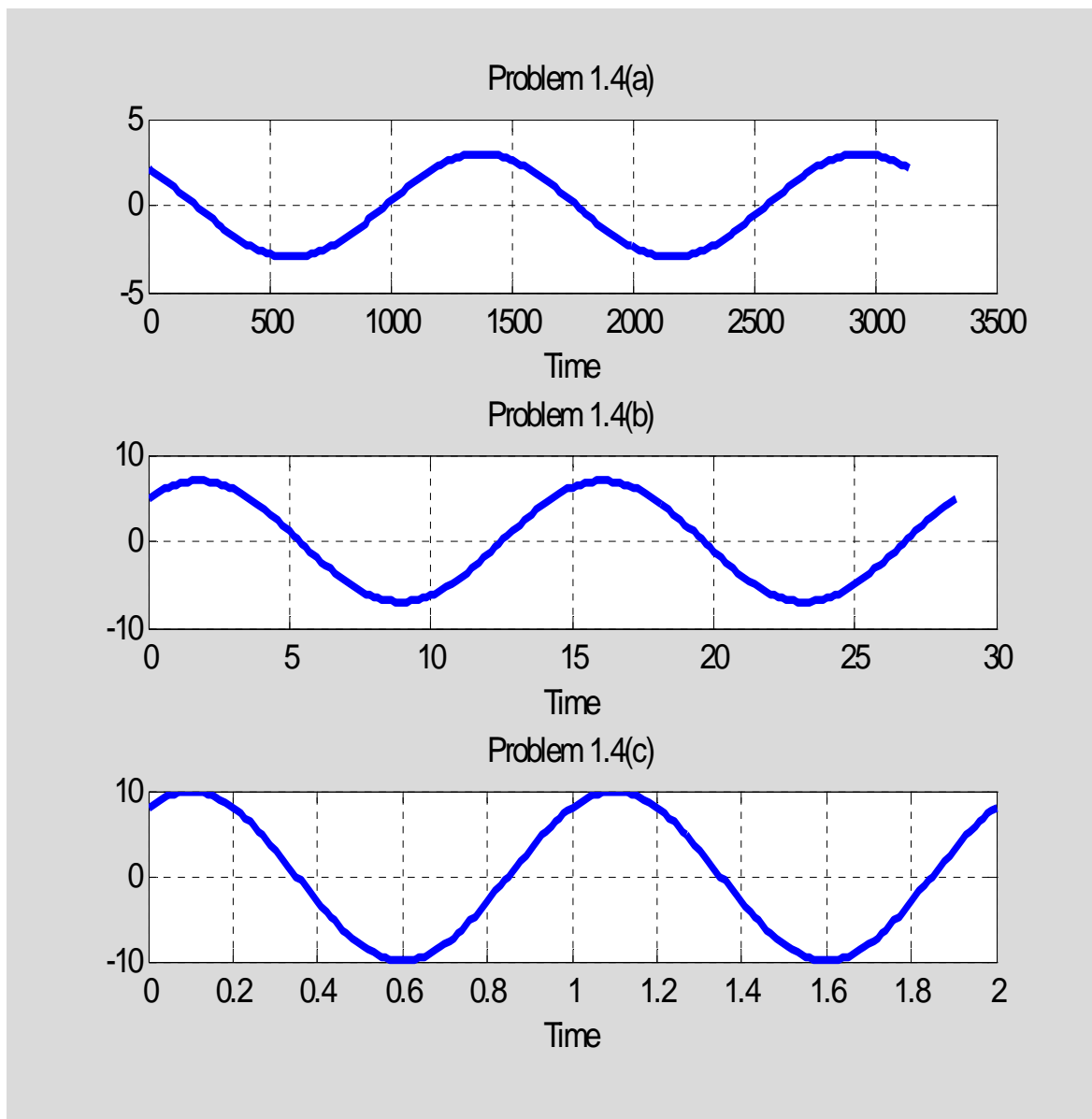
**Approach-2:** The second approach is to make a sketch by hand. In this case, the period must be found from the frequency; the time location of a positive peak is already given in the formula because the argument of the cosine has a term,  $(t - 0.1)$ , so a positive peak will occur at  $t = 0.1$ .

**Answer:** The time shift is  $t_m = 0.1$  s, so that is the time location of a positive peak. The period is  $T = 2\pi / \omega = 2\pi / (2\pi / 1) = 1$  s. It's easy to verify these exact values on the MATLAB plot below, so the plot is correct. If the direct MATLAB evaluation approach is used, then the sinusoidal formula is used as given. The range of times to give two periods is  $2T = 2$  s.

```

%- Problem 1.4
T = 500*pi; A = 3; phi = 0.7854;
tt = 0: T/100: 2*T;
subplot(3,1,1)
plot(tt, A*cos(2*pi/T*tt + phi),'LineW',2.5), grid on
title('Problem 1.4(a)'), xlabel('Time')
%
ww = 7*pi/50; A = log(exp(7)); phi = -pi/4;
T = 2*pi/ww;
tt = 0: T/100: 2*T;
subplot(3,1,2)
plot(tt, A*cos(2*pi/T*tt + phi),'LineW',2.5), grid on
title('Problem 1.4(b)'), xlabel('Time')
%
ww = 2*pi; A = pi*pi; tm = 0.1;;
T = 2*pi/ww;
tt = 0: T/100: 2*T;
subplot(3,1,3)
plot(tt, A*cos(2*pi/T*(tt-tm)),'LineW',2.5), grid on
title('Problem 1.4(c)'), xlabel('Time')

```



### **Problem 1.5:**

(a)

**Approach-1:** Express the numerator as  $pe^{j\pi/2}$  and the denominator as  $de^{j\theta}$ . This requires that we assume  $p$  is positive. Then compare angles, i.e., the angle of the numerator minus the angle of the denominator equals the angle of the righthand side (RHS), which leads to  $\theta = \pi/3$  as the angle of the denominator. Next, work with the denominator which has an imaginary part equal to 36, and find the real part of the denominator to get  $q$ . Finally, compare magnitudes, i.e., the magnitude of the numerator ( $p$ ) divided by the magnitude of the denominator ( $d = 36/\sin \theta$ ) equals the magnitude of the righthand side (RHS) which is 0.1.

NOTE: if you start out with  $p$  being negative, there is no solution.

**Approach-2:** Cross multiply by the denominator of the left-half side. Expand the RHS in rectangular form. Then equate the real and imaginary parts. The result is two linear equations in the two unknowns.

**Answer:** The results are:  $p = 2.4\sqrt{3}$ ,  $q = 12\sqrt{3}$

(b) **Approach:** Compare magnitudes: 5 times the magnitude of  $(3 + j\beta)$  must equal 25, so  $\beta = \pm 4$  to get  $|3 + j\beta| = 5$ . Then compare angles:  $\alpha$  plus the angle of  $(3 \pm j4)$  must equal  $\pi$ . Work out the two cases.

**Answer:** The results are:  $\{\alpha, \beta\} = \{2.214, 4\}$ , or  $\{\alpha, \beta\} = \{4.069, -4\}$ , or  $\{\alpha, \beta\} = \{-2.214, -4\}$  because  $\alpha$  is angle which can be changed by integer multiples of  $2\pi$  radians.

(c) When  $\{\alpha, \beta\} = \{2.214, 4\}$ , the two vectors are  $5e^{j2.214}$ , and  $3 + j4 = 5e^{j0.927}$ .

When  $\{\alpha, \beta\} = \{4.069, -4\}$  the two vectors are  $5e^{j4.069} = 5e^{-j2.214}$ , and  $3 - j4 = 5e^{-j0.927}$ .

(d) The magnitudes are equal because the magnitude of the RHS is 25, and the magnitude of the first term is 5, and the product of the magnitudes on the LHS must also equal 25.

### **Problem 1.6:**

**Approach:** The general procedure for getting a formula for the sinusoid from a plot is to (1) measure the period (between two successive positive peaks) and calculate the frequency from the period. Then (2) measure the time location of a positive maximum, and calculate the phase from this time and the frequency. Finally, (3) measure the height of a positive maximum to get the amplitude.

**Answer:** From the figure, we can measure the Period ( $T$ ), amplitude ( $A$ ) and time of a positive peak ( $t_m$ ). The (approximate) values are:

$$A = 10$$

$$T = 0.01 \text{ s}$$

$$t_m \approx -0.004 \text{ s}$$

So, we convert to frequency and phase in the following way

$$f = 1/T = 1/0.01 = 100 \text{ Hz}$$

$$\omega = 2\pi f = 200\pi \text{ rad/s}$$

$$\varphi = -\omega t_m = -200\pi(-0.004) = 0.8\pi \text{ rads}$$

NOTE: The true value of the phase is actually  $\varphi = 3\pi/4$ , which is equivalent to a time shift of  $t_m = -\varphi/\omega = -(3\pi/4)/200\pi = -0.00375$  s. However, it is very hard to read this precise value from the graph.