GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2011
Problem Set #2

Assigned: 21-Jan-11
Due Date: Week of 31-Jan-11

Quiz #1 will be held in lecture on Friday 4-Feb-2011. It will cover material from Chapter 2, as represented in Problem Sets #1 and #2.
Closed book, calculators permitted, and one hand-written formula sheet ($8\frac{1}{2}'' \times 11''$, both sides)

Reading: In *SP-First*, all of Ch. 2, and start reading in Chapter 3: *Spectrum Representation,* Section 3-1.

The web site for the course uses **`t-square`**: `https://t-square.gatech.edu`
$\Longrightarrow$ Please check **`t-square`** daily. All official course announcements will be posted there.

ONLY the **STARRED** problems should be turned in for grading; a random subset of these will be graded.

Some of the problems have solutions that are similar to those found on the SP-First CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 2.1:

Each of the following signals may be simplified, and expressed as a sinusoid of the form: $A\cos(\omega t + \varphi)$, where φ **is the phase in radians**. For each signal, draw a vector diagram of the complex amplitudes (phasors), and use vector addition to estimate the amplitude A and phase(s) φ of the resultant sinusoid. Then use the phasor addition theorem (and a calculator) to find the exact numerical values for A and φ.

(a) $x_1(t) = 3\cos(999t - 0.2\pi) + 5\cos(999t + 0.6\pi)$

(b) $x_2(t) = 7\cos(\pi t - \pi/8) + 7\cos(\pi t - 5\pi/8) + 8\cos(\pi t - 9\pi/8) + 8\cos(\pi t + 3\pi/8)$

(c) $x_3(t) = \sqrt{2}\cos(6\pi t - 45) - \sqrt{3}\cos(6\pi t - 60) + \sqrt{5}\cos(6\pi t - 90)$

PROBLEM 2.2:

Complex exponentials obey the expected rules of calculus when doing integrals and derivatives, i.e., when c is a complex number

$$\frac{d}{dt}e^{ct} = ce^{ct} \qquad \text{and} \qquad \int e^{ct}\,dt = \frac{1}{c}e^{ct}$$

(a) Evaluate the definite integral, and express the answer in polar form: $\int_0^{0.5} e^{j\pi t}\,dt = re^{j\theta}$?

(b) Recall that the magnitude magnitude squared $|z|^2$ of a complex number z is equal to $(z^*)z$, where z^* is the conjugate of z. When $z(t) = (1-j)e^{j\pi t}$ evaluate the following definite integral: $\int_0^1 z^*(t)z(t)\,dt$

PROBLEM 2.3*:
Define $x(t)$ as

$$x(t) = 7\cos(33\pi t + 1.1\pi) + 5\cos(33\pi t + 0.4\pi)$$

(a) Find a complex-valued signal $z(t)$ such that $x(t) = \Re\{z(t)\}$. Simplify $z(t)$ as much as possible, so that you can identify its complex amplitude (in polar form).

(b) Find a complex-valued signal $z_1(t) = (Ae^{j\varphi})e^{j\omega t}$ such that $\Re\{\frac{d}{dt}z_1(t)\} = 7\cos(33\pi t + 1.1\pi)$.

(c) When $s(t) = 5\cos(33\pi t + 0.4\pi)$, find a complex-valued $z_s(t)$ such that $\Re\{z_s(t)\} = s(t - 0.01)$.

PROBLEM 2.4*:
The following identity is true:

$$\sum_{k=0}^{N-1} e^{j2\pi k/N} = 0$$

A generalized restatement of this identity is that the sum of N equal-length vectors whose angles differ by $2\pi/N$ radians is zero. For example, when $N = 5$ the angular separation has to be $2\pi/5$ rads. which is 72°.

(a) Simplify the expression: $\sum_{k=0}^{N-2} e^{j2\pi(k+\frac{1}{2})/N}$, which is the sum of $N-1$ complex exponentials.

(b) Plot the complex numbers $z_1 = e^{-j0.2\pi}$, $z_2 = e^{j3.4\pi}$, $z_3 = -1$, and $z_4 = e^{j6.6\pi}$ as vectors, and then *use a variation on the identities above* to obtain the sum $z_1 + z_2 + z_3 + z_4$. Explain your work.

(c) The MATLAB code below adds many sinusoids whose phases differ by $2\pi/N$.

```
tt = 0:1:1000;
xx = 0*tt;
for kk=5:11
   xx = xx + 99*cos(0.006*pi*tt + 0.25*pi*kk);
end
plot(tt,xx), title('SECTION of a SINUSOID'), xlabel('TIME (sec)')
```

The plot made from the vector `xx` is a single sinusoid, which can be written as $A\cos(\omega_0 t + \varphi)$. Use the identities above and analyze the code to determine the parameters for the sinusoid in the vector `xx`. Also, identify the value of N needed to apply the identity.

PROBLEM 2.5*:
Solve the following simultaneous equations by using complex amplitudes, i.e., phasors,

$$\begin{aligned}
1.1\cos(\omega_0 t + 2) &= A_1\cos(\omega_0 t + \varphi_1) + A_2\cos(\omega_0 t + \varphi_2 + \pi/2) + A_3\cos(\omega_0 t + \varphi_3 - \pi/2) \\
1.2\cos(\omega_0 t - 2) &= A_1\cos(\omega_0 t + \varphi_1) + A_2\cos(\omega_0 t + \varphi_2 - \pi/2) + A_3\cos(\omega_0 t + \varphi_3 + \pi/2) \\
1.3\cos(\omega_0 t) &= A_1\cos(\omega_0 t + \varphi_1) + A_2\cos(\omega_0 t + \varphi_2) + A_3\cos(\omega_0 t + \varphi_3)
\end{aligned}$$

where the amplitudes (A_k) are positive, and the phases (φ_k) lie between $\pm\pi$.

(a) Show how to convert the sinusoidal equations into simultaneous complex-number equations.

(b) Use MATLAB to solve the (complex-number) equations with the backslash operator, or with `inv`. Give your answers as real numbers for A_1, A_2, A_3, φ_1, φ_2, and φ_3.

(c) Draw a vector diagram for the addition of the three complex amplitudes corresponding to the three sinusoids. Then explain how this vector diagram verifies that you might have the correct answer. NOTE: the SP-First function `zcat` in MATLAB will draw head-to-tail vectors.